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Dialogic Learning as an Alternative Approach for Mathematics Classrooms

A Aprendizagem Dialógica como Estratégia Alternativa para Aulas de Matemática

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ABSTRACT

This article characterizes the Dialogic Learning (DL) approach, presenting its constitutive elements using the literature related to the studies that have dealt with this theme in the specific context of practices and learning in math classes. In this way, the conception of dialogue that is practiced lies in the interaction between educator and learner through written language. The text also reports and shows two illustrative records that are used to establish a discussion about the importance of the activity and to emphasize how significant would be to disseminate the methodology amid math teachers at all educational levels. The DL approach aggregates important dimensions of communication and interaction between participants that are necessary to construct a differentiated idea about making mathematics, replacing the restrictive image disseminated over time by the classic lecture classes.

KEYWORDS: Dialogic Learning. Written records. Methodologies.

RESUMO

Este artigo traz uma caracterização da Aprendizagem Dialógica (AD), por meio da apresentação de seus elementos constitutivos e da literatura relacionada aos estudos que tratam do tema no contexto das práticas e da aprendizagem em salas de aula de Matemática. Nesse sentido, a concepção de diálogo praticada fundamenta-se na interação entre educador e educando, por meio da linguagem escrita. O texto ainda relata e mostra dois exemplos que são usados para estabelecer uma discussão sobre a importância da atividade dialógica e, ao mesmo tempo, para enfatizar sua importância para a disseminação como uma estratégia instrucional entre professores e futuros professores de Matemática em todos os níveis educacionais. A AD agrega, em suas práticas, as dimensões essenciais da comunicação e da interação entre os participantes, que são fundamentais para a construção de uma ideia diferenciada do fazer Matemática em contrapartida à imagem restritiva e disseminada, através dos tempos pela clássica aula expositiva.

PALAVRAS-CHAVE: Aprendizagem Dialógica. Registros Escritos. Metodologias.

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Introduction

Mathematics educators have dedicated themselves to develop methodological alternatives to promote classroom spaces that foster discussions, exchange of information, and mathematical experiences in which students and teachers can interact and engage collaboratively in a mathematical investigation process. As a consequence, they expect to be able to see better learning outcomes in their classes, which are related not only to the content covered but to remarkable experiences in the practice of mathematics that will hopefully not be forgotten after the evaluations.

These teachers expect, also, that these exploratory approaches will bring meaning to the knowledge acquired, helping students to pursue successful learnings, and, at the same time, to acquire new knowledge and the mathematical foundation that is necessary for their subsequent studies, creating and increasing their interest in mathematics or related topics, as well as knowing how to use mathematics in everyday situations.

The so-called didactic-pedagogical strategies based on dialogic and investigatory learning may be considered as possibilities to these educators' aspirations. This paper focuses on these strategies which, generally, are a set of alternative and complementary actions to the lecture classes of mathematics. By applying them, it is aimed to make it possible the formation of mathematical investigative spaces, facilitating interaction between students and teachers characterized by the development of an exploratory attitude, critical thinking, creativity, and communication (LUTZ-WESTPHAL, 2019; DÖRR; LUTZ-WESTPHAL, 2020).

This text addresses some activities in the mathematics classroom that are based on the concept of dialogic learning according to Ruf and Gallin (GALLIN, 2012). This approach is originated in the work of interdisciplinary cooperation and didactic-pedagogical dialogue between a mathematics teacher and a German language teacher, namely the Swiss teachers and scientists Peter Gallin and Urs Ruf. As a result of their dialogic and exploratory interaction, a peculiar methodology has been developed which is based on a truthfully and inquiringly dialogue and is shown as a counterpoint to a classical instructional mathematics class. Among its most remarkable aspects is the advantage that its practical application could be implemented at any educational level and it is adaptable to different areas of knowledge (RUF; KELLER; WINTER, 2008).

In this context of instructional practice, dialogue and investigation are simultaneously established in the classroom using exploratory tasks that could instigate the students' scientific curiosity. These tasks should induce them to research, examination and study from an initial question that will trigger the so-called mathematical research dialogue. Thus, dialogue and research should be expected to occur simultaneously during the learning process. With this in mind, we will often mention only dialogic learning in this paper. However, we reinforce that, implicitly, we consider establishing a correlation of dependency between mathematical dialogue and an environment of exploratory activities.

When using dialogic and investigative learning in classes, the written records with the responses to students' tasks are essential elements, as well as the reactions and considerations of the educators, that is, their written feedback to the students' responses (GALLIN; RUF, 2011).

Behind this working mechanism lies the unique opportunity given to teachers to foster the students' interest not only in responding to tasks but also in formulating questions, creating exploratory mathematical situations, making discoveries, verifying patterns, formulating conclusions, communicating and exchanging mathematical ideas with their colleagues, among others. In other words, as Lutz-Westphal (2019) affirms, this investigative movement helps to create a cycle of actions based on insights, discoveries, and the formation of conclusions, aiming for more authentic teaching and learning of mathematics.

Therefore, considering the importance of the dialogic methodology as a pedagogical approach that can promote mathematical learning, this article has two main objectives. Firstly, it aims to present the theory related to the Dialogic Learning approach. Secondly, it shows and discusses some reactions of teachers and future teachers when they are challenged to perform a typical initial task of this didactic concept using the topic Function as mathematical subject. Most of them have not had previous contact with the theory. To illustrate their reactions, two written records are given that were selected from a group of participants who were brought together during an introductory practical workshop on the topic.

To address these objectives, the following sections characterize the Dialogic Learning (DL) approach, presenting its constitutive elements using the literature related to the studies that have dealt with this theme in the specific context of practices and learning in math classes. Then, the research methodology is put together with the presentation of some illustrative records. Hence, from the

methodological point of view, the investigation combines the bibliographical research to a qualitative method to describe, interpret and illustrate an experience associated to the DL approach (TRIVINÕS, 1987).

Finally, this text concludes with considerations about the activity carried out, emphasizing how important it is to disseminate the methodology among math teachers at all educational levels, and proposing some points related to themes that may guide future discussions and researches.

Dialogic Learning with Emphasis on Written Records

Today, multiple methodological strategies are available to educators through different media, with emphasis on the use of technologies (CLARK-WILSON; ROBUTTI; THOMAS, 2020) and the so-called active methodologies (DÖRR; LUTZ-WESTPHAL, 2020; NEVES; LOPES; NASCIMENTO, 2020). These possibilities point to discussions related to the choice of the most appropriate for each group. In this case, the availability of resources and materials, the teacher's skills and interests, and the students' needs should be considered. Schoenfeld (2014) expands the debate on this topic by considering that this decision-making is still linked to the teacher's inclinations and beliefs and the established educational objectives, as long as they meet and are in line with the previous elements.

Considering its historical development, the DL's methodological perspective has some influences of Discovery Learning inspired by the pragmatic ideas of the epistemologist John Dewey that have been passed on in the educational context. According to his conceptions, we only think when we face problems. Thus, the educational process must be active and constructive. This approach suggested that curricula should consider the students' interests and they should be conducted through practical and interdisciplinary activities. For Dewey, the main education problem lies in the experiences that could develop creativity and be useful in subsequent events, that is, the educational process would be determined by the construction and reconstruction of significant experiences (DEWEY, 1938; ORD, 2012; LESSANI et al., 2017). In this way, Ord (2012) puts it:

> Experience for Dewey is our 'lived' experience. The experience at the heart of experiential learning therefore is not something separate or additional but something which embraces the lives of individuals. (ORD, 2012, p.61)

The emphasis on students' practical experiences is one of the propositions of the pedagogical approaches associated with constructivism. Constructivism proposes to put learners as active subjects in the construction of their knowledge,

both individually and socially. One of its premises is that all knowledge is constructed, thus, in part, mathematical knowledge can be significantly built with a process of reflection and abstraction (NODDINGS, 1990).

In constructivism, the active construction of new information in the mental structures of learners who cooperate for significant learning could be promoted by the use of open-ended questions that provoke reflections and discussions among peers. To this end, group activities are used, fostering the autonomy and initiative of those involved. This methodological path does not separate knowledge construction and discoveries, meaning that they should advance together (HANLEY, 1994).

Knowledge construction through discoveries and investigation are such interconnected phenomena that they often seem to coincide. The use of problem-solving is one of the important mechanisms for reaching them. So that applying it to school practice, we can consider mathematical investigation and problem solving as indissoluble processes (SCHOENFELD; KILPATRICK, 2013).

In George Polya's words, we have: “a great discovery solves a great problem, but there is a grain of discovery in the solution of any problem” (POLYA, V, 1957). In his classic work on problem-solving, Polya (1978) established four fundamental steps that guide the process of solving mathematical problems. They are: Understanding the proposed problem; formulating a resolution plan; performing this plan; and, finally, acting towards making a retrospective of the resolution. These steps contribute to the mathematical investigation to occur and bring gains for the students’ learning and discoveries.

The educators’ mediating role is one of the characteristic components of investigative learning. In this context, their actions should encourage situations in which learners are engaged and knowledge discovery is stimulated. In math classes, this needs to occur so that the process includes interactions between all subjects with a focus on the mathematical dialogue (TAPLIN, 2006) to prevent the teacher’s speech prevailing.

Generally speaking, we are talking about a math dialogue that occurs like an exchange of ideas and opinions on a math topic between two or more people. This math dialogue can be constructed in the classroom by creating discussion spaces within the group. The group is expected to talk about mathematics to the same extent that they hear or write about it. In this environment, students are encouraged to ask and answer questions proposed by the teacher or by colleagues. In short, it is a place with different forms of expressions and conversations about mathematics

(COMMITTEE ON THE TEACHING OF UNDERGRADUATE MATHEMATICS, 1979).

The word “dialogue” in its strict meaning refers to orality and verbal communication between two or more individuals. This dialogic action, inherent and fundamental in human and school relations, can be developed in classes using different strategies, such as the posing of questions or group discussions.

The oral dialogue is an essential element for teaching and learning mathematics, as it helps to formulate thoughts and to consolidate, clarify and elaborate the students’ ideas in a more significant manner (MARINO, 2005). However, to organize an effective opportunity for discussions about mathematics stands as one of the challenges connected with the teaching activity (SCHOENFELD, 2019).

The students’ written expression, pointing out their ideas and findings, is an important way of promoting dialogue in mathematics classroom practices. As a common factor, studies emphasizing this idea show positive implications brought for the development of mathematical thinking and learning from its use. Wille (2017), for example, reports experiences with the construction of imaginary dialogues with students in both basic and higher education. In this exercise, students were given the individual task to create an imaginary dialogue between two protagonists who discussed a mathematical activity or question.

Among the possibilities for mathematical teaching and learning that are seen in the use of math dialogues as a didactic-pedagogical support strategy, the study of Wille (2017) highlights that, initially, dialogues are capable of connecting the spoken and written mathematical communications in such a way that they make it possible to understand those thoughts processes associated with the development of both written and oral aspects of students’ mathematical language. In addition to that, they can also provide the student with an alternative of expression in an individual, open, and honest way in which he does not feel threatened by making mistakes. Finally, written dialogues are like open spaces for the written presentation of attempts and alternatives for solving mathematical problems that may be modified throughout the process. In the dialogic learning approach presented in the following, conversations between educator and student will be done with notes so that the written dialogues are established as a central methodological element for the learning process. Through the students’ writings, you can expect the occurrence of an investigative communication, which will allow teachers to know the thoughts, emotions, and other

expressions of the students' feelings, all of which are related to the mathematical theme under study (DÖRR; LUTZ-WESTPHAL, 2020; GALLIN *et al.*, 2008).

This additional instrument for teaching mathematics should be considered as we usually find no room for everyone to participate in conventional oral dialogues in classrooms. Thus, the use of writing promotes the participation and individual expression of all members of the group. This will result in diverse papers full of expression of ideas, visions, discoveries, and creations from the learners. These are subjective moments that will provide the teacher with a more precious and reliable overview of the students' mathematical knowledge construction (DÖRR; LUTZ-WESTPHAL, 2020; GALLIN *et al.*, 2008; WILLIE, 2017).

Taking into account the importance of the Dialogic Learning approach for math classes, the next section presents the theoretical framework related to it.

Dialogic Learning and its Components

The Dialogic Learning (DL) approach discussed in this paper has been developed and propagated by initial studies of Ruf and Gallin (1998a,1998b), Gallin (2010), and Gallin and Ruf (2011). Considered as a didactic-pedagogical conception that emerges from joint actions between the classroom practices and investigation, it has the dialogue between the educator and the students as one of the main components that constitutes the class.

Thus, the conception of dialogue that is practiced lies in the interaction between educator and learner through written language (GALLIN, 2012). When presenting writing as an effective learning strategy, Emig (1977) defends that this is a unique modality of knowledge acquisition in which process and product add a set of unique attributes at the same time. Among these specificities, what stands out is the fact that the written result is a visible graphic product. Or, as it involves a final product, the written work tends to be more conscious and more committed when compared to speech. Moreover, it is an active, individual, and customized engagement activity for participants.

In this methodological context, while teachers conduct their activities considering the curricular guidelines and peculiarities of their subject, one of the main goals of their classroom work is to transform their specific knowledge in the area, skills, and competencies in tasks that instigate participation, involvement and promote the awakening of students' scientific interest.

In turn, students should concentrate their efforts on the most realistic and complete documentation of the activities. So, while implementing this pedagogical

model, the idea is that a movement should be led to evaluate the learning that will generate successful knowledge acquisitions. Moreover, using the Self-Determination Theory formulated by Deci and Ryan (2000) as a theoretical basis, Gallin and Ruf (2011) indicate that the DL model should promote students' development in three levels of fundamental and motivational experiences:

* the experience of autonomy in which the student is becoming independent in his studies;

* the experience for social engagement in which the student feels heard by colleagues;

* the experience of competence, in which the student experiences progress in learning.

In this didactic and dialogic learning conception in which the organization and planning of the class follow certain definite assumptions, teachers must be aware of their students' capabilities and, conversely, that students are capable of valuing their teachers' potential so that effective communication occurs. The classroom practice is associated with another assumption which concerns the valorization of all students' written productions for the tasks, considering their abilities to express themselves and their previous knowledge (GALLIN; RUF, 2011).

To implement this learning conception in the classroom, a cycle of four basic actions is considered, starting with the development of a core idea that will be the initial focus to develop activities and that should be in line with the rules, norms, and peculiarities of the mathematical content to be addressed. Then, based on this core idea, the teacher introduces a task in the form of a challenging, provocative question, in agreement with the content being studied, which can attract interest and curiosity and also lead to discussion and investigation. In this step, the teacher acts as a mediator of the activities.

The core ideas embedded in the tasks are linked to the motivating elements that drive the students' interests, as well as to individual and investigative work. In short, according to Ruf and Gallin

> Core ideas have to be phrased in such a way that they arouse questions in the singular world of the student, which in turn direct attention to a certain subject area of the lesson. (RUF; GALLIN, 1998c, p.37, apud GALLIN, 2012, p. 6)

The core idea becomes a task and, therefore, the next practical step is that each student constructs what was named a journal. The journal may be, for example, a notebook with the written responses for the task and should include, in addition to

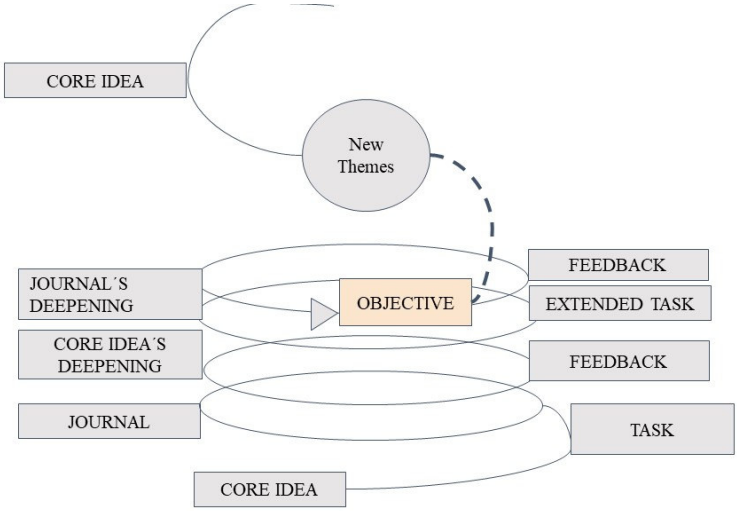
personal data, all written records of the student regarding activities proposed during the process with their name, the task's statement, the responses, the questions that arose during the resolution, among others. After completing this step, the teacher suggests that the students check their colleagues' responses, which can be done with a variety of modalities of individual or group activities.

After that, the teacher reads each journal, and the dialogue with the student is effectively put into practice. To activate the dialogues, teachers will make comments on the journal, using questions, observations, or suggestions of a more in-depth study, if necessary. They can also propose challenges that inspire students to look into the subject they are studying. In other words, the teachers' job is not only to correct the responses, but to analyze them, looking for understanding, clarification, and amplification of the acquired knowledge. Thus, the educator has the opportunity to bring other questions and try to extract from the written productions the "pearls", as the authors designated. These "pearls" can be both mistakes and successes, or even the mathematical discoveries that occurred during the learning process (GALLIN, 2019).

In the last step of the cycle, we have the feedback, which is when the teacher brings the discussion to the group after analyzing all the journals. At the end of this stage, the teacher will have the subsidies to make any necessary adjusts when exposing the content, after having verified by the written records the development of each student's core ideas, understanding, and learning.

Klimke and Lutz-Westphal (2018) represented the four instruments of Dialogic Learning through a spiral, namely, core idea, task, journal, and feedback. They say that this is because it is a better expression of the dynamism of the actions that resulted from the methodology (Figure 1). Their interpretation is based on the cycle representation of Gallin and Ruf (2011). This illustration shows that new core ideas may arise during the dialogic process and that this task can be reformulated, expanded, deepened, or modified. This representation also shows a dynamic movement around the core objective to be achieved. Furthermore, in this ascending path, it can be observed the cyclical stages of the learning process through dialogue and its advance toward a higher dimension of knowledge appropriation.

Figure 1 – DL component's



Source: adapted from Klimke and Lutz-Westphal, (2018, p. 2)

Report on an Initial Experience with Dialogic Learning

This section reports and illustrates a practical activity of Dialogic Learning constructed and implemented within a four-hour workshop on the topic, divided into two days, carried out in a Brazilian public university. Among the 15 participants, mathematics undergraduates and teachers from all levels of education were included. The activities were carried out by the author and Professor Brigitte-Lutz Westphal[2].

In the first part of the workshop, the methodology was presented from a practical and theoretical point of view. All participants had not had previous contact with the didactic concept of Dialogic Learning. Following, it was given a task to the participants to illustrate the approach.

The proposed task was individual and its initial question was: “What is a mathematical function? Write down your thoughts.” It contains an open-ended question, that is, whoever faces it will be free to respond in the way that they interpret it, as the statement has no indications of expectations or specificities in the response. The use of open-ended tasks in math classes has an intrinsic exploratory character (PEHKONEN, 1997).

The group was asked to put in their responses what they considered most important to know about functions. As everyone already had some previous knowledge of the mathematical theme, they were requested to try to answer the

[2] Professor Brigitte Lutz-Westphal coordenates the didactis of mathematics research group in the mathematics department at the Free University Berlin, Germany.

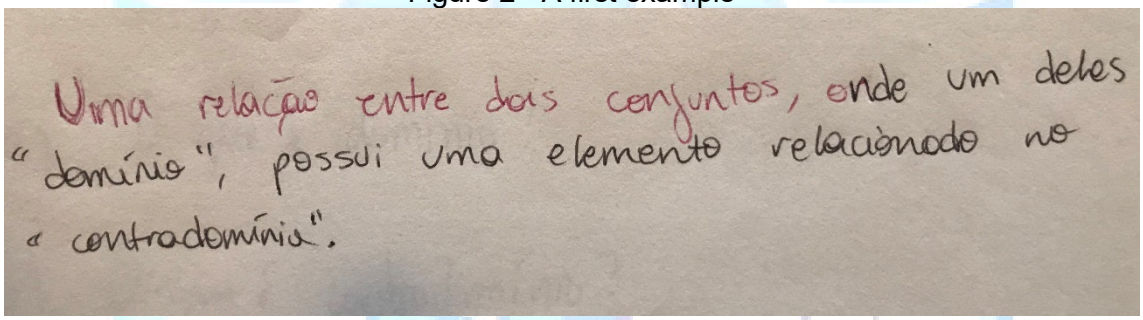
question in a way that they would rather formulate their explanations using words more than symbols or mathematical expressions.

This activity could be used, for example, to start a mathematical dialogue on the topic functions with students from the ninth grade, middle school, who in the previous year were introduced to linear functions and were currently starting to study quadratic functions. It can also be used with high school or college students as an instigator for further deepening in the study, or for an introduction of a specific type of function.

The following images (Figures 2 and 3) show two responses presented by members of the group of participants. The first was written by a future teacher. To the image from Figure 2 corresponds the following text:

> One relation among two sets, where one of them, the “domain”, has a related element in the “contra-domain”.

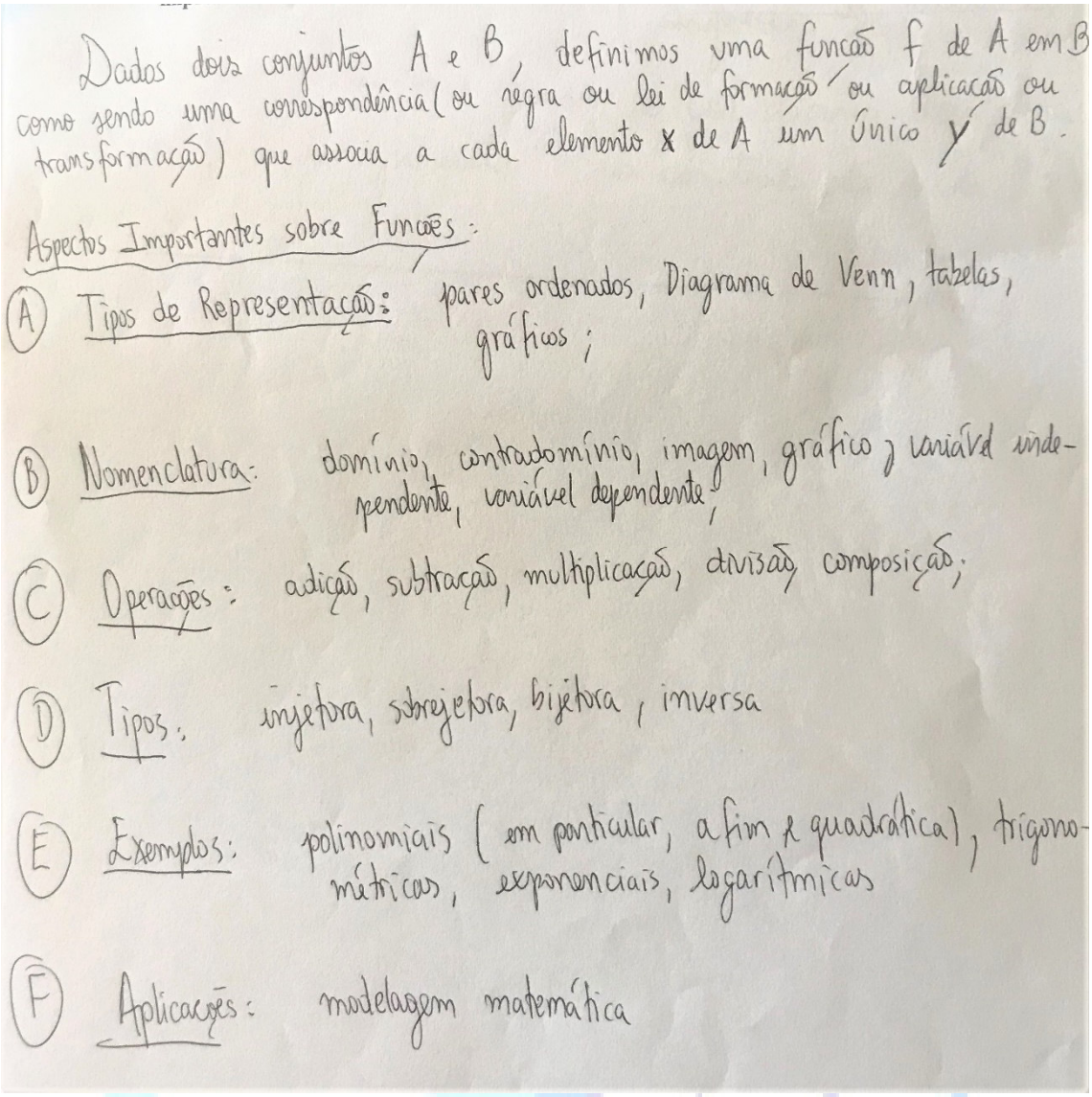
Figure 2 - A first example



Source: the author

As can be seen from Figure 2, this example shows us a concise, objective, direct, and limited answer. It is mostly formed by specific terms of the area, such as relation, sets, domain, and contra-domain, which only individuals who already know their meanings will understand.

Figure 3 - A second example



Source: the author

This response shows the teacher's commitment to transmit complete and well-elaborated information. It is written in an organized way, following a model of a didactic text, adding the pieces of information which are considered to be relevant on the topic, their formal definition, etc. One can see the existence of some experience with the topic and the writer also highlights examples and applications.

Afterward, all the participants' responses were placed on a table so that everyone could read them and choose two or three to make comments about to practice the beginning of a math dialogue. At this stage, they were requested to include in their responses, encouraging words and questions that would help with the initial written communication.

For these both illustrations, questions or comments that could be used are:

- What does a domain of a function mean?
- Could you show an example of a function?

- I like your explanation!

- I do not understand…

At this point, the activity should follow the same strategy in the classroom, that is, before giving feedback, the teacher gives the possibility to the students to see each other's responses, hiding their names (or not, depending on the group) and also comment on them. This part of the action could bring interesting discussions and group engagement.

Generally speaking, the exercise of proposing an open-ended question surprises the respondents, who at first need some time, each one according to their own pace, trying to find the best way to express their thoughts. Besides, this type of activity usually is not practiced in math classes. Considering this fact, along with individual characteristics and, sometimes, individual's limitations in their written expression, it would be recommended that teachers should prepare their students before carrying out the task and also motivate them to exercise their writing as a form of expression and sharing of ideas, thoughts, knowledge or mathematical discoveries.

The mathematical subject of Functions considered in this practice report is extensive and involves different possibilities to be addressed. For instance, we can represent a function by an expression, a table, or a graph. Thus, this initial question may be answered in many different ways. As the public was composed of teachers and future teachers, we noticed an inclination to formalism and to summarize ideas using more technical terminology.

Final Considerations and Prospects for Future Research

The Dialogic Learning approach as described in this text uses the written expression of students to promote different possibilities of interactions among participants in a way that enables dialogic communication. Among these interactions, we highlight that teachers and students become closer, students get more engaged in the process of mathematical investigation, and written communication is developed, providing students with freedom of expression. Consequently, it is expected that the activities could contribute to successful learnings (DÖRR; LUTZ-WESTPHAL, 2020; GALLIN *et al.*, 2008; WILLE, 2017).

In this context, applying this approach in the classroom will instigate questions about how evaluation processes would be developed, how long it would take to apply it in order to achieve the pre-established educational objectives, or also, how to prepare students for its implementation. All these subjects and others are seen as

productive research fields in the area (KLIMKE; LUTZ-WESTPHAL, 2018). These researches are directly linked to fostering the dissemination of this methodological strategy. Thus, we could consider it as a developing methodological approach and as long as it could be put into practice, more practices, new versions, and extensions may be implemented.

It is known that amid the various duties schools have, one of them would be to contribute to citizens' formation, enabling students to develop better understandings in their process of decision-making. Such requirements and expectations include understandings in the fields of abstraction, conceptualization, communication, or others that must be adapted to the contemporary teachers' work, who also has the challenge of being a facilitator, mediator, and support in sharing new knowledge (D'AMBROSIO, 2003).

The didactic-pedagogical support strategy of Dialogic Learning, namely, the use of writing in Mathematics, may certainly enable significant contributions to meet these demands. Such demands share the understanding that it is necessary to promote and encourage mathematical educators to create significant tasks that could connect math concepts to routine practical situations, to other areas of knowledge, and offer resources for mathematical discovery, discussion, and critical reflection (LUTZ-WESTPHAL, 2014; SKOVSMOSE, 2001, 2007).

Students' written productions could be crucial for the good development of activities because they could reveal a lot of content on their mathematical activity and their particular ways of interpreting statements. Moreover, the students' background can be verified concerning a specific mathematical topic, their learning advances, the questions they commonly have, etc. To do so, the teacher should encourage and guide students to write about their ideas, believe in their creative possibilities, and, finally, he or she could carefully analyze the written productions (GALLIN, 2012).

From the perspective of Dialogic Learning, mathematics is considered not only a topic of memorization but of investigation. It must be conducted in an environment that could enable the development of creativity, freedom of expression, and a willingness to learn.

No one methodology could be magical or transformative, but some have the power of changing teachers' thoughts, positioning, and attitudes regarding knowledge construction (D'AMBROSIO, 2003).

In short, the Dialogic Learning approach aggregates important dimensions of communication and interaction between participants that are necessary to construct

a differentiated idea about making mathematics, replacing the restrictive image disseminated over time by the classic lecture classes.

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