REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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Sensitive-Computational Thinking of Pre-Service Mathematics Teachers on Nested Loops

Pensamento Computacional-Sensível de Professores de Matemática em Formação Inicial sobre Nested Loops

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ABSTRACT

In this paper we emphasize the aesthetic-sensitive dimension of computational thinking in a scenario of pre-service mathematics teacher education. Through the development of teaching experiments we investigate aspects (skills/concepts/affordances) of computational thinking of mathematics majors emergent in the exploration of a task based on coding with the online application based on a coding application developed using Blockly. The findings highlight the processes of experimentation with technology in which the command named *repeat* was used in several manners in the attempt of creating nested loops to solve the task. The sensitive-computational thinking points to the perception and modification of aesthetic elements such as the form of objects, colors, symmetries, patterns, sounds, etc. and its relation to commands that compose the code and the overall structure of the code.

KEYWORDS: Computational Thinking. Aesthetics. Digital Technology. Coding.

RESUMO

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Neste artigo enfatizamos a dimensão estética-sensível do pensamento computacional em um cenário de formação inicial de professores de matemática. Por meio do desenvolvimento de experimentos de ensino, investigamos aspectos (habilidades/conceitos/propriedades) sobre o pensamento de estudantes de graduação em matemática emergentes na exploração de uma tarefa baseada em programação com um aplicativo on-line desenvolvido usando o Blockly. Os resultados destacam os processos de experimentação com tecnologia em que o comando *repeat* foi usado de várias maneiras na tentativa de criar *loops* para resolver a tarefa. O pensamento sensível-computacional aponta para a percepção e modificação de elementos estéticos, como a forma de objetos, cores, simetrias, padrões, sons etc., e sua relação com os comandos que compõem o código e a estrutura geral do código.

PALAVRAS-CHAVE Pensamento Computacional. Estética. Tecnologia Digital. Programação.

Introduction

Computational thinking and its relationship to mathematical learning have been discussed since the work of Papert (1980). Based on the use of the software LOGO, articulated to the learning theory named constructionism, Papert (1993) proposed the use of computer programming for children's pedagogical activities. Authors such as Borba, Scucuglia and Gadanidis (2014) argue that the use of LOGO during the 1980s is a characteristic of the first phase of the use of digital technologies in mathematics education in Brazil. The access to personal computers in schools and the development of research on educational technology have begun at the time, i.e. 1980s. From this perspective, since 2010s, the current phase is marked by the use of mobile technologies, high-speed Internet, social networks, production of digital videos, development of virtual and augmented reality, and more. In fact, computational thinking is still an outstanding aspect of the current phase regarding the pedagogical use of different types of online applications such as Scratch and those developed using Blockly.

Curricular documents such as the Brazilian National Common Curricular Base (BRASIL, 2017) highlight computational thinking as an important skill for mathematical activity at elementary and high school levels. Computational thinking is therefore an important and relevant component in the preparation of pre-service mathematics teachers. Mathematical processes such as problem solving and project-based learning "are potentially rich for the development of fundamental skills for mathematical literacy (reasoning, representation, communication and argumentation) and for the development of computational thinking" (BRASIL, 2017, p. 266). In fact, curricular documents, such as the National Curriculum Parameters – Mathematics, proposed the use of information technologies as a resource for teaching mathematics since the 1990s (BRASIL, 1997).

The use of digital technologies fosters transformations in mathematics education, potentially attributing pedagogical specificities to the dynamics of classrooms. One of the aspects highlighted in this scenario concerns the process of experimentation-with-technologies (BORBA; SCUCUGLIA; GADANIDIS, 2014; ENGELBRECHT; LLINARES; BORBA, 2020). The nature of mathematical problems elaborated and solved with paper and pencil, for example, is qualitatively different from problems explored with mathematics software. The literature has thus argued that knowledge is not produced only by humans, but by collectives of humans-with-media (BORBA; VILLARREAL, 2005). The mathematical activity in classrooms can be reorganized through the development of hands-on tasks based on the use of technologies that allow the elaboration, refutation, and/or confirmation of conjectures and the exploration of multiple solutions of problems regarding the open-ended design of tasks. This kind of inquiry-based approach fosters the emergence of a maker culture, where an emphasis is placed on risk-taking and learning through mistakes in a collaborative community. The mediating role of teachers is also transformed by the use of new media. The Internet, for example, offers access to wide information and applications, modifying pedagogy, didactics and power relations of school scenarios.

In this article we discuss the results of a research project in which the objective was to investigate, through the development of teaching experiments (STEFFE; THOMPSOM, 2000), aspects of the sensitive-mathematical-computational thinking of majors in mathematics[6] when exploring an investigative task based on the creation of virtual (artistic) objects elaborated through coding. Thus, we invite the reader to try the application available at http://mathsurprise.ca/apps/patterns/v2/ (GADANIDIS, 2017a). Specifically, we investigate aspects related to the exploration conducted by undergraduate students in mathematics (or mathematics majors) on nested loops, which is a relevant concept of computer science (CHAMSKI, 1993). According to Cooper (2014, para. 1), "[a] nested loop is a loop within a loop, an inner loop within the body of an outer one". Gibbs (2020, para. 4) states that nested loops "are used to cycle through matrix and tabular data and multi-dimensional arrays. Since a table is a matrix of rows and columns, you need one loop to loop through the row, then across the column". Thus, in particular, we discuss aspects of mathematics majors' thinking when exploring a task based on coding focusing on the use of loops.

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[6] Due the structure of some undergraduate courses in mathematics in Brazil, undergraduate students in mathematics (mathematics majors) may be also pre-service mathematics teachers (*licenciandos em matemática*).

The expression “aspects of computational thinking” is conceptualized in this research as a set of skills, concepts, and affordances related to computational thinking. The ISTE/CSTA (2011) state that the skills of computational thinking are data collection, data analysis, data representation, problem decomposition, abstraction, algorithms and procedures, automation, simulation, and parallelization. Brennan and Resnick (2012) argues that the concepts of computational thinking are sequences, loops, parallelism, events, conditionals, operators, and data. Gadanidis (2017b) points out that the affordances of computational thinking are agency, accesses, abstraction, automation, and audience. Therefore, as proposed by Barbosa (2019), the aspects of computational thinking in this study may be comprised as related to the following categories: algorithmic thinking, decomposition and generalization, patterns and abstraction, representation and automation, and evaluation.

On Sensitive-Computational Thinking

According to Abbagnano (2007, p. 751), the concept of thinking concerns at least two main meanings: “1 - any mental or spiritual activity; 2 - activity of the intellect or reason, as opposed to the senses and the desire”. In contrast, Boal (2009) argues that there are two complementary forms of thinking: the sensitive and the symbolic[7].

> [Sensitive thinking] is a non-verbal way of thinking (...) articulated and resolutive, which guides the continuous act of knowing and commands the dynamic structuring of sensitive knowledge. (...) to be understood, even when they are expressed in words, thoughts depend on the way those words are pronounced or on the syntax in which the sentences are written - that is, they depend on Sensitive Thinking (BOAL, 2009, p. 27).

Based on this perspective, one may consider that mathematical and computational thinking are fundamentally symbolic, but have their genesis in sensitive elements. Machado (2013, p. 88) states that “[thinking] develops, therefore, on a sensory-material basis. It aims at the rational but its genesis is sensory, not rational”. Thus, it is important to highlight that the pedagogical activities developed in this research involve aesthetic elements so that the sensitive thinking is educationally enriched in the constitution of the mathematical and computational ways of thinking[8].

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[7] According to Machado (2013, p. 88), “[the] sensorial and the rational are two elements of the knowledge development process, but they do not constitute independent or even successive steps. At each point in the process, they participate as a unit”.

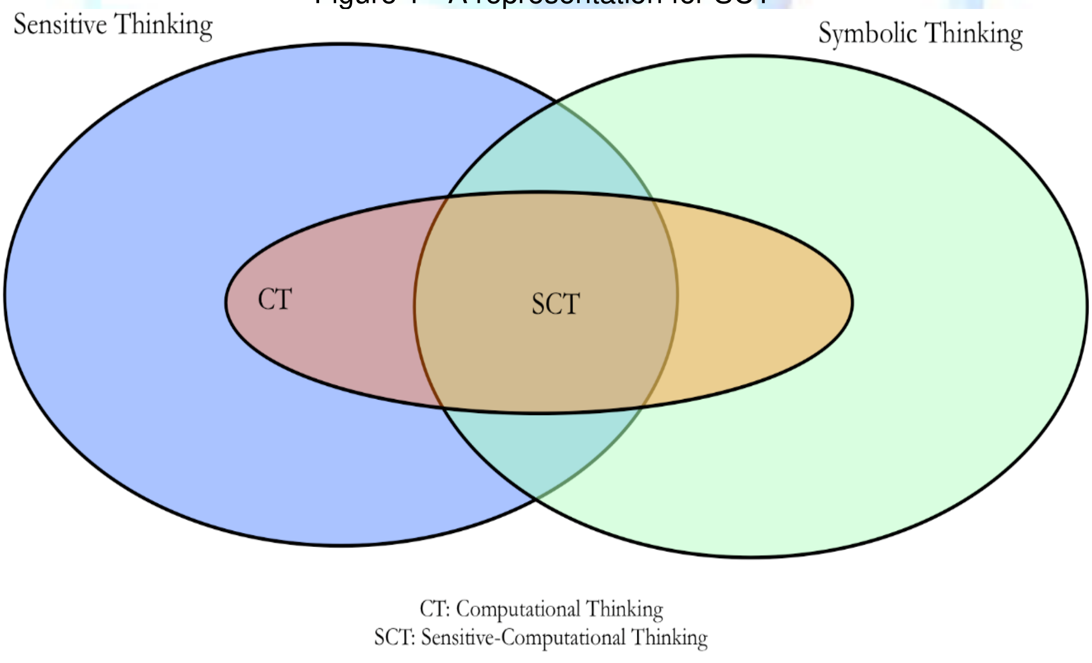
[8] According to Dewey (2010), the arts are not the only means in which aesthetic experiences emerge.

Mannila *et al*. (2014, p. 2) state computational thinking "is a term encompassing a set of concepts and thought processes from [computer science] that aid in formulating problems and their solutions in different fields". According to Wing (2008),

> Computational thinking is a kind of analytical thinking. It shares with mathematical thinking in the general ways in which we might approach solving a problem. It shares with engineering thinking in the general ways in which we might approach designing and evaluating a large, complex system that operates within the constraints of the real world. It shares with scientific thinking in the general ways in which we might approach understanding computability, intelligence, the mind and human behaviour. (WING, 2008, p. 3717).

Particularly, within this framework, we propose the notion of sensitive-computational thinking (SCT) from the interface between symbolic and sensitive ways of thinking. SCT refers thus to the perception, exploration, and emphasis of aesthetic elements of coding such as forms and colors of shapes, symmetry of objects, sounds, multiple types of patterns etc. and its relation to the commands and the overall structure of the code. In Figure 1 we show a representation of SCT in relation to the ways of thinking.

Figure 1 - A representation for SCT



Source: Elaborated by the authors

In this perspective, thinking involves necessarily symbolic and sensitive elements (BOAL, 2009). However, regarding the nature of the tasks and the development of the educational scenarios in different approaches, we have argued the pedagogic enterprise may emphasize one type of thinking particularly. In this research, we intend to highlight the sensitive dimension of thinking through the exploration of patterns, symmetries, shapes, colors, and sounds. The pedagogic nature of the proposed tasks intends to engage students in the exploration of these elements. Therefore, the tasks investigated in this research were designed focusing on sensitive-computational ways of thinking, without disregard the relevance of symbolic endeavours for mathematical and computational activity.

Methodology

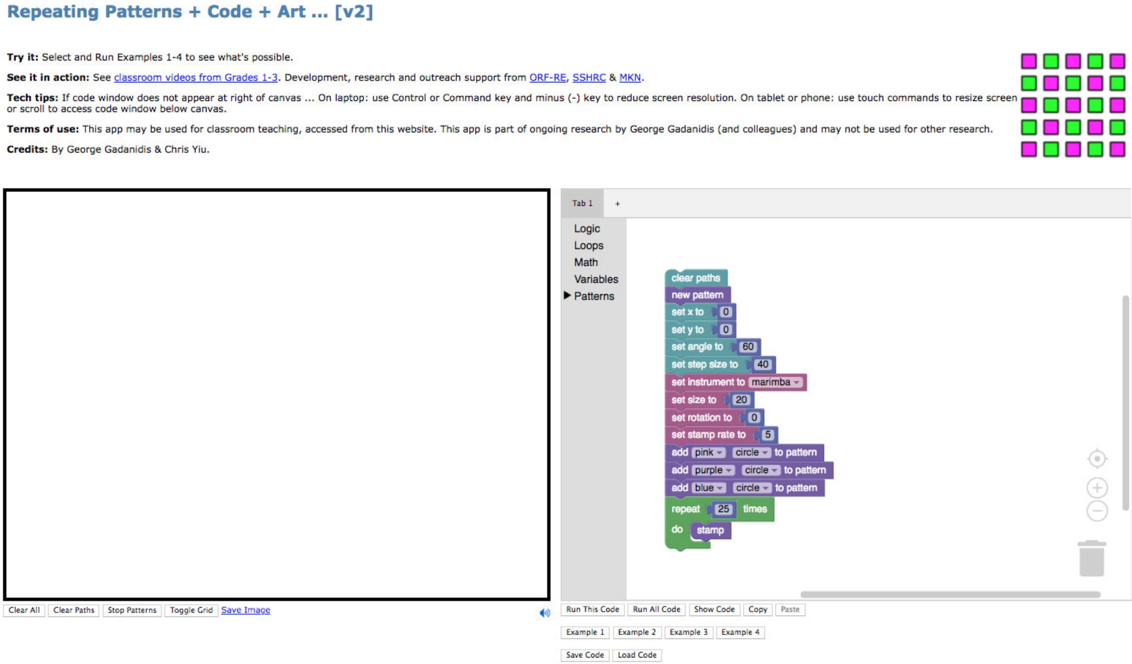
This research is qualitative in nature (BICUDO, 1993) and seeks to address the question: what aspects of SCT emerge when pre-service mathematics teachers explore mathematical-artistic-computational tasks? Specifically, we investigate in this paper aspects related to the concept of nested loops.

To discuss the research question, sessions of teaching experiments were held with pairs of mathematics majors at a state university in São Paulo in 2019. According to Steffe and Thompson (2000, p. 267) the purpose of teaching experiments "is for researchers to experience, firsthand, students' mathematical learning and reasoning". Moreover, "[a] teaching experiment involves a sequence of teaching episodes […], [and a] teaching episode includes a teaching agent, one or more students (…), and a method of recording what transpires during the episode" (p. 273). The records "can be used as in conducting a retrospective conceptual analysis of the teaching experiment" (STEFFE; THOMPSON, 2000, p. 273).

The sessions of teaching experiments in this research were filmed with a digital camera and the use of the software Flashback Pro5, which is a desktop computer recorder that also captures the audio of the participants' voices and the images from the computer's webcam. One of the authors of this paper participated as an instructor in the sessions, and two authors elaborated field notes as well (PATTON, 2002). The task proposed to the students consists of four activities (GADANIDIS, 2017a). All activities are based on the use of an application named Repeating Patterns created by Gadanidis and Yiu (2017) using Blocky (Google)[9] (see Figure 2).

[9] https://developers.google.com/blockly

Figure 2 - The Blockly application



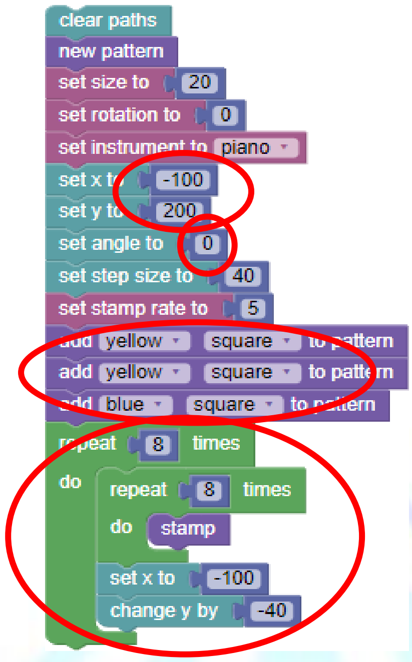
Source: Research data

The first activity engages students in the use of initial commands and codes of the app. Students may get familiar with the interface overall, including commands such as set x to, set y to, set angle to, set step size to, set instruments to, and so on. Elaborating/manipulating and running a code using these commands organized algorithmically, students may create virtual objects. The second activity engages students in the creation of multiple codes that can run simultaneously. Activity 3 explores the notion of nested loops through the use of the command repeat to create codes. Finally, the last activity proposes the exploration of a puzzle that comprises all the concepts explored in the activities 1, 2, and 3.

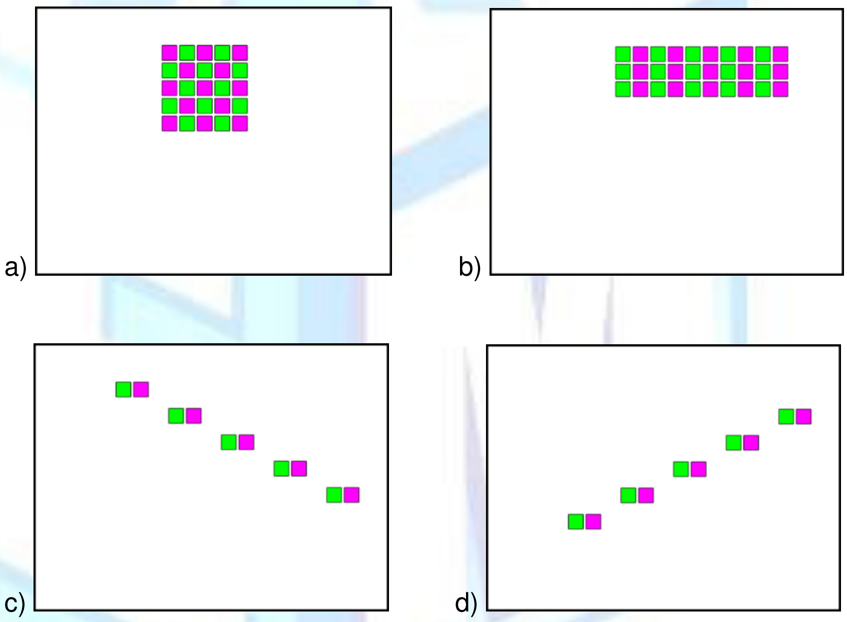
Next, we present activity 3 of the task explored by the participants of the study, which is the focus of analysis in this article.

Activity 3

3.1 Make edits to the code (Example 3 - http://mathsurprise.ca/apps/patterns/v2/), according to the image below and observe its execution. What is new in this code?



3.2 Edit the code to create the following patterns.



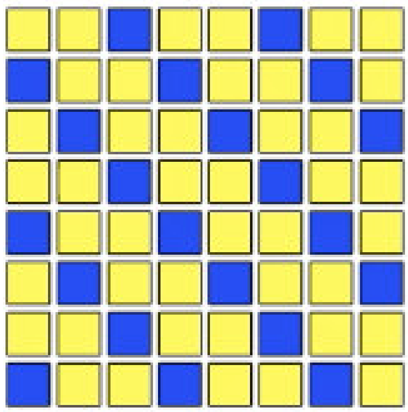
For this article, based on both convenience sampling and theoretical sampling (MARSHAL, 1996), we selected for analysis and discussion the events that occurred in one 2-hour session of teaching experiments carried out with a pair of mathematics majors. For this qualitative case study (STAKE, 2005; PONTE, 2006), we use the nomenclatures *Student A* (fourth year of the course) and *Student B* (sixth year of the course) to refer to students who are members of the pair. We use Instructor to refer to the teacher-researcher who conducted the teaching experiment.

The videos were analyzed based on the model proposed by Powell, Francisco and Maher (2004), consisting of the following procedures: observation, description, identification of critical events, transcription, coding, elaboration of episodes, and composition of the narrative. We also consider the notion of data triangulation for the trustworthiness of the analytic process (ARAÚJO; BORBA, 2004).

SCT on Nested Loops

The pair of mathematics majors started to carry out Activity 3 by changing the command settings as shown in 3.1. Thus, when executing the configured code, they obtained the object shown in Figure 3.

Figure 3 - Execution of 3.1



Source: Research data

In comparison to previous activities of the task, activity 3 introduces a computational concept of *nested loop*, which refers to the use of a loop inside a loop. When asked what was new in activity 3, the majors identified the command *repeat* of Blockly, which refers to the loop. Thus, the instructor explained some characteristics related to that concept as described in the following transcription.

> ***Instructor:*** *What's new about this...*
>
> ***Student A:*** *... is the repeat.*
>
> ***Instructor:*** *Yes! There is a repeat inside a repeat. And notice, we have also "changed dimensions". Until then, the objects were "linear"; the elements were juxtaposed. Now, there are moments in which the elements change from one line to another. So, in a way, there is a modification from the "one-dimensional" to "two-dimensional".*

Analyzing the settings of the algorithm introduced in activity 3, in terms of SCT, we notice that there is a sequence of squares as yellow / yellow / blue. In addition, both *repeat* commands are configured with 8 units (8x8). Comparing the commands at the beginning of the code in relation to those inside the *repeat*, the value of x is mandated at -100 and the value of y is changed to -40[10]. Thus, using Y

[10] The commands *set x to #* and *set y to #* determine the coordinate of the starting point of the virtual object. The commands *change x to #* and *change y to #* determine the variation of the new coordinates generated by the repeat command in relation to the original coordinate.

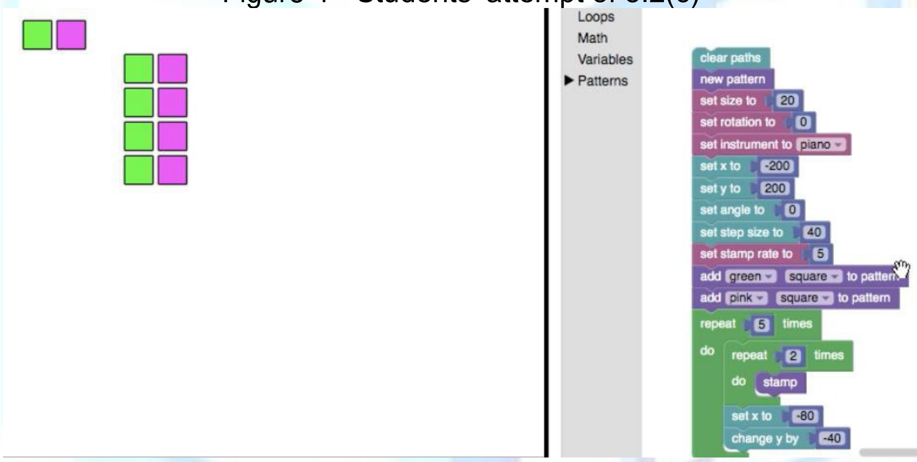
to name yellow and B to determine blue, we have in the first line of the object the sequence YYBYYBYY (eight elements defined inside *repeat*). After the execution of the first 8 elements there is a change in the position of the ninth element to another line, which refers to the use of the nested loop. The x-axis is maintained at -100 and the y-axis is changed to -40, which means that the position of the ninth element will be (-100, 160) instead of (-100, 200). Note that the value -40 is used because *set size to* is 20, which refers to ½ of the side of the square. If the shape were a circle, the value of *step size to* would refer to the radius of the circle. The command *set step size to* also plays a determinant role in this positional configuration. Thus, since the intention was to position the ninth element on a line below the first one, the value of y was decreased using the command *change y by* -40. If a value higher than -40 was described, such as -30, and the value of *set step size to* was maintained, the elements or shapes would overlap. If one inserted a value like -50, for example, there would be a gap between the lines that would aesthetically alter the constructed object. Therefore, the aesthetic-sensitive component assumed a conditioning role in the process of solving the activity. In this case, the form of the shapes (squares), the size of the shapes, and their (x, y) positions (of each shape itself and of it each shape in relation to others) conditioned the majors' SCT.

Colors also played an aesthetic-sensitive role in the exploration of the activity 3.1. When asked about what kind of patterns they would identify in the executed object (Figure 3), the students mentioned "blue diagonals" and "yellow stairs". In specific, Student A stated that those were "blue diagonals to the left", which reveals an interesting aspect about the student's perception. The sensitive thinking in this moment supported the thinking that the diagonals were created from the right to the left, expressing an "upward motion". However, if one considers the execution of the code itself, the motion of the shapes is from the left to the right and, thus, the blue diagonals are "top to bottom". We argue that in this situation, Student A's perception mainly regarded the static final object constructed instead of the dynamism of the execution of the object.

After the instructor's explanation about nested loop and the identification of patterns in 3.1, the pair of majors proceeded with the development of Activity 3. To create the object displayed in the item 3.2 (a), the students changed the values of the *repeat* commands from 8x8 to 5x5, deleting one of the lines of the elements - in this case, they deleted *blue* / *square* - and changed the colors of the other two left objects, respectively to *pink* and *green*. They also made a change in the *set x to* command from -100 to -80, as they intended to “center” the object. Similarly, they created the object in item 3.2(b), changing the parameters of the *repeat* commands from 5x5 to 3x5. Aesthetically, we highlight in 3.2(a) and 3.2(b) not only the modification in the dimension of the matrix through the nested loop (8x8, 5x5, and 3x5) and the way students modified the colors of the shapes, but also their desire to “center” the object, which refers to a sense of symmetry (SINCLAIR, 2004). Therefore, SCT also play a role in this perspective.

The exploration of item 3.1(c) took a lot of time during the session. There were several attempts, elaboration of hypotheses and conjectures made by the pair of mathematics majors. From a pedagogical point of view, it was an interesting episode that revealed different kinds of experimentation-with-technology and conflicts in think-with-technology (BORBA; SCUCUGLIA; GADANIDIS, 2014). Basically, at first attempt the mathematics majors were able to change the displacement up to the second line of the object. However, they had significant difficulty in changing the position of the objects from the third row onwards (see Figure 4).

Figure 4 - Students’ attempt of 3.2(c)



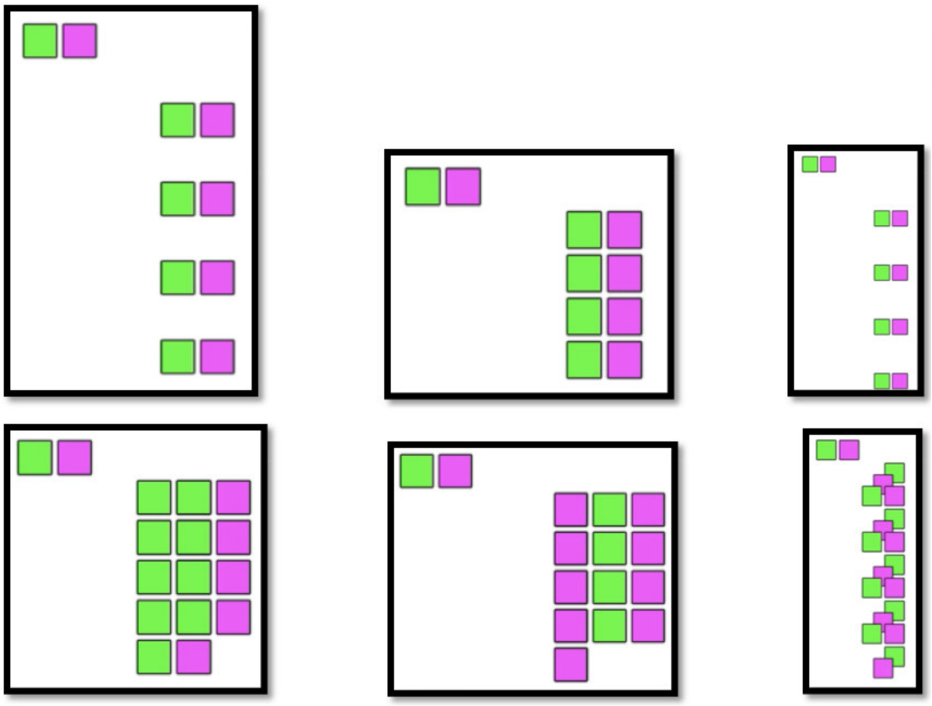
Source: Research data

The mathematics majors made 16 different attempts. They changed the values of *set x to*, inserted a new *repeat* command, changed the position of *repeat* in the code, changed the values of the commands *set x to* and *change y by* inside the *repeat*, deleted one of the *repeat* commands and inserted a *stamp* command, changed the position of the *stamp* in the code, among other changes. As in the software LOGO (PAPERT, 1993), the description window of the commands in Blockly, that display the code, can be considered representations of the students’ thinking, and in this case, those representations reveal the students’ sensitive-computational thinking. Although the application does not create a history of executions, we had access to the history of codes created by the makers because the computer's desktop was being recorded on video. In Figure 5, we display some of the loops that were configured by the pair of mathematics majors and, in Figure 6, the respective objects generated from the execution of these codes.

Figure 5 - Configuration of the *repeat* command in codes - Activity 3.2(c)

Source: Research data

Figure 6 - Object created from the codes - Activity 3.2(c)



Source: Research Data

Eventually, during this process of experimentation-with-Blockly, the mathematics majors made relationships to contents explored in courses on introduction in computer science (ICC) offered in the undergraduate course in mathematics. For example, the initiative to insert and change the position of the *stamp* command in the code was based on the following statement made by Student A: "*we are going to change the stamp as we do in ICC;, it has the beginning and the end*", referring to commands of programming in C. However, after these several "unsuccessful" attempts, the instructor made the following suggestion:

> ***Instructor:*** *I think those were good attempts. But, please, note that you are using the commands set x to and change y by. Look at the library of commands. Is there something to change the x in the same way you change the y?*
>
> ***Student B:*** *[accessed the library / position commands]. Like this?*
>
> ***Instructor:*** *Yes. Note that they are set to and change by for x and for y as well.*

Despite this mediation by the instructor, the pair of majors continued to carry out various experiments for another 30 minutes in the teaching session. *Repeat* commands were inserted and modified again in the code, 3 *repeat* commands were inserted in a single repeat, and 4 *repeat* commands were inserted "one inside another". At a certain moment, the major made changes in the code that did not result in changes in the executed object. After another 27 different attempts, the pair decided to "restart the problem", that is, they ran example 3 of the application again. Two more attempts were made using 3 *repeat* commands until the pair managed to complete item 3.2(c) according to the dialogue transcribed below.

> ***Instructor:*** *The set x to concerns the position of the object. Set x and set y refers to the (x, y) coordinate [in which the execution of the object starts].*
>
> ***Student B:*** *And change to refers to new position regarding the initial (x, y), isn't it?*
>
> ***Student A:*** *So we are going to put change x by also and see if it works.*
>
> ***Student B:*** *Now we need to change the puzzle.*
>
> *[The majors still made some experimentation using 3 repeat commands]*
>
> ***Student A:*** *Wait, I think we should go straight, because if y is changing automatically, x is going to have to change. Will it not? How long will it walk [referring to steps]? It will walk 40, 80, hum, 120, 160 to the right. Let's insert it, just to check it up.*
>
> ***Student B:*** *But 160 is not much? Maybe 80.*
>
> ***Student A:*** *Let's go for 80 so if you think so.*

[The students execute the code].

Student A: *Oh! Now it has changed two by two.*

Student B: *But it is too far one from another. Let's go for 40.*

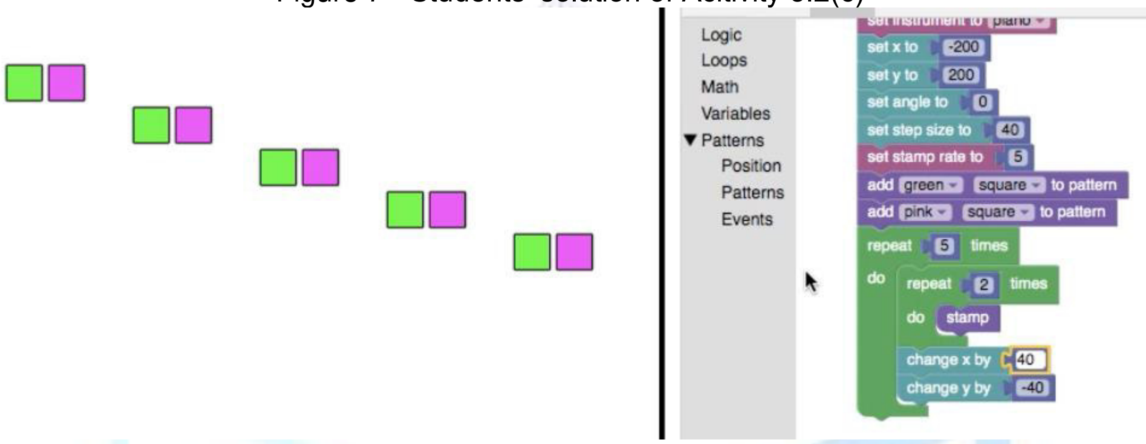
[Execute the code - see Figure 7]

Student B: *Okay! Wow!*

Student A: *Cool!*

Instructor: *Very good!*

Figure 7 - Students' solution of Acitivity 3.2(c)



Source: Research data

Based on the resolution of item 3.2(c), the pair of majors easily completed item 3.2(d). The pair chose to change the values of *set x to* from -200 to 280 and *set y to* from 200 to 140 at the beginning of the code. Also, the *set angle to* command was changed from 0 to 180 and the colors were inverted (from *green* / *pink* to *pink* / *green*). When asked why the angle was changed, the majors replied that the resolution executed the object from right to left. To execute from left to right they would have to keep the angle value as 0 and the *green* / *pink* sequence as it was, and change the initial values of x and y and change the values of x and y inserted in the *repeat* command. In this sense, each of the Activity 3's items could be solved in different ways. In each of these possible solutions perception plays different roles for the students' CT thinking and aesthetic experience. The possibility of exploring multiple resolutions or multiple strategies for coding, that is, the investigative / open-ended design of tasks based on the use of computers, is an important affordance about the use of digital technologies in mathematical education (BORBA; SCUCUGLIA; GADANIDIS, 2014; da SILVA, 2019).

Conclusions

We have investigated the aesthetic-sensitive dimension of computational thinking in mathematics education. We thus elaborated in this paper a framework to emphasize aspects of sensitive thinking in the exploration of investigative tasks based on coding in scenarios of pre-service mathematics teacher education. In previous research, we have argued for the significance of artistic elements such as colors, symmetries, and sounds for computational thinking and modeling (GADANIDIS *et al.*, 2019). In this study, we explored these elements regarding the development of teaching experiments conducted with mathematics majors.

Overall, participants explored in the sessions aspects about the sounds available in the examples and in the elaboration of codes. However, sounds were not deeply explored in Activity 3 of the task, as discussed in this paper. Even though the code employed the use of the command *set instrument to*, the students did not try it as they did in Activity 4, for instance. One may notice that the form of the shape was not modified in their exploration as well as they kept using squares. Overall, from an aesthetic-sensitive point of view, the experimentation-with-coding in this study emphasized the modification of colors and positions of the shapes. The colors were significant to the formation of patterns and symmetries and the position in terms of symmetry as well.

The process of experimentation with technology revealed students' difficulties in understanding not only the role/function of nested loops, but also the execution of this combination of commands. The activity 3.1 introduces the concept of nested loop, changing the dimension of the matrix. Then, activities 3.2(a) and 3.2(b) explore variations of values of commands, maintaining the construction of rectangles. The activities 3.2(c) and 3.2 (d) helped student to understand the difference between *set x to* and *change x by*, and the role of these commands in the code. The several attempts conducted by the participants in the teaching experiment exposed aspects related to the mathematics majors' difficulties in comprehending the function of these commands.

Among other reasons, sensitive thinking played a significant role in the process of experimentation-with-coding when the mathematics majors refused the result of most of their attempts. Their perception led them to recognize that the execution displayed in Figure 6, for instance, was different than expected for the solution of 3.2(c). Through the sensitive-computational analysis about the qualitative differences between the objects and the configuration of the respective codes possibilities to conjecture and execute new strategies may emerge, offering ways for new attempts and the finding of solutions through experimentation-with-technology.

Overall, regarding the large number of the attempts conducted by the students, we consider that the mathematics majors could be more analytical in the way they were thinking-with-technology. In many descriptions-executions made by them, we may identify that the participants were not deeply analyzing the functions of the commands and the functions of combinations of commands. They were trying to create “complex codes” with a focus on the usage of different types of nested loops, that is, with many overlaps and combinations of repeat commands. Eventually, these combinations did not make sense in terms of the execution of the code.

We argue this research contributes to the current phase on the use of digital technology in mathematics education (BORBA; SCUCUGLIA; GADANIDIS, 2014; ENGELBRECHT; LLINARES; BORBA, 2020). Computational thinking is not only an important endeavour over decades in the area, but the emphasis on aesthetic elements of computational thinking also offers ways to make explicit the potential complexity of students’ and teachers’ thinking-with-technology in mathematical activities. SCT may play a relevant pedagogical and investigative role in mathematics education.

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References

ABBAGNANO, N. **Dicionário de filosofia**. São Paulo: Martins Fontes, 2007.

ARAÚJO, J. L.; BORBA, M. C. **Construindo Pesquisas Coletivamente em Educação Matemática.** In: BORBA, M. C.; ARAÚJO, J. L. (Org.) Pesquisa Qualitativa em Educação Matemática, Belo Horizonte: Autêntica, 2004.

BARBOSA, L. M. **Aspectos do Pensamento Computacional na Construção de Fractais com o software GeoGebra**. Dissertação Mestrado em Educação Matemática). Universidade Estadual Paulista (Unesp), Instituto de Geociências e Ciências Exatas, Rio Claro: 2019.

BICUDO, M. A. V. **Pesquisa em educação matemática**. Pro-posições, Campinas, v. 4, n. 10, p. 18-23, mar. 1993. Disponível em: https://www.fe.unicamp.br/pf-fe/publicacao/1755/10-artigos-bicudomav.pdf. Acesso em: 20 abr. 2020.

BOAL, A. **A estética do oprimido**. Rio de Janeiro: Garamond, 2009.

BORBA, M. C.; SCUCUGLIA, R. R. S.; GADANIDIS, G. **Fases das Tecnologias Digitais em Educação Matemática**: sala de aula e internet em movimento. 1. ed. Belo Horizonte: Autêntica, 2014.

BORBA, M. C.; VILLARREAL, M. E. **Humans-With-Media and the Reorganization of Mathematical Thinking**: information and communication technologies, modeling, experimentation and visualization. v. 39, New York: Springer, 2005.

BRASIL, Secretaria de Educação Fundamental. **Parâmetros Curriculares Nacionais**: Matemática. Brasília: MEC, 1997.

BRASIL. **Base Nacional Comum Curricular**. Brasília, DF: MEC, 2017.

BRENNAN, K.; RESNICK, M. **New frameworks for studying and assessing the development of computational thinking**. AERA 2012, Vancouver, Canada, 2012. Disponivel em http://web.media.mit.edu/~kbrennan/files/Brennan_Resnick_AERA2012_CT.pdf. Acesso em: 01 jul. 2018.

CHAMSKI, Z. **Nested loop sequences**: towards efficient loop structures in automatic parallelization. [Research Report] RR-2094, INRIA. 1993.

COOPER, M. **Advanced Bash-Scripting Guide: An in-depth exploration of the art of shell scripting.** Available at: https://www.tldp.org/LDP/abs/html/index.html, 2014.

da SILVA, R. S. R. **On music production in mathematics teacher education as an aesthetic experience**. ZDM Mathematics Education, 2019. https://doi.org/10.1007/s11858-019-01107-y.

DEWEY, J. **Art as experience**. São Paulo, Brazil: Martins Fontes, 2010.

ENGELBRECHT, J.; LLINARES, S.; BORBA, M. C. **Transformation of the mathematics classroom with the internet**. ZDM Mathematics Education, 2020. https://doi.org/10.1007/s11858-020-01176-4.

GADANIDIS, G. **Repeating Patterns V. 6**. 2017a. Available at: https://wordpress.oise.utoronto.ca/robertson/files/2018/04/repeating-patterns-tutorial.pdf.

GADANIDIS, G. **Five Affordances of Computational Thinking to Support Elementary Mathematics Education**. Journal of Computers in Mathematics and Science Teaching, v.36, n.2, p.143-151, 2017b.

GADANIDIS, G.; YIU. C.. **Repeating Patterns + Code + Art ... [v2],** 2017. Available at: https://mathsurprise.ca/apps/patterns/v2/.

GADANIDIS, G.; HUGHES, J.; NAMUKASA, I.; SCUCUGLIA, R. **Computational modeling in elementary mathematics teacher education**. In: LLINARES, S.; CHAPMAN, O (Eds.). International Handbook of mathematics teacher education: Volume 2 (Second edition). The Netherlands: Koninklijke Brill NV, 2019. p. 197-222.

GIBBS, D. **Nesting Loops & Statements in C Programming.** 2020. Retrieved from https://study.com/academy/lesson/nesting-loops-statements-in-c-programming.html.

ISTE/CSTA. **Computational Thinking Teacher Resource**. 2ed., 2011. Disponivel em: http://csta.acm.org/Curriculum/sub/CurrFiles/472.11CTTeacherResources_2ed-SP-vF.pdf. Acesso em: 01 jul. 2018

MACHADO, N.J. **Matemática e realidade**: das concepções às ações docentes. 8a. ed.. São Paulo: Cortez, 2013.

MANNILA, L.; DAGIENE, V.; DEMO, B.; GRGURIN, N.; MIROLO, C.; ROLANDSSON, L.; SETTLE, A. **Computational Thinking in K-9 Education**. In: Proceedings of the WorkingGroup Reports of the 2014 on Innovation & Technology in Computer Science Education Conference. Uppsala, Sweden, p. 1-29, 2014.

MARSHAL, M. N. **Sampling for qualitative research**. Family Practice, [s.l.], v. 13, n. 6, p. 522–526, 1996.

PATTON, M. Q. **Qualitative Research & Evaluation Methods**. 3rd edition. Sage Publications, Inc., 2002.

PAPERT, S. **Mindstorms**: children, computers and powerful ideas. New York: Basic Books, Inc., 1980.

PAPERT, S. **The Children's Machine**. New York: Basic Books, 1993.

PONTE, J. P. **Estudos de caso em educação matemática**. Bolema, v. 25, p. 105-132, 2006.

POWELL, A. B.; FRANCISCO, J. M.; MAHER, C.A. **Uma abordagem à Análise de dados de vídeo para investigar o desenvolvimento de Idéias e Raciocínios Matemáticos de Estudantes**. Bolema, Rio Claro, v. 21, n. 17, p. 81-140, 2004.

SINCLAIR, N. **The roles of the aesthetic in mathematical inquiry**. Mathematical Thinking and Learning, v. 6, n. 3, p. 261–284, 2004.

STAKE, R. E. **Qualitative Case studies**. In: DENZIN, N. K.; LINCOLN, Y. S. (Eds.). The Sage handbook of qualitative research. London: Sage Publications, 2005

STEFFE, L. P.; THOMPSON, P. W. **Teaching experiment methodology:** Underlying principles and essential elements. In: LESH, R.; KELLY, A.E. (Eds.). Research design in mathematics and science education. Hillsdale, NJ: Erlbaum, 2000. p. 267-307.

WING, J. M. **Computational thinking and thinking about computing**. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, [s.l.], v. 366, n. 1881, p. 3717–3725, 2008.

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