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Remaining Capacity Estimation of Lead-Acid Batteries Using Exponential Decay Equations

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SUPPLEMENTARY MATERIAL

We present here the results of other batteries that were not presented in the article.

The empirical equations employed for the modelling of the behavior of the remaining capacity, C_r (Ah), with the discharge current, I_d (A), were the following:

Empirical Peukert equation

$$C_\mathrm{r}(I_\mathrm{d}) = K \times I_\mathrm{d}^{(1-n)} \tag{1}$$

where K and n are Peukert empirical parameters.

One-phase exponential decay (ExpDec1)

$$C_\mathrm{r}(I_\mathrm{d}) = C_0 + C_1 \exp\left(\frac{-I_\mathrm{d}}{I_{C_1}}\right) \tag{2}$$

where C_0 (Ah) is the part of the capacity that is not sensitive to the discharge current, C_1 (Ah) is the amplitude of the part of the capacity that decays exponentially with the discharge current at a rate determined by the characteristic current I_{C_1}.

Two-phase exponential decay (ExpDec2)

$$C_\mathrm{r}(I_d) = C_0 + C_1 \exp\left(\frac{-I_\mathrm{d}}{I_{C_1}}\right) + C_2 \exp\left(\frac{-I_\mathrm{d}}{I_{C_2}}\right) \tag{3}$$

where C_0, C_1, I_{C_1} have the same meaning as in ExpDec1 and the second phase of the decay is represented by C_2, I_{C_2}.

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Stretched exponential decay

$$C_{\mathrm{r}}(I_{\mathrm{d}}) = C_{\max} \exp\left[-\left(\frac{-I_{\mathrm{d}}}{I_{C_1}}\right)^{\alpha}\right] \quad (4)$$

where $C_{\max}$ (Ah) is the capacity when the discharge current tends to zero, I_c (A) is a characteristic current that, together with the exponent α, dictate the dynamics of the decay.

The following tables present the nominal capacity of the batteries, C_{nom} (Ah), their application, the parameters of the equations normalized by C_{nom} (since n and α are dimensionless, they were not normalized) and the accuracy by the $\chi 2$ test and the Akaike information criteria (AIC). The application of the presented batteries are: main and standby power supplies, standby power supplies, main power supplies and pitch backup systems in wind turbines. Details about the project, characteristics, discharge curves and Peukert plots can be found in reference [25] of the article.

Table 1. LABs for main and standby power supplies from 1.3 to 3.4 Ah.

Empirical Model	Battery	LC-R061R3P	LC-R121R3P	LC-R122R2P	LC-R063R4P	LC-R123R4P
	C_{nom} (Ah)	1.3	1.3	2.2	3.4	3.4
Peukert	K	0.606	0.599	0.600	0.710	0.709
	n	1.182	1.200	1.231	1.199	1.210
	AIC	-417.7	-480.6	-440.3	-337.4	-402.5
	$\chi 2$	0.061	0.033	0.038	0.012	0.020
ExpDec1	C_0	0.469	0.483	0.447	0.544	0.506
	C_1	0.484	0.516	0.557	0.518	0.533
	I_{C_1} (h^{-1})	0.592	0.473	0.347	0.259	0.329
	$C_0 + C_1$	0.953	0.998	1.004	1.062	1.039
	AIC	-463.3	-459.1	-399.7	-234.3	-323.6
	$\chi 2$	0.026	0.043	0.069	0.115	0.083
ExpDec2	C_0	0.378	0.393	0.381	0.502	0.439
	C_1	0.219	0.299	0.329	0.737	0.335
	I_{C_1} (h^{-1})	0.130	0.114	0.090	0.034	0.101
	C_2	0.433	0.420	0.422	0.398	0.379
	I_{C_2} (h^{-1})	1.286	1.147	0.735	0.487	0.764
	$C_0 + C_1 + C_2$	1.031	1.112	1.131	1.637	1.153
	AIC	-664.3	-693.4	-625.2	-386.6	-516.9
	$\chi 2$	0.001	0.001	0.002	0.005	0.002
Stretched	$C_{\max}$	1.298	1.896	2.705	8.719	5.016
	I_c (h^{-1})	1.911	0.405	0.044	3.4E-6	0.001
	α	0.319	0.210	0.170	0.080	0.109
	AIC	-682.4	-687.8	-673.6	-308.0	-500.2
	$\chi 2$	0.001	0.001	0.001	0.022	0.003

Table 2. LABs for main and standby power supplies from 7.2 to 33.0 Ah.

Empirical Model	Battery	LC-R067R2P	LC-R0612P	LC-RA1212P	LC-X1220P	LC-R1233P
	C_{nom} (Ah)	7.2	12.0	12.0	20.0	33.0
Peukert	K	0.843	0.963	0.963	0.890	1.014
	n	1.168	1.161	1.169	1.113	1.191
	AIC	-212.0	-284.1	-229.6	-173.2	6.7
	$\chi 2$	0.401	0.221	0.191	0.480	4.563
ExpDec1	C_0	0.469	0.560	0.544	0.534	0.376
	C_1	0.468	0.462	0.476	0.316	0.490
	I_{C_1} (h^{-1})	0.808	0.510	0.503	0.867	0.794
	$C_0 + C_1$	0.937	1.022	1.020	0.850	0.866
	AIC	-222.7	-224.2	-203.2	-165.8	-25.0

	χ2	0.314	0.307	0.319	0.512	2.711
ExpDec2	C_0	0.355	0.492	0.480	0.442	0.206
	C_1	0.246	0.264	0.272	0.167	0.257
	I_{C_1} (h^{-1})	0.173	0.162	0.161	0.214	0.150
	C_2	0.416	0.332	0.338	0.285	0.493
	I_{C_2} (h^{-1})	2.290	1.307	1.249	2.882	2.560
	$C_0 + C_1 + C_2$	1.017	1.088	1.090	0.894	0.956
	AIC	-440.5	-474.6	-407.6	-330.4	-298.1
	χ2	0.008	0.010	0.012	0.048	0.081
Stretched	$C_{\max}$	1.318	2.187	2.269	1.154	1.149
	I_c (h^{-1})	2.705	0.285	0.212	10.195	2.088
	α	0.281	0.156	0.156	0.232	0.346
	AIC	-441.5	-429.0	-337.7	-377.8	-223.9
	χ2	0.011	0.038	0.036	0.030	0.266

Table 3. LABs for standby power supplies (SbPS) from 7.2 to 28.0 Ah and 28.0 Ah main power supplies (MPS).

Empirical Model	Battery	LC-P067R2P	LC-P127R2P	LC-X1228P	LC-XC1228AP
	C_{nom} (Ah)	7.2	7.2	28.0	28.0
	Application	SbPS	SbPS	SbPS	MPS
Peukert	K	0.850	0.846	1.131	1.130
	n	1.169	1.168	1.162	1.188
	AIC	-249.9	-219.5	-11.5	4.3
	χ2	0.453	0.413	2.359	2.723
ExpDec1	C_0	0.475	0.471	0.512	0.452
	C_1	0.469	0.469	0.505	0.562
	I_{C_1} (h^{-1})	0.768	0.785	0.743	0.643
	$C_0 + C_1$	0.944	0.941	1.018	1.014
	AIC	-255.3	-226.9	-119.9	-117.4
	χ2	0.391	0.341	0.415	0.225
ExpDec2	C_0	0.372	0.381	0.381	0.423
	C_1	0.244	0.236	0.184	0.348
	I_{C_1} (h^{-1})	0.155	0.144	0.170	0.030
	C_2	0.416	0.414	0.504	0.556
	I_{C_2} (h^{-1})	2.020	1.921	1.693	0.774
	$C_0 + C_1 + C_2$	1.032	1.032	1.068	1.327
	AIC	-462.2	-407.7	-283.1	-224.4
	χ2	0.019	0.018	0.026	0.035
Stretched	$C_{\max}$	1.359	1.340	1.193	1.179
	I_c (h^{-1})	2.415	2.574	3.380	2.111
	α	0.272	0.275	0.427	0.475
	AIC	-477.7	-441.9	-294.0	-145.9
	χ2	0.018	0.012	0.024	0.171

Table 4. LABs for pitch backup systems in wind turbines (PSPBSWT) from 7.2 to 12.0 Ah and 50 Ah LAB [26].

Empirical Model	Battery	LC-WTV127R2	LC-WTP1212	LC-WTV1212	US PATENT
	C_{nom} (Ah)	7.2	12.0	12.0	50.0
	Application	PBSWT	PBSWT	PBSWT	-
Peukert	K	0.844	0.944	0.943	1.511
	n	1.191	1.174	1.170	1.218
	AIC	-128.1	-50.6	-50.9	21.3
	χ2	0.790	2.162	1.748	0.865
	C_0	0.397	0.355	0.357	0.437

ExpDec1	C_1	0.540	0.584	0.569	0.609
	I_{C_1} (h^{-1})	1.039	1.518	1.521	1.085
	$C_0 + C_1$	0.936	0.939	0.927	1.046
	AIC	-105.8	-72.7	-73.9	3.1
	$\chi 2$	0.859	0.948	0.747	0.091
ExpDec2	C_0	0.142	0.104	0.083	0.342
	C_1	0.438	0.387	0.308	0.324
	I_{C_1} (h^{-1})	0.087	0.062	0.081	0.509
	C_2	0.629	0.737	0.748	0.412
	I_{C_2} (h^{-1})	3.728	3.699	3.966	2.710
	$C_0 + C_1 + C_2$	1.209	1.229	1.138	1.078
	AIC	-303.1	-198.3	-194.4	-8.6
	$\chi 2$	0.013	0.065	0.043	0.003
Stretched	$C_{\max}$	1.395	1.090	1.073	1.390
	I_c (h^{-1})	1.765	3.829	3.997	2.489
	α	0.286	0.475	0.479	0.362
	AIC	-163.7	-111.6	-114.0	0.7
	$\chi 2$	0.294	0.476	0.341	0.065