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Lesson Study in Matematics: Means, Aims and Obstacles

Lesson Study em Matemática: Meios, Objetivos e Obstáculos

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ABSTRACT

This paper, reflecting my talk at the XIIth summer workshop in mathematics, has three related aims: (1) to give a concise introduction to the phenomenon called "Lesson Study", particularly in mathematics, based on a selection of scientific literature and the authors' experience (mainly from Japan and Denmark); (2) to discuss the potentials of lesson study and some of the obstacles which the activity has met when transferred to schools outside of Japan; (3) to introduce and briefly discuss a research question which has been latent in research on Lesson Study since the late 1990's, and that is currently being addressed in the framework of the Anthropological Theory of the Didactic.

KEYWORDS: Lesson Study. Paradidactic Infrastructure. Anthropological Theory of the Didactic.

RESUMO

Esse artigo, baseado em minha palestra no XII *Workshop* de Verão em Matemática, tem três objetivos relacionados: (1) dar uma introdução concisa ao fenômeno denominado "*Lesson Study*", particularmente em matemática, com base em uma seleção da literatura científica e da experiência de autores (principalmente do Japão e Dinamarca); (2) discutir os potenciais do *Lesson Study* e alguns dos obstáculos que a atividade encontrou quando transferida para escolas fora do Japão; (3) apresentar e discutir brevemente uma questão de pesquisa que tem estado latente nas pesquisas sobre *Lesson Study*, desde o final da década de 1990, e que, atualmente, está sendo abordada no âmbito da Teoria Antropológica do Didático.

PALAVRAS-CHAVE: *Lesson Study*. Infraestrutura Paradidática. Teoria Antropológica do Didático.

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Introduction

In research on Didactics of Mathematics, the role of teachers has been increasingly emphasized since the late 1990s. Mathematics teachers' knowledge about mathematics and its teaching is certainly an extremely complex object of study, but research has succeeded in developing qualitative and quantitative metods to measure it which, even if they are relatively rough, can be significantly related to student outcome (see e.g. MONK, 1994).

Two books, incidentally published in the same year, have been particularly influential in pointing to East Asia as a source of inspiration when it comes to this role. The first of these is Ma's (1999) comparison of Chinese and American teachers' grasp of elementary school mathematics and its teaching. Despite the longer and more academic background of the American primary school mathematics teachers, their knowledge is decisively inferior to that of their Chinese counterparts. The most famous example from the study is about division of fractions. Only 4% of the American teachers were able to come up with a correct story or model that would illustrate the division 1¾ ÷ ½, against 90% of the Chinese teachers; and while all Chinese teachers knew at least how to carry out this division, only 52% of the Americans succeeded with this. Ma credits the advantage of the Chinese teachers to the more practice oriented and school based training that they get not only prior to teaching but also all along their career as teachers.

A very similar point was made by Stigler and Hiebert (1999) in order to explain certain striking findings from the TIMSS video study. When analysing a large number of Japanese, German and American mathematics lessons (grade 8), it became clear that Japanese lessons were systematically different and in several ways superior to those observed in Germany and the USA. The authors explain this by the fact that Japanese teachers participate in school-based professional development all along their career, with an activity called "lesson study" as one of the most central components.

Lesson Study in Mathematics: the case of Japan

It is important to understand that "lesson study" did not arise as an invention by educators or other persons outside of the involved school institutions. In fact, lesson study in Japan evolved from collective teacher practices at schools over more than a century (ISODA, 2007). Despite this "bottom up" history, or perhaps because of it, lesson study is a very widespread practice in Japanese schools, both in

mathematics and in other subjects (LEWIS, 2002). It should also be noted that lesson study in Japan exists in many variations, some of which have their own terms in Japanese (e.g. MIYAKAWA; WINSLØW, 2019). Teacher students meet lesson study both in their initial education (e.g. ELIPANE, 2012), in induction programmes (PADILLA; RILEY, 2003) and as a common form of in-service development (LEWIS; YOSHIDA, 2004).

The literature offers many detailed descriptions of lesson study, written to introduce non-Japanese teachers to the practice (e.g. STEPANEK et al., 2007). Some of these reflect adaptations to educational systems outside Japan, and include details which are not always (or never) present in Japan. Here we shall give a more minimal description of what is included in any form of lesson study:

1. One or more teachers formulate a teaching problem which may be meaningfully investigated with the frame of a lesson, although it may also have a more general form. For example, "how to help students discover how to solve quadratic equations" could be a specific instance of a wider goal (or problem), "to let students experience new mathematical methods as meaningful and justified continations of what they already know".

2. The teacher(s) elaborate a written lesson plan, in which the problem is formulated along with considerations of experiences from previous teaching, to motivate the actual "script" for a lesson, which specifies not only what the teacher does (including task(s) given to the students to work on, time plan for the lesson) but also concrete hypotheses about students' work in the lesson.

3. A group of teachers observe the research lesson, taught by one of those who were involved in crafting the lesson plan. Their focus is on recording details of student work that can help elucidating how and to what extent the hypotheses are realised, if students develop strategies not foreseen, etc. - and, above all, how the lesson sheds light on the teaching problem. The observation takes place in the classroom, not via video or other forms of "recording". Among the observers, some are "external" in the sense of not having participated in the planning. At least one of the external observers is an experienced lesson study participant and takes on the role of "knowledgeable other" (see 4. and TAKAHASHI, 2014).

4. The teacher of the research lesson, and the observers, meet to share observations and reflect on them. One of the observers directs the meeting, which generally lasts about an hour. The teacher shares his impressions first. Other observers join in with observations, questions and remarks, which are focused on

some element (problem, hypotheses etc.) from the lesson plan, supported by observations from the research lesson. The knowledgeable other concludes the meeting by summarizing important remarks and putting them in perspective.

It is crucial that it is the lesson which is observed, not the teacher and the students as individuals. The goal is for all participants to learn from practice about elements of answers to the problem posed in the lesson plan, based on sharing observations and experiences.

In many cases, some form of report is drafted by the planning group, to sum up what has been learnt. Here, the contribution of the knowledgeable other may be important. In Japan, such reports can take on forms like "papers" in professional journals, parts of a book for teachers etc., which allow the outcome of the lesson study to be shared more widely with colleagues who have not participated in the activity. In fact, texts resulting from lesson study are important to the development of Japanese teachers' shared professional knowledge, not only by contributing to an extensive professional literature, but also as input to the drafting of new textbooks, curriculum revisions etc. These are then, unlike what is the case in many other countries, the result of documented practice experiments carried out by teachers in schools, rather than derived merely from theoretical assumptions of education researchers or policy makers.

Lesson study also have some particular affordances and caracteritics in the special case of mathematics. In many other countries - such as those involved in the TIMSS lesson study (STIGLER et al., 1999) - mathematics lessons often take the form of student work with drill type exercises, for which the students have been given a method beforehand. Japanese mathematics lessons are, on the other hand, typically centred on students' solving a single, challenging problem, for which they can and must develop one or (frequently) several methods. A substantial part of the lesson is spent on students presenting, comparing and assessing methods developed by students. Such lessons are interesting to observe, unlike lessons where students are merely expected to use known methods on routine tasks. Research lessons are typically based on a challenging problem, and with a large space offered for students to craft, formulate and validate strategies of solution, in order hypotheses about student strategies to be real hypotheses, that can materialize or not, along with student strategies not foreseen (which are perhaps the most important sources of learning for all participants).

The experiences gained from observing such lessons also inspire teachers to teach ordinary lessons which are more ambitious in terms of student work. Participation in lesson study, therefore, has a considerable impact on mathematics teaching in general: more focus on students' independent development, communication, and justification of solutions. Thus, lesson study, and teaching mathematics through problem solving, are "two whells of a chart" (FUJII, 2018).

Lesson Study in Mathematics outside of Japan

Following the first widely published presentations cited above, lesson study has been implemented, in a variety of forms, outside Japan. The United States was probably the first major scene for such implemetations, organised by funded projects such as the Chicago Lesson Study Alliance and the Lesson Study Centre at Mills College in California (e.g. LEWIS, 2002), and involving hundreds of schools accross the country.

Japanese mathematics teachers and scholars have been crucially involved both in these efforts and in development projects carried out in other countries, particular in South-East Asia and South America, often funded as development projects under ASEAN or UNESCO (e.g. BALDIN; ISODA; OLFOS; ESTRELLA, 2018).

Accounts and analyses of recent implementations are provided, for example, in Quaresma et al. (2018), covering experiences from Malaysia, Ireland, Portugal, Chile and Denmark. Other papers and books offer well published experiments with lesson study in a large number of further countries around the world, including Brazil (BADLIN et al., 2018; DE MACEDO; BELLEMAIN; WINSLØW, 2019). In fact, the "International Journal for Lesson and Learning Studies" was founded in 2012 and is fully devoted to sharing such experiences.

Of course, there are many variations in the way lesson study has been introduced and implemented in all of these countries. While a more in-depth and comprehensive analysis of these variations could certainly be worthwhile, there are also rather evident commonalities. I will mention three that I consider important.

First, lesson study is as a matter of fact introduced and implemented by agents from outside of the school, in contrast to being developed by the teaching profession itself, like in Japan. In most countries the initiative has come from researchers and teacher educators employed at tertiary education institutions. Given the nature of these institutions, it is not surprising that lesson study has very often been experimented to a form of practicum in teacher education, or as integrated in in-

service courses offered by tertiary institutions (for example, Rasmussen, 2018). In other cases, lesson study experiments have been implemented as part of research projects including doctoral projects (for example BAHN, 2018).

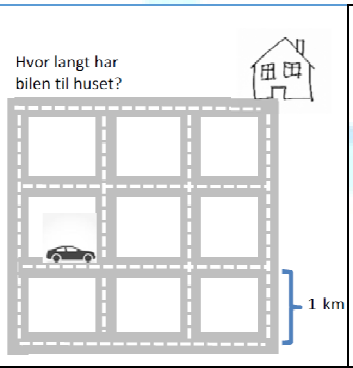
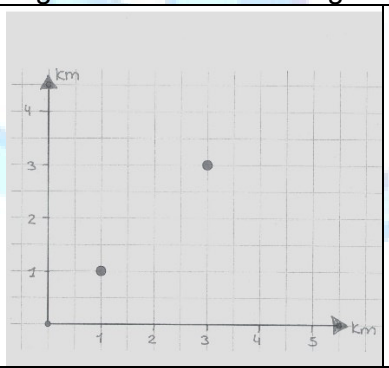
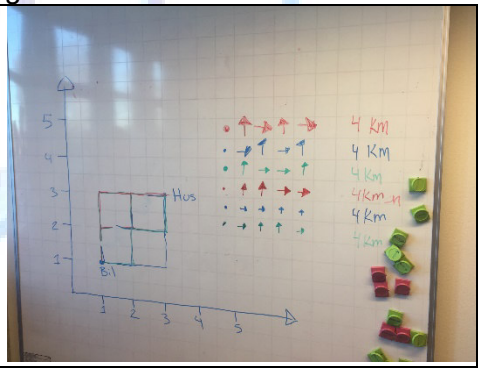
Secondly, and in spite of this external origin, teachers in Western countries tend to view lesson study as a form of emancipation of the profession, which allows teachers to recover some of the autonomy that bureaucratic regulations, educational experts and political agendas seem to take away. This viewpoint is forcefully expressed by Stigler et al. (1999, p. 174) who talk of how the teaching profession in the US has been "robbed [...] of the opportunity to participate in the development of new knowledge about teaching". The general impression from reports on lesson study in Western countries is that this format is received, by many teachers, with great enthousiasm, but also that many difficulties arise, especially to carry on with the activity without external support. We return to this problem in the last section.

Thirdly, one specific reason that lesson study in mathematics appears to be particularly attractive to both teachers and university professionals, is that this format appears to focus attention on students' mathematical productions from lesson observed, and that this focus enables complementary reflections from both teachers and university faculty, whether they specialize in mathematics, didactics or education. I will close this section by outlining an example from my own experience that illustrates this point (more details on this example are given in ØSTERGAARD; WINSLØW, 2018, to appear).

For some years, I have been occasionally invited as a "knowledgeable other" to lesson studies at schools in and around Copenhagen. One recent grade 5 research lesson was built around what the teachers considered a problem related to coordinate systems. Considering the map shown in Figure 1, the students were first asked to find the road trip distance from the car to the house, and then to find all the possible routes of minimal distance that the car can take. The teachers had themselves found a total of six such routes and now wanted to see if the students could do the same. It turned out that most students could not - only group of students found exactly six correct routes. In Figure 3, we see the teacher's whiteboard notes from this groups' presentation. The arrow notation was invented by the students and used to systematically count different routes, such as up-right-up-right (the first in Figure 4). With this notation they are able to find all possible routes (each with two "up" and two "right", arranged in different order. The other students fail because they draw the routes on top of each other and loose track of what is possible.

As an "knowledgeable other", I first summarized the many interesting observations of student stragies, and on how various materials for the lesson could be improved. For instance, the handout shown in Figure 2 misled many students as the grid seems to "show" twice as many roads as Figure 1. I finally provided a perspective on the lesson from the point of view of mathematics: the second problem is a "counting" problem, related more to the field of combinatorics that to cartesian geometry. In such problems it is essential to device a model of the problem which allows for systematic counting. Moreover, the students' that the minimal routes are in one-to-one correspondence with symbolstrings of type URRU (with two U's and two R's) can be used to put this problem in a wider mathematical context. In fact, all that matters is the choice of two out of the 4 places (where we write "U"). Thus, the number of possibilities is K(4,2) = 4!/(2!2!) = 6 and we can also easily see how to find the number of minimal paths between other positions in the grid. Of course, combinatorial combinations and binomial coefficients are not taught in grade 5 and the teachers may not even remember this material from upper secondary school. Nevertheless, the wider mathematical perspective on the problem (and its generalization) is highly relevant to the teachers' research lesson, as they would certainly want to know that their own six paths were in fact the only ones possible - and be able to consider variations of the problem without having to draw ad-hoc lists of paths every time, as they did in their own preparation. Moreover, the mathematical elucidation of the problem might inspire research lessons for higher grades.

Figure 1 - Lessons for higher grades

		
How far from the car to the house?	A handout for students.	Student work on counting paths.

Source: produced by author

In fact, lesson study provides an excellent interface between academics and teachers and help to close the gap between universities and school which is perhaps more acute in the West than in Japan, in part due to lesson study. Both bring complementary ressources to the common aim of learning from observing the

reserach lesson: the teachers, their professional experience from mathematics classrooms. University faculty can hep putting observations in perspective and link particular observations to more general (mathematical, educational) knowledge which, when communicated in other situations (e.g. "courses") that do not include a concrete classroom context, is usually much less helpful by teachers.

The sustainability question and paradidactic infrastructure

From the time of the very first experiments with lesson study outside of Japan, in the 1990's, a crucial problem has become evident (cf. LEWIS, 2002): unlike what is the case in Japan, the normal conditions of teachers' work in schools outside Japan do not seem to enable them to engage in lesson study beyond proects, as a normal part of their job. For the experiments, the normal conditions do not apply, since external support (financial as well as intellectual) is provided from outside the school. Almost all experiments reported on in the literature show teachers' enthousiasm and real benefits from the activity. However, when the external support ceases - and it is always limited in time - it becomes difficult or even impossible for the teachers to continue doing lesson study. In recent years, this has been formulated as the "sustainabillity problem" of lesson study (QUARESMA et al., 2018; ØSTERGAARD et al., to appear).

Already Fernandez and Yoshida (2004) pointed out that lesson study in Japan must be understood as part of a "system" for professional development of teachers. We already noted above that lesson study itself exhibits many variations and is met by Japanese teachers already during teacher education. Moreover, young teachers will always have the opportunity to engage in lesson study with more experienced teachers, as the practice is well established in all schools. But lesson study is also related to the existence of other conditions in Japan, such as the existence of a lively literature (including journals and commercially sold books) devoted to disseminate professional knowledge among teachers, including knowledge developed during lesson study. Even the textbooks and curricula used in schools reflect such practice (and lesson study) based knowledge, unlike the textbooks and curricula that are available to teachers outside of Japan. Teachers in Japan have many opportunities to attend "open lessons" in other schools (MIYAKAWA; WINSLØW, 2013), and lesson study reports and activities also appear at teacher conferences at many levels (municipal, regional and even national). There is a sufficient number of people with experience from lesson study and in particular with serving as "knowledgeable other" (TAKAHASHI, 2014). So it is evident that many conditions which support lesson

study in Japan are simply not available, at the outset, in other countries. How can we understand the nature of these conditions?

In the anthropological theory of the didactic (CHEVALLARD, 1999) mathematics teaching is modeled in terms of institutions and praxeologies. Indeed, school institutions in any country offer a certain set of conditions and constraints that to a large extent determine the actions of the teacher. We can certainly perform fine-gradined descriptive studies of the praxeologies (the practice and discourse that unfold in mathematics classrooms) and we can compare them accross different school systems, societies and even cultures. However, we cannot understand the differences observed without taking into account higher levels of influence, that is, conditions and constraints which come not only from the school institution but also from outside (ARTIGUE; WINSLØW, 2010; WINSLØW, 2012).

The key to understand the conditions and constraints for teachers' work - including the conditions and constraints for carrying out lesson study - is to realize that this work is to a large extent "paradidactic", that is, carried out in other contexts that the didactic (classroom) context itself (WINSLØW, 2012). We can, in fact, talk about a "paradidactic infrastructure" for teachers' work, consisting of all the material, intellectual and institutional conditions that determine the work that teachers do when they are not teaching: preparing for lessons, choosing textbooks, meeting with other teachers, and so on. Modeling paradidactic infrastructures explicitly, and analysing their impact on teachers' work, is a very recent development in the anthropological theory, and it has especially been applied to study the sustainability question for lesson study (WINSLØW, 2012; MIYAKAWA et al*.,* 2019; ØSTERGAARD et al., to appear). We anticipate that this direction of research holds large potential for going beyond naïve positions (whether protagonist or sceptical) towards the lesson study phenomenon outside of Japan.

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