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Exploring Torricelli's theorem with Arduino

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Abstract. We explore the dynamics of the water in a PVC pipe during the drain using two independent sensors simultaneously. We track the height of the water column and the discharge velocity of the water through the hole made in the lower part of the pipe. The ultrasonic distance sensor and the flow meter used as sensors were controlled by an Arduíno board. The acquired data follows the theoretical models but, with a coefficient of discharge smaller than 1.

Keywords: Ardiuin, fluid dynamics , Torricelli's theorem.

1. Introdução

A very common question/problem presented in physics textbooks [1] related to fluid dynamics is: What is the speed of water flowing from a hole in the lower side of a vessel? Evangelista Torricelli (1608-1647) discovered the answer showing that the speed of the fluid that leaks through a small hole is proportional to the square root of the water column, and later, its equation was demonstrated as a particular case of Bernoulli´s principle. Mathematically, that problem can be solved easily using the Bernoulli´s and continuity equations, and the result is:

$$v_d = \frac{1}{\sqrt{1-\left(\frac{A_d}{A}\right)^2}}\sqrt{2gh} = f\sqrt{2gh} \qquad (1).$$

were A_d and A are the hole and the vessel cross-sectional areas, respectively, g is the gravitation acceleration, and h is the height of the water column. The term that multiplies the square root (written as f) is a factor that considers the velocity of descent of the fluid expressed in terms of the ratio areas. Considering that the vessel has a large border related to the hole size factor f can be considered equals to 1, and we can express the result observed by Torricelli:

$$v_d = \sqrt{2gh} \qquad (2).$$

However, in a realistic situation, we will not fit Equation (2) to the experimental data. The discharge velocity will be smaller than predicted. First because $f \neq 1$, and second, it occurs due to irrecoverable losses associated with the liquid flow, for example, the viscosity of the liquid, the contraction of streamlines close to the hole, any rotation and turbulence inside the pipe, the shape and position of the hole, etc. [2] Thus, we can consider a more general form of Torricelli's law:

$$v_d = C_d \cdot \sqrt{2gh} \qquad (3)$$

were C_d is the coefficient of discharge (the ratio of the measured discharge to the theoretical discharge), a number smaller than one. This coefficient includes all factors cited that are responsible for decreasing the real discharge velocity.

Another fact is that Equations (2) and (3), as presented, are valid only if the water level in the vessel is constant. If we are interested to observe the fluid drain out the tube and follow the dynamics of the system, with a little calculus we can derive, from Equation (3), the temporal equations for the height ($h(t)$) and discharge velocity ($v_d(t)$) [3-4]. The equations for height and discharge velocity as a function of the time, with the water column beginning at height h_0, are:

$$h(t) = \left(-C_d\left(\frac{A_d}{A}\right)\sqrt{\frac{g}{2}}t + \sqrt{h_0}\right)^2, \qquad (4)$$

$$v_d(t) = -{C_d}^2\left(\frac{A_d}{A}\right)g\,t + C_d\sqrt{2gh_0}, \qquad (5)$$

Many authors present different ways to show the time evolution of Torricelli's law measuring only the height of the water column [3, 5], the discharge velocity [6], or measuring the time to drain the vessel [4]. In this work, we use an Arduino connected to two sensors: one to measure the water column height, another the discharge velocity. Measuring these two parameters independently, we fitted Torricelli's law equation, considering the correction due to the system not being ideal.

2. Materiais e Métodos

The apparatus consists of a 1.5 m long, and 100 mm in diameter, PVC tube with a cap in the lower extremity (Figure 1a). At the lower end, in the lateral of the tube, we drilled a hole. In the hole, we attached a valve followed by a flowmeter (YF-S201 Hall Effect Water Flow Sensor). The end drain pipe is 10 mm in diameter. At the top end of the tube, we fixed an ultrasonic sensor (HC-SR04). $f = 1.005$. The details of the flowmeter and the ultrasonic sensor are presented in Figures 1c and 1d, respectively.

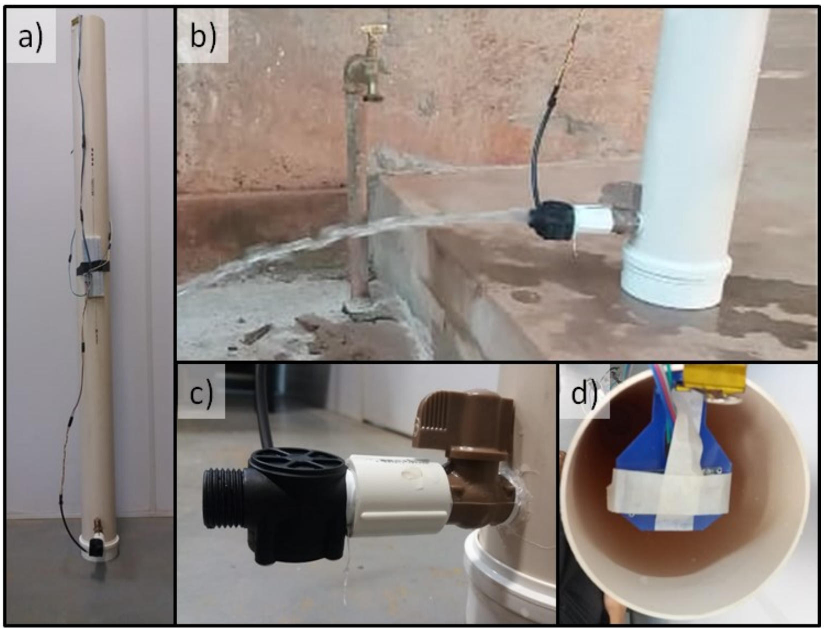

Figure 1. a) Picture of the tube used to realize the experiment. b) A picture showing the moment that the valve was opened, and the water starts to discharge. c) and d) Close-up in both extremities showing the flowmeter and the ultrasonic sensors.

The YF-S201 has three wires that were connected then to the Arduino board: red (5 VDC power) that was connected to the VCC pin, black that was connected to GND, and yellow to the digital pin D2. The yellow wire is the Hall effect pulse output, by counting the pulses, it is possible to measure the water flow. The HC-SR04 ultrasonic sensor has four pins: VCC, TRIG, ECHO, and GND. They were connected, by wires, to the Arduino board pins: VCC, D8, D5, and GND, respectively. Using the ultrasonic.h library, we got the information of the distance between the sensor (on top of the tube) and the water surface. Figure 2 shows an electronic diagram of the circuit used in the experimental apparatus.

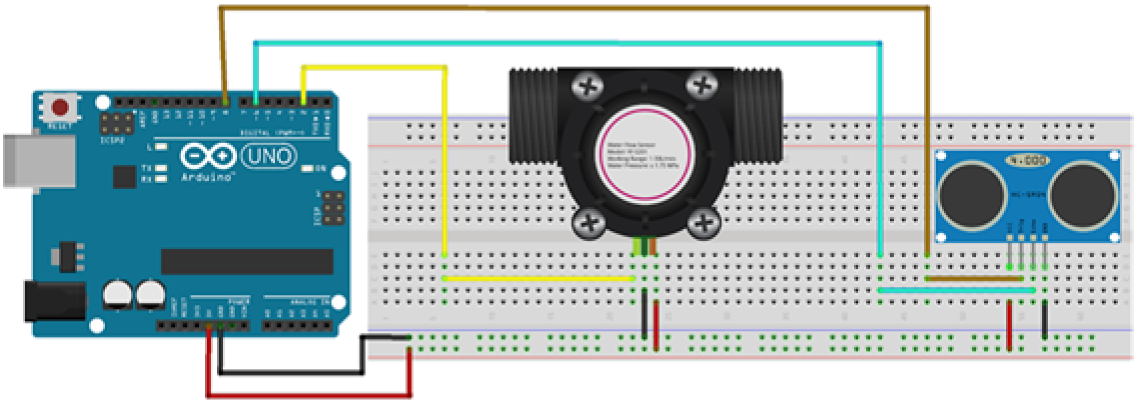

Figure 2. Sketch of the circuit with Arduino [7].

We wrote source code in the Arduino IDE, version 1.8.42.0. In the resume, it collects the data from both sensors and sends it to the Serial Monitor (Figure 3). The Arduino converts the frequency count (f) to flow rate (Q) employing the calibration equation:

$$Q = A \cdot f + B (6)$$

Instead of using the datasheet calibration, we decided to make it ourselves. We attached the flow meter sensor to a faucet and opened it at different flow rates. With a graduated bucket, we measure the time necessary to fill it with 2 liters of water and the average frequency (pulse counts per second) given by the sensor. By a linear regression of the data (Figure 4), we found $A = 2.36 \times 10^{-6}\, m^3$ and $B = 3.15 \times 10^{-6}\, m^3/s$ on Equation (6).

```
#include <HCSR04.h>
HCSR04 hc(8, 6); //Initialisation of HCSR04 sensor
float FlowRate;
int PulseCount, Time = 0;
void setup() {
  Serial.begin(9600);
  pinMode(2, INPUT); //Set the pin as an input
  attachInterrupt(0, Flow, RISING); //Configures interrupt 0 to run the function "Flow“
  Serial.println("time (s)\tdistance (m)\tFlow (10^{-6}m^3/s)"); //Print the headers interrupts
}
void loop () {
  PulseCount = 0;   //Turn the variable equals to zero
  sei(); //Enables interrupts on the Arduino
  delay (1000); //Wait 1 second
  cli(); //Disable the interrupts on the Arduino
  FlowRate = PulseCount * 2.36 + 3.15; //Convert pulse count in flow rate
  //For sensor callibration uses: FlowRate = PulseCount;
  Serial.print(Time); //Print time
  Serial.print("\t");
  Serial.print(hc.dist() * 0.01, 2); //Print distance
  Serial.print("\t");
  Serial.println(FlowRate); //Print flow rate
  Time++; //Increment "Time" by 1
}
void Flow () {
  PulseCount++; //Increment "PulseCount" by 1
}
```

Figure 3. Arduino source code.

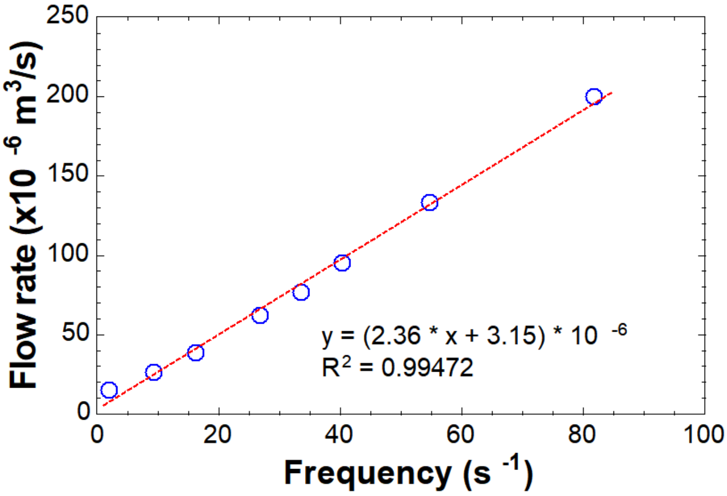

Figure 4. Graph of flow-meter calibration.

3. Resultados e discussões

We filled the PVC pipe until the height reached close to the open end. With the pipe in a vertical position, we started to collect the data and turned on the valve to begin the water discharge (Figure 1b). We collected the data until the water efflux stops. The distance data increases with time and stops to increase when the tube is empty. We subtract from position data the final value of distance to obtain the initial height $h_0 = 1.17\,m$. The discharge velocity is calculated by taking the flow rate and dividing it by the cross-sectional area of the drain tube.

Figure 5 shows the graph of discharge velocity as a function of the height. We fitted Equation (3), which can be represented by the red line in the graph. As can be observed, the equation fits well the data. We found a coefficient of discharge equals to $C_d = 0.4628$ (using $g = 9.81\,m/s^2$). In Figure 5 we plotted Equation (3) for the ideal case, $C_d = 1$, showing that the discharge velocity, in the ideal case, is bigger than the real one.

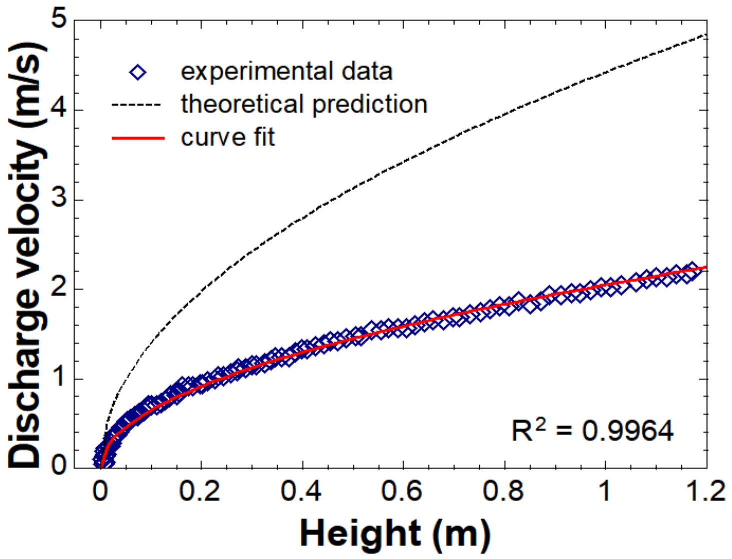

Figure 5. Graph of the discharge velocity given by the flowmeter as a function of height from the ultrasonic sensor. The black dashed line is the theoretical prediction without correction and the red line is a fit of Equation (2).

We plotted the time series for the data collected from both sensors: height (Figure 6a) and discharge velocity (Figure 6b), and Equations (3) and (4), for ideal ($C_d = 1$) and real ($C_d = 0.4628$) situations. We use the same parameters of the experiment realized: initial height $h_0 = 1.17\ m$, hole ($d = 0.01\ m$), and tube ($D = 0.1\ m$) diameters, . As expected, the time of discharge for the ideal situation is smaller than the real, and the initial discharge velocity is higher in the ideal situation. The differences are related to viscosity, turbulence, and other factors of a real fluid.

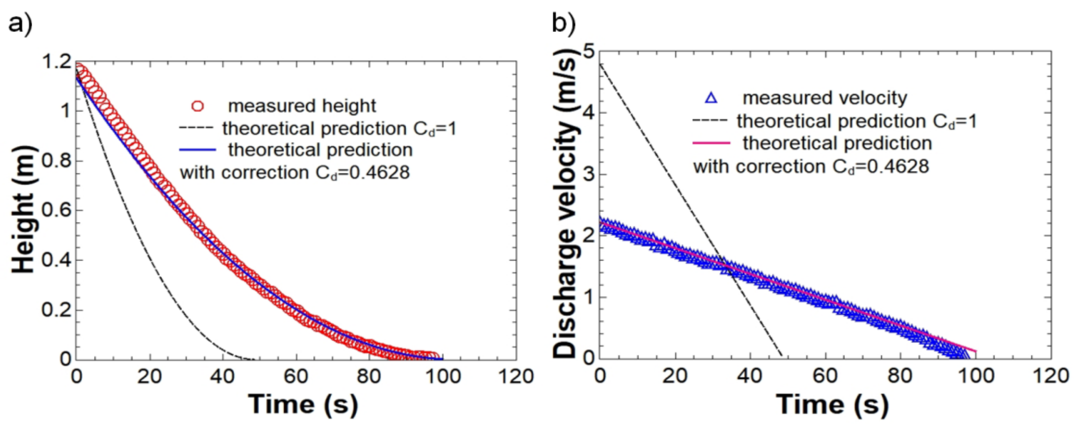

Figure 6. Graphs of: a) discharge velocity and b) height as a function of time. On both graphs, we present the theoretical prediction using $\alpha = 1$(no correction) and $C_d = 0.4628$.

Since we measure here the discharge of a $1.17\ m$ water column the velocities are higher and it can promote a lot of turbulence. Another responsible for the turbulence is the flow meter. Since it works with fluid passing through a rotor, it generates more friction than just a hole. Although the discharge coefficient found is small, the system behaves like in theory, i. e., the velocity of discharge is proportional to the square root of the height.

4. Conclusões

In this manuscript, we showed the use of two different sensors to collect the discharge velocity and the water column height to show the Torricelli relationship: $v_d = C_d \cdot \sqrt{2gh}$. The results present flow resistance observed by a small value of the coefficient of discharge, $C_d = 0.4628$.

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