Algebra and its Grammar: Philosophical and Educational Reflections

A álgebra e sua Gramática: Reflexões Filosóficas e Educacionais

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ABSTRACT

In this theoretical essay, I intend to reflect on the philosophical foundations of algebra and its teaching possibility from the Epistemology of the use of Arley Moreno, debtor of Wittgenstein's philosophical therapy. Initially I bring up a discussion of how Wittgenstein changes his thinking, abandoning an idea of a rationale for understanding a grammar based on human praxis. I then discuss the linguistic nature of algebra, where I argue that it is a language game that has a grammar, in the Wittgensteinian mold, and point out consequences in teaching based on this understanding. From the idea that meaning is in use, I argue that language should be given greater prominence in teaching, as the use of algebraic language is what allows meaning to be obtained, and this is possible when the student takes possession of grammar immersed in the algebraic language game.

KEYWORDS: Algebra. Grammar. Wittgenstein. Teaching. Epistemology of Use.

RESUMO

Neste ensaio teórico, pretende-se realizar uma reflexão sobre os fundamentos filosóficos da álgebra e sua possibilidade de ensino a partir da Epistemologia do uso de Arley Moreno, devedora da terapia filosófica de Wittgenstein. Inicialmente, trago uma discussão sobre como Wittgenstein muda seu pensamento, abandonando a ideia de um fundamento lógico para a compreensão de uma gramática apoiada na práxis humana. Em seguida, discuto a natureza linguística da álgebra, na qual argumento que esta é um jogo de linguagem que possui uma gramática, nos moldes wittgensteinianos e aponto consequências no ensino a partir deste entendimento. A partir da ideia de que o significado está no uso, defendo que a linguagem deve tomar maior destaque no ensino, pois o uso da linguagem algébrica é que permite que se obtenha significado, e isso é possível quando o aluno se apropria da gramática e imerge no jogo de linguagem algébrico.

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PALAVRAS-CHAVE: Álgebra. Gramática. Wittgenstein. Ensino. Epistemology of Use.

Introduction

In this theoretical essay, I intend to reflect on the philosophical foundations of algebra and its teaching possibility based on Arley Moreno's Epistemology of Use (2005), indebted to Wittgenstein's philosophical therapy. The intention is to reflect on the linguistic nature of algebra, which was not discussed by Moreno or Wittgenstein, but which, in their philosophical approaches, indicate ways to understand human activities as language games and how they have a grammar that governs such practices.

Initially, I bring up a discussion about how Wittgenstein changes his thinking, abandoning an essentialist philosophy that had an ideal and unique language as a biunivocal reference of the facts of the world, having logic as its foundation. Wittgenstein comes to understand in his second philosophy that there is not just one language, but a diversity, which he calls language games, and no longer takes logic as a foundation, but each game presents a set of rules – a grammar – generated in the process history and accepted by the community.

Arley Moreno, with his Epistemology of Use, undertook efforts to realize what Wittgenstein did not, a theory of knowledge. The Austrian philosopher did not seek to make theories, but a therapy of existing theories, in which he sought to show the confusions of these theories by seeking semantic foundations and having language only as a reference. However, Moreno understands that it would be possible to extract a theory based on Wittgensteinian pragmatism in order to avoid dogmatism, such as foundationalist theories, and to understand that the meaning is in the use that is made in the various language games.

In this regard, a reflection on education based on Wittgenstein's therapy is possible. It is noticed that some authors have already been using Wittgenstein in educational discussions, such as Cristiane Gottschalk (2008), Antônio Miguel (2016), Gelsa Knijnik (2014), Marisa Silveira (2020) etc. Thus, in this text, I approach algebra, its linguistic nature and also a discussion of the consequences for the teaching of the concept taken. From the idea that meaning is in use, I argue that language should be given greater prominence. The use of algebraic language is what allows meaning to be obtained and the idea of an algebra as a mental/ideal/abstract product, regardless of language, leading to a teaching that comprises a potential knowledge present in the

student, which often does not occur. Language needs to take precedence and from there make relationships with other areas of mathematics and life, allowing the student to be immersed in the language game of algebra and, consequently, be introduced to its grammar.

From logic to grammar

Wittgenstein is known to have produced two philosophies. The first would be in his only published work, the Tractatus Logico-Philosophicus of 1921, and the second, in the work Philosophical Investigations, published posthumously in 1953. In the "Tractatus Logico-Philosophicus", Wittgenstein argued that there was an underlying logic that sustained/connected language in/to our world. In the transition phase (the 1930s), Wittgenstein begins to try to escape the referentialism of his first phase, in other words, the idea of a biunivocal connection, names to objects, propositions to facts, language to the world. Wittgenstein starts looking for a more structural context, which are called rule systems, which is what will lead him, in the future, to propose language games (AZIZE, 2004). The philosopher leaves a conception of the general criterion of signification and adopts a conception of the criteria of signification. According to Azize (2004), Wittgenstein moves from univocity to vagueness, from the idea of a proposition, as a figure (image, mirror) of reality, to an idea of an autonomous grammar, in which rules begin to stand out more in a use, in ordinary language, of what is supported by an ideal logical language, when we will see linguistic pragmatics revealing itself as a source of meanings in Wittgensteinian philosophy, in short, in the work Philosophical Investigations.

According to this new conception, the task of philosophy does not consist in correcting the ordinary use of language, but in understanding its functioning correctly, what we call philosophical therapy or even self-therapy, since already in the preface to the Philosophical Investigations, the "second" Wittgenstein, recognizes that there are "gross mistakes" in the "Tractatus Logico-Philosophicus", and changes the way of analyzing the problems of language, taking as a basis his first work, his old way of thinking, as a background for writing the Philosophical Investigations, in other words, this work can only be understood with the "Tractatus Logico-Philosophicus" as a starting point.

According to Schmitz (2004), Wittgenstein moves away from the "Tractatus Logico-Philosophicus" by changing two conceptions. Firstly, regarding the independence of elementary propositions, since he starts to admit that there are logical

relations between them and, thus, there would not be a one-to-one correspondence between propositions and facts, but the relation occurs between the systems of language and the systems of the world, no longer having a limited logic as a background, in other words, it is not limited to the truth functions presented in the table of aphorism 5.101 of the "Tractatus Logico-Philosophicus".

Wittgenstein thus abandons the referential conception of language. This conception is represented by the Augustinian conception of language, which Wittgenstein considers to have dominated philosophy from ancient times to modernity, reaching the "Tractatus Logico-Philosophicus". This conception is already presented in the first aphorism of the Philosophical Investigations, when Wittgenstein quotes a passage from chapter 8 of The Confessions of St. Augustine.

According to Wittgenstein, for Augustine, words name objects, and each word has a meaning, thus, first would come the designation of names, the nouns, and then the designation of activities, in a kind of labeling, or one-to-one correspondence between the propositions of language and the facts of the world. Wittgenstein is based on a criticism of Augustine's referentialist conception for two reasons: the first concerns the fact that Wittgenstein perceived in the Augustinian conception a portrait of what he himself had done in the "Tractatus Logico-Philosophicus" and, thus, he noticed that he had also been stuck to a conception that dominated all philosophy, but which he now perceived as a mistake.

The second reason refers to his own new philosophical project. Wittgenstein saw in the Augustinian conception the relationship between language and its use, in other words, Augustine believed that we learn language, and in this way we express our ideas, we describe facts, from this referential use of language, which is to label objects and facts. For the medieval philosopher, we learn language as children by pointing to an object and saying: "this is a chair", "this is a lamp", "this is a door" etc., when they saw a situation, they would tell us: "this is a war", "this is the love of a father for his son", and so we were learning. Wittgenstein recognizes that we learn through use, but that this process does not take place only in a referential way, in other words, language is not just a reference to the facts of the world.

The second change made by Wittgenstein was in the meaning of propositions. In the "Tractatus Logico-Philosophicus", the truth condition of the propositions depends on the truth functions of the elementary propositions. Wittgenstein realizes that, in order to understand a proposition, we do not resort to ultimate truths. The meaning of a proposition does not depend on its truth conditions, but on the "context in which it is

stated and the use made of it in that context" (SCHMITZ, 2004, p. 140). According to Moreno (2006), the concept of proposition is no longer an exact model of reality and is now understood as a hypothesis, which fits reality, which can be reformulated depending on the circumstances in which it is used.

The "first" Wittgenstein understood that meaning only shows itself, but cannot be said, so philosophical problems only arise because we would not understand the logic of our language. And it is, paradoxically, from this idea that Wittgenstein begins to understand that he was wrong. It is not logic that is at the bottom, but language itself (MORENO, 2004, p. 292) and we can understand a proposition by simply using it.

In the Philosophical Investigations, Wittgenstein deals with language no longer from a logical point of view, or rather, he considers it from another perspective, since it is no longer, purely and simply - in a single perspective -, which would account for everything, but it still exists in the internal relations of language. According to Moreno (2005, p. 246), "the conditions of meaning will be situated, after the 'Tractatus Logico-Philosophicus', in the uses of language and no longer in its logical form; the logical form itself will be considered one of the possible uses, coming to lose its old privilege".

Wittgenstein comes to understand that the search for an essence in language is doomed to failure, since there is no essence to be discovered. The second Wittgenstein is opposed to essentialism, or rather to the type of essentialism that he practiced, because, in a way, he still continues to believe in the existence of an essence, however, no longer a priori and no longer external to language, but an essence. essence that is constructed in language.

Wittgenstein does not abandon the fundamental conception of the "Tractatus", that "what we say makes sense in terms of a certain number of restrictions" (SCHMITZ, 2004, p. 146), however, the restriction to which Wittgenstein was referring was logic, in this way, there would only be meaning within a logical delimitation. Wittgenstein, however, comes to believe that such delimitation is the very context of human practice, and this is how he fails to understand the existence of just a single language, but the existence of several languages, whose understanding depends on the context and of rules, which he now considers as what regulates our doing and thinking, in other words, they fully take the place previously reserved for logic, but now, an essence that is at the basis of all and any activity is not assumed. Wittgenstein, then, replaces logic with what he now calls grammar.

> Here is a therapeutic result that will only be fully assimilated by Wittgenstein's own thought well after its formulation in the early 1930s

> - when he can finally say about grammar what he once said about logic: that it is not to be accountable against reality and, especially, that the relationships between language and reality are defined within language (MORENO, 2009, p. 161).

For Wittgenstein, there is superficial and deep grammar. The first is systematic or formal (WITTGENSTEIN, 1999, §572[2], §573, §664), it is the one that provides formal rules, in other words, that refer to facts, that focus on the immediately evident characteristics of expressions, to the detriment of its general use. Deep grammar (WITTGENSTEIN, 1999, §464) is its practical use in a given language, it is what provides the rules of use of language in its inner workings, it concerns the rules of use that cannot immediately reveal themselves in the form surface of our grammar. According to Silva (1996, p. 120), "when Wittgenstein speaks of deep grammar, he does not speak of a grammatical system (which would include rules to be transformed, etc.) [...] linguistic use always refers to a praxis". Wittgenstein was well aware that our grammar lacks clarity, and that philosophical problems arise from this (SILVA, 1996).

In the context of Wittgenstein's philosophy, grammar can be understood as the set of rules for language games. Therefore, Wittgenstein decides to use the term game, because games are characterized by the existence of rules. Thus, the philosopher moves away from a single conception of language and starts to speak in languages or language games, a variety of uses, which are techniques that we develop to manipulate language in different situations. Wittgenstein decides to call a language game "the totality formed by language and the activities with which it is intertwined" (WITTGENSTEIN, 1999, §7). These other activities would be a larger context, the forms of life, which condition the ways of playing games more broadly. Wittgenstein also adds that, among language games, there is not an essence, as he understood in the "Tractatus Logico-Philosophicus", but family resemblances, when he makes an analogy with the resemblances between members of the same family, who are not the same, however they have some similarities which are not fundamental characteristics in all games, but an a posteriori perception performed by a user inserted in several language games.

Wittgenstein understands that meaning is determined by use. However, Moreno (2006) warns that the notion of use does not have the function of foundation in the same way that the notion of reason has in traditional philosophy, since use is an

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[2] When citing Wittgenstein's works, I use the number of the aphorism from which the quotation is taken, as this philosopher wrote most of his works in this format. The aphorism will be indicated by the symbol "§", but some works are divided into parts, so in these I will indicate such parts, with Roman numerals, before the aphorism.

unstable foundation. The rules define the uses and we use the words according to the rules, and it is in this use that the meaning expands.

Language games contain grammars that provide the rules for their use, from which we infer that language becomes at some point freed from the need for external justifications. This shows us that grammar is autonomous. The second Wittgenstein no longer sees language only as a function of referring to a meaning external to it, but as the very source of meanings. In addition to being autonomous, grammar is arbitrary because, as Schmitz (2004) points out, we choose rules (just as we could have chosen others). For example, Wittgenstein compares a rule to a unit of measurement because it is chosen arbitrarily and we use it. However, the author makes it clear that life forms indicate possibilities.

Based on these discussions arising from Wittgenstein's therapy, the Brazilian philosopher Arley Moreno formulated an Epistemology of Use, in which he argues that grammar is autonomous and arbitrary and its meanings are not exterior, in other words, it denies any exteriority to language, whether it is ideal, mental or empirical, as the foundation of meaning.

The linguistic nature of algebra

Is mathematics a language, a language game, a grammar or a way of life? For Bouveresse (1987), Wittgenstein did not understand mathematics as a language, but as a preparation of one, and a simple system of rules cannot be considered a language, it can only be considered in its application. Mathematics, then, would be the opposite of a language, which is not about anything, and which can, in fact, apply to everything. Therefore, for Bouveresse (1987), mathematics is a system of rules, which becomes language only in its application.

On the other hand, Gottschalk (2004a) understands mathematics as a language game, or even a language, and even says, referring more closely to school mathematics, that it "can be seen as another image of the world, in the sense that it gives us conditions of meaning to our experience of a certain point of view" (GOTTSCHALK, 2004a, p. 5). In another article, Gottschalk (2004b, p. 323) understands that "mathematics is also one of our 'Grammars'" and also reveals that mathematical propositions constitute an image of the world. When referring to the episode of the slave Meno, Gottschalk (2007, p. 27) understands that he had access to another grammar. As for algebra, the same author understands it as a language game.

On the other hand, Glock (1998) understands arithmetic and geometry as systems of rules, but does not mention algebra.

I do not see a contradiction in these different definitions by authors who rely on Wittgenstein, as the philosopher did not want his concepts to define, but language games are criteria for comparison, in other words, we look at mathematics and perceive in the various activities linked to it the Wittgensteinian concepts.

Mathematics can be thought of as a set of related language games. In terms of mathematical activity, one can think of, as language games, the activities of substituting values in an equation, solving an algorithm, interpreting a problem, finding a point in the Cartesian plane, given its coordinates, etc., and others mentioned in the aphorism 23 of the Philosophical Investigations by Wittgenstein.

So it can be said that there are mathematical language games or that mathematics itself is a language game. Mathematics is a game, which has games, such as algebra, arithmetic or even any mathematical activities could be considered language games. But, in making a definition of this type, one runs the risk of dogmatization, seeking an ultimate foundation. There are no predefined limits on what a language game is, because mathematics can be considered a language game, with algebra in its interior, as well as within algebra itself considering different language games, such as games that involve equations or function, for example.

Wittgenstein clarifies this issue by making it very clear that he carries out a therapy and, therefore, dogmatic definitions are not necessary, but grammatical clarifications.

> Why does mathematics need a foundation?
>
> It needs one just as little, I think, as propositions dealing with physical objects or those dealing with sense impressions need an analysis.
>
> Although both mathematical propositions and others need clarification of their grammar[3] (WITTGENSTEIN, 1987, VII, §16, author's translation).

However, I choose to speak of the grammar of algebra, because, in the Wittgensteinian sense, grammar is the set of rules for the uses made of a word within a language game or even in different language games. It is necessary to clarify the grammar of algebra, in other words, its rules of application. Wittgenstein

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[3] "¿¡Para qué necesita la matemática una fundamentación?! La necesita tan poco, creo, como las prosicones que tratan de objetos físicos o las que tratan de impresiones de los sentidos, necesitan un análisis. Aunque si precisan, tanto las proposiciones matemáticas como las otras, de una clarificación de su gramática".

(WITTGENSTEIN, 2014, §108) tells us that arithmetic is not a game, but says that there are arithmetic games. He seems to give arguments to say that algebra has a grammar, when he says that arithmetic is not a game in the same way that the displacement of chess pieces is not a game, in other words, the rules make a game, a game. If we consider arithmetic as a set of rules, when we talk about the arithmetic game, we are already talking about its grammar.

As already said, there are grammar rules that govern language games, which are autonomous and arbitrary. It is in this regard that I understand the matter of essence, in other words, no longer dependent on factors external to language, but dependent on grammar, which is now considered a priori. "The essence is expressed in grammar" (WITTGENSTEIN, 1999, §371). However, such a need occurs in the use of language, denying the possibility of understanding before the contact with certain rules of grammar, by individuals in the learning process.

Our thesis is that algebra has a grammar, a set of rules on how to perform the calculation with other symbols, which often present similarities with the rules of arithmetic, geometry and everyday situations, but which has its own arbitrariness and autonomy, in addition to today having a meaning in itself, from the development of algebraic structures. With the different uses, the concept of algebra is expanded and activities are added to its concept. In this regard, any definition of algebra is a posteriori. Thus, by understanding algebra as a language game, and which, consequently, has a grammar, we are taking a reference criterion, as in Wittgenstein's therapy, in other words, to show that it has conventional nature and foundations.

I understand that there is a grammar of algebra, because its construction was based on arbitrariness and became autonomous, in other words, it potentially became a language with defined rules. As for its transmission/teaching, it takes place by rules, by insertion in uses, in algebraic language games.

Algebra may have been born from a development of arithmetic, but it gained its own characteristics of a grammar, which developed itself and then allowed that one could, today, see in algebraic rules supports for understanding in other areas of mathematics, as well as in practical experience itself.

For Wittgenstein, mathematical propositions are grammatical in nature. This understanding is fundamental to understand the positioning of Wittgenstein's philosophy of language on all aspects of mathematics, since it ranges from its nature to its pedagogical aspects. Algebra would not be generalization, but formalization, in other words, much more than a natural development in which letters came to represent

numbers and symbols represent more complex operations, algebra is part of a development of formalization of concepts, and it is from that, that it can be understood as having an autonomy. This is usually considered to be the development in the algebraic content of mathematics, as it is considered to be one that deals with abstraction and generalization, but which, in fact, is the most developed formalization of mathematics, which already brings a range of other developments, both in practice and in their self-development.

Through the Epistemology of Use, supported by the philosophy of language of the second Wittgenstein, it can be understood that there is no essence for algebra, because its meaning is in its use, however, it is valid to say that there are algebraic language games, which have a grammar, in other words, a set of rules that governs algebraic language games, and these have family resemblances to other language games (mathematics, logic, everyday life, etc.) and not an essence. In aphorism 23 of the Philosophical Investigations, Wittgenstein before citing some language games says:

> How many kinds of sentences are there? Affirmation, question and command perhaps? – There are innumerable such different kinds of employment of what we call "star sign", "words", "phrases". And this plurality is not fixed, a data that is unchangeable; but new types of language, new language games, as we might say, they are born and others grow old and are forgotten. (A picture of this can give us the math modifications.) (WITTGENSTEIN, 1999, §23)

Algebra makes such "new language games" possible, as it allowed the existence of some new ones from the various modifications that mathematics has experienced throughout history. Wittgenstein continues in aphorism 23: "The term 'language game' should here emphasize that speaking of language is a part of an activity or a way of life". The human way of living, Western or Eastern culture, the way of doing mathematics, the need to use symbols to abbreviate concepts, to facilitate calculations, promoted algebraic language games.

> The expression "language game" emphasizes the role that our ways of life play in the use of our words. Every language game involves a grammar of uses, which are anchored in a praxis, in a way of life. In this regard, the semantic link between language and reality is not only given by the rules that govern language, but by the language games themselves, since the rules only make sense against the background of a given language game. Therefore, language games take precedence over rules. With the concept of "language game" Wittgenstein clarifies how we assign meaning to our words. According to him, these only acquire meaning when we operate with them, therefore, within a language game, which would be for Wittgenstein,

> the totality formed by language and the activities with which it is intertwined (GOTTSCHALK, 2004b, 317-318).

It's not just the algebraic rules that determine what it means, but the usage in a language game that can be, for example, solving an equation. There are several mathematical and particularly algebraic language games, where rules of arithmetic do not apply to algebra, so it is at this point that the language game offers a delimitation.

By repeating some language games listed by Wittgenstein in aphorism 23, such as commanding and acting, according to commands, reporting an event, exposing a hypothesis and proving it, reading, singing a nursery rhyme, solving riddles, counting, solving an example of applied calculus, translating from one language to another, greeting, praying (and I add) "and so many others". Among these "many others" can be: calculating with letters, solving algebraic equations, modeling situations, perceiving functional relationships, etc. Or one can even use some of the ones already mentioned by Wittgenstein, because, in algebraic problems, one has to act according to commands, which could be formulas or translate from one language to another, if we understand that understanding expressions and formulas with letters or some types of exercises require a translation into the natural language or simply solve an example of applied calculus.

Algebra grammar in teaching

The grammar of algebra, like any grammar, is arbitrary, so it is rash to think of it as a reference to some supersensible knowledge and/or that such knowledge would exist in the student's mind. The notion of arbitrariness, highlighted by Moreno in his Epistemology of use, emphasizes that grammatical description is carried out through exemplification and through the relationship with symbolic activity. Based on Moreno (2005), I understand that the grammatical description or the description of the rules of algebra, from the exemplification is a possibility, however, that they are from the internal relations of meaning, because, in this case, the exemplification would provide the ideal context for transitions of meaning to appear.

Wittgenstein states that "Every sign by itself seems dead. What gives you life? In use it lives" (WITTGENSTEIN, 1999, §432), in other words, the symbol by itself has no meaning, it comes to have meaning in use. The algebraic language becomes meaningful when used in examples, applications, comparisons and, therefore, it cannot be said that it exists prior to its use, in the mind, in reality, etc. For Moreno (2005), the use of examples expands linguistic practice, since they will use different

instruments, such as analogies and situations. Thus, I understand that, in an attempt to solve learning difficulties, one must seek a path based on algebraic content and how it can be transmitted to the student.

The aspect of symbolic activity must be understood as a reflection on meaning, internal to grammatical symbolism and as a symbolic ritual. The internal meaning connections of language do not have causal explanations, but are realized in the symbolic use we make in different situations. Moreno (2005) understands that the symbolic object is autonomous and, from its creation and acceptance, it becomes independent. The symbol gains meaning due to its use, and its use begins to define its meaning and, thus, symbols are used to explain facts in the world, as formulas, equations and algebraic functions do in relation to many situations.

Algebra, considered as a formal language game, would be as arbitrary and autonomous as a non-formalizable language game, like an unwritten dialect or some human attitudes, such as gestures, that are understood in some context. In any language game, there are internal relations of meaning, which are only understood when one is immersed in the symbolic system, that is, in the case of algebra, there would be no way to expect understandings, without the presentation of the rules, without the use, without the immersion in algebraic syntax and its consequent uses.

The so-called areas that form mathematics, such as arithmetic, algebra and geometry, can be considered as containing a grammar, as they have their own set of rules, which enable language games and which, because they have conventional contents, could even be learned separately, but that are part of a broader grammar that is that of mathematics. When a student starts to learn arithmetic in basic education, they are being inserted into this grammar which, for them, is all the mathematics there is. Then they are introduced to geometry and then to algebra, which are different games. They are different grammars, with family similarities between their language games, but they need to have their limits of comparison, due to the particularities of each content. Over time, the concept of mathematics itself will be expanded and the similarities and connections made will allow more relationships to be built between the areas, making it appear that everything is part of the same grammar, the same game - as if there was an essence behind all mathematical content.

In fact, the very dimension of the game changes, it starts to encompass new rules. It would be okay to see mathematics in this way, as a single game – because I understand these areas as part of a grammar called mathematics –, but it is problematic to think that a student who has not yet had contact with other areas can,

for example, from only from arithmetic, intuit algebraic notions. Realist and idealist philosophical conceptions point in this direction. The concept of mathematics for a student who is in the 3rd grade of elementary school I is different from a student who is in the 9th grade. In 3rd grade they could boil math down to numbers, but in 9th grade they will be able to talk about many more objects. Over time, they will be able to see the contents already mastered as belonging to a single grammar. The essence is expressed in grammar.

Generally, it causes surprise to teachers when they realize that the student does not perform generalizations or abstractions in the expected manner. The problem is that the weight of this falls on the student and not on the teacher's teaching method, or when this occurs, cognitive theories are resorted to rather than language. The teacher's work must be about the algebraic language and its own structure of a game with its own rules, which does have similarities with other areas, but does not depend on them for understanding.

Algebra is public, it is a social consensus, and it has techniques for using signs that are usually letters, variables, unknowns, in operations, equations, functions, etc. Algebra demands skills specific to a language that we must know to master this area, and this does not involve a private mental process, but a linguistic one. Hebeche (2002) clarifies that grammatical illusions arise when the word representation or image is conceived as objects, which is the position of traditional metaphysics, which originates in the Augustinian conception of language. Thus, language would just be a medium that carries information about something that is not originally found in it. For Hacker (1990 apud HEBECHE, 2002, p. 199), the only criterion is externalization and thus, "a mental representation is not an internal sample that can be compared with reality".

So how is it possible to use certain rules that do not have an empirical application? We learn to follow the new rule, just as we learn to add new sums that we had not yet learned. I understand that for a student who has never seen a calculation with letters, a simple sum of the type "a + a" seems to him something strange and about which we cannot expect or demand an intuition from the student, because operating with letters requires new rules, of a new set of rules, therefore, of a new grammar.

There is no essence of language game, so it is the variety of uses that favors meaning. Accordingly, it is necessary to present the student with the various possibilities of using letters in mathematical content, for example, and not be expected to understand by some "natural rationality" that the use of a letter as a variable is

different from its use as an unknown or that the use made of an arithmetical sum is different from the algebraic sum. There is no essence behind the activities that can be considered algebraic and arithmetic, but only family resemblances.

Someone who knows only arithmetic will see algebraic equations and expressions as a tangle of letters written on the board or paper, so they need to be taught to see it another way, to see a 2nd degree equation that has certain roots as a solution, for example. This new way of seeing depends on mastering the rules of the new game presented, which are not obvious, deducible, or discovered, in general, but must be learned, as Gottschalk (2008, p. 81) shows:

> It is not a matter of discovering something that already exists in some way: there is nothing to be discovered before we have a method that allows us to search. The propositions of mathematics do not refer to something to be discovered, they do not have a descriptive function, but rather a paradigmatic one, in other words, they are seen by Wittgenstein as rules on how to proceed.

It is not possible to construct meanings only through explanations, but through use, in other words, it is no use just explaining how an equation is solved or a student cannot be expected to make abstract relationships or generalize arithmetic concepts, in an algebraic scope, if you have not yet been given opportunities to use this new language. It is in the use that a student begins to construct meanings and it is in it that a teacher manages to transmit content.

Final considerations

The teacher needs to be prepared to teach mathematics with an emphasis on language. It is necessary to learn to teach, emphasizing the different languages that interact in the classroom, namely, natural language, mathematical language, teacher and student language. Each of these languages has its own characteristics that can influence each other and collaborate with the interaction necessary for the student to learn mathematics at school. The teaching of mathematics with an emphasis on language provides communication between student and teacher, in this regard, it is necessary that the teacher's language clarifies the meanings of mathematical symbols, as well as the rules that govern the texts in which these symbols are inserted.

A large portion of the problems of learning mathematics content is related to the comprehension of the writing of its statements in which the meanings of the symbols must be interpreted. I understand that we do not have access to the student's thinking, we only have access to what is expressed in their words, spoken and written, that's why greater attention to language is so necessary.

Meaning can – and should – be constructed, through the use of similarities with other contents, but against an essentialist conception, a student cannot be expected to learn algebraic rules because they already know arithmetic rules or from any other areas of mathematics or knowledge. The different areas of mathematics can be seen as autonomous language games if we consider that each game has its grammar. It is its relationship to the world and its applications that make it so important in our ways of life. We learn through use and with practice we expand the limits of the use of our concepts. Understanding a word is simply being able to follow one or more rules of grammar, that is, when we use a linguistic symbol we are already attributing a meaning to it.

I seek to present, in this text, a reflection on the grammar of algebra, as an understanding of this discipline, which indicates a non-dogmatic, non-essentialist and non-referential view, in other words, it is not intended as a single definition, but which is susceptible to changes; it does not have an a priori essence, but it is autonomous, and its language is not a reference to something extralinguistic. This expands the range of pedagogical possibilities, placing in language the perspective of a construction and transmission of algebraic knowledge, as something that must be done, because this is a historically constructed content and that we, teachers, have the duty to transmit to our students, understanding that it is in fact new content, which must have its grammar presented, using all possible resources to facilitate learning.

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