REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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# The use of GeoGebra Classroom in Elaborating Conjectures in the Study of Trigonometric Functions: An Investigation with Undergraduate Mathematics Students

# O uso do GeoGebra Classroom na Elaboração de Conjecturas no Estudo de Funções Trigonométricas: Uma Investigação com Licenciandos em Matemática

*Jean Lucas Acelino de Aguiar*[1]

*Roberto Seidi Imafuku*[2]

*William Vieira*[3]

*Emanoel Fabiano Menezes Pereira*[4]

## ABSTRACT

In this article, we present the results of a research in which we analyzed if and how the use of GeoGebra allows future teachers to elaborate and validate conjectures about the influence of parameters on the behavior of the graph of the cosine function. To this end, a workshop was applied using the Google Meet and GeoGebra Classroom platforms, for undergraduate Mathematics students. At the end, the participants evaluated the potentials and limitations involved in the activities by answering an evaluative questionnaire. The Three Worlds of Mathematics are the theoretical ideas that underlie the analysis of the participants' productions. It was found that GeoGebra helped in the creation of conjectures about the influence of parameters on the domain, image, amplitude, and period of cosines defined by f(x) =

[1] Undergraduate student of Mathematics at the Federal Institute of Education, Science and Technology of São Paulo (IFSP – Guarulhos - Brazil). Undergraduate Research Fellow at PIBIFSP. E-mail: jean.lucas@aluno.ifsp.edu.br. ORCID: https://orcid.org/0000-0002-8988-7677.

[2] Professor and member of the Center for Research and Innovation in Mathematics Education and Teacher Training (CEPIN) at the Federal Institute of São Paulo (IFSP – Guarulhos - Brazil). E-mail: roberto.imafuku@ifsp.edu.br. ORCID: https://orcid.org/0000-0002-4047-9533.

[3] Professor and member of the Center for Research and Innovation in Mathematics Education and Teacher Training (CEPIN) at the Federal Institute of São Paulo (IFSP – Guarulhos - Brazil). E-mail: wvieira@ifsp.edu.br. ORCID: https://orcid.org/0000-0002-5592-891X.

[4] Professor and member of the Center for Research and Innovation in Mathematics Education and Teacher Training (CEPIN) at the Federal Institute of São Paulo (IFSP – Guarulhos - Brazil). E-mail: emanoel.pereira@ifsp.edu.br. ORCID: https://orcid.org/0000-0002-6070-6561.

cos (x+d). The future teachers were in favor of using technologies and activities such as those applied in the workshop to teach trigonometry.
**KEYWORDS:** Teaching trigonometry. GeoGebra. Three Worlds of Mathematics.

## RESUMO

Neste artigo, apresentam-se os resultados de uma pesquisa em que foi analisado se, e como, o uso do GeoGebra possibilita que futuros professores elaborem e validem conjecturas acerca da influência dos parâmetros no comportamento do gráfico da função cosseno. Para tal, aplicou-se uma oficina por meio das plataformas *Google Meet* e GeoGebra *Classroom*, para estudantes de Licenciatura em Matemática. Ao final, os participantes avaliaram as potencialidades e limitações envolvidas nas atividades, respondendo a um questionário avaliativo. Os Três Mundos da Matemática são as ideias teóricas que embasam a análise das produções dos participantes. Verificou-se que o GeoGebra auxiliou na criação de conjecturas acerca da influência de parâmetros no domínio, imagem, amplitude e período de cossenoides definidas por f(x) = cos (x + d). Os futuros professores manifestaram-se favoráveis ao uso de tecnologias e de atividades como as aplicadas na oficina para ensinar trigonometria.
**PALAVRAS-CHAVE:** Ensino de Trigonometria. TDIC. GeoGebra. Três Mundos da Matemática.

## Introduction

Throughout history, trigonometry has proved to be a useful tool in solving various human issues in the most varied areas of knowledge. In ancient Egypt, it was used in the demarcation of lands; in the era of great discoveries, it was applied to astronomy as a location tool in ocean navigation; and in cartography, it was present in the triangulation of territories. Nowadays, a person's heart rate, the orbit of celestial bodies and the frequency of sound waves are some of the periodic phenomena modeled by trigonometric functions.

The Parâmetros Curriculares Nacionais (PCNs, which are National Curriculum Parameters) (BRASIL, 2002) is a document prepared by the Brazilian government with guidelines for education in the country. It recognizes trigonometry as a knowledge responsible for technological advancement at different times, however, it points out that, despite being an important topic, trigonometry is not worked in a way so that its applications are explored and, consequently, much time is spent on algebraic calculations of trigonometric identities to the detriment of important aspects of trigonometric functions and the analysis of their graphs (BRASIL, 2002).

The geometric thinking stimulated in Elementary School develops skills that allow the student to interpret and represent the location and displacement of a figure in the Cartesian plane (BRASIL, 2018). Technological advances impact the contemporary world, especially high school students. The Base Nacional Comum Curricular (BNCC, which is a Brazilian Common Core Curriculum) is another document target at crating guidelines for education in the country and it highlights the importance of using these technologies and applications as a resource to develop students' computational thinking while developing research activities and establishing conjectures regarding different concepts and properties in Mathematics (BRAZIL, 2018).

The Mathematics curriculum of the State of São Paulo (SÃO PAULO, 2012) recognizes the potential of trigonometric functions in the representation of periodic phenomena, guiding an approach to these functions in High School. This curriculum indicates that students must know the main characteristics of the basic trigonometric functions (sines, cosines, tangents), associate them with periodic phenomena and construct their graphs so that students understand the impacts of the coefficients on their transformations.

The popularization of digital technologies in society has opened space for discussions about their use in the classroom, at all levels of education. Borba and Lacerda (2015) surveyed government projects and actions that sought to introduce technologies into Brazilian public schools. These projects culminated in the installation of computer labs in public schools, but several difficulties were encountered such as "the infrastructure of schools, lack of preparation of teachers to plan activities, in addition to problems in the configuration of computers and internet speed in schools" (BORBA; LACERDA, 2015, p. 496 – our translation).

In an attempt to identify how computers from state public schools were being used by Mathematics teachers in the final years of Elementary School, Chinellato (2014) interviewed teachers in the city of Limeira (state of São Paulo) and concluded that "the lack of equipment, the lack of interest on the part of students , the difficulty of accessing the internet, [...], the deficiency of the teacher's educational background" (CHINELLATO, 2014, p. 91 – our translation) are some of the reasons that make it impossible for teachers to use technology in their classes. Other reason that also contribute to the non-use of technologies is the initial and continuing education of Mathematics teachers, which are neither satisfactory nor effective for teachers to be trained and able to incorporate digital technologies into their pedagogical practices (CHINELLATO, 2014).

Digital Technologies of Information and Communication (DTICs) are increasingly present in people's lives, and the school, as an institution that is part of society, cannot ignore the effects that DTICs have brought to human beings, and should encourage studies that seek to incorporate these technological resources in the classrooms.

Moreno-Armella and Hegedus (2009) argue that mathematical objects previously defined and explored only in pencil and paper learning environments can now, through Dynamic Geometry software, be significantly explored in an environment that produces a digital representation of these objects, allowing students to investigate their properties while performing actions on them. The concept of *co-action* created by the authors describes how users and dynamic environments mutually exert actions on each other, so that Dynamic Geometry software transforms the interaction that the student has with Mathematics, as it allows him/her to perform actions on representations of mathematical objects, manipulating their characteristics, so that, through investigation, the student can perceive and understand properties, create concepts, meanings, and conjectures, expanding their mathematical knowledge (MORENO-ARMELLA; HEGEDUS, 2009).

Research such as those by Costa, Figueiredo, and Llinares (2019), Costa and Allevato (2019), Braz, Castro, and Oliveira (2019), and Neto (2010) have presented new possibilities for teaching trigonometric functions with the assistance of Dynamic Geometry software, which we briefly summarize below.

Costa, Figueiredo, and Llinares (2019) investigated how a teaching experiment promoted student learning about the periodicity of trigonometric functions in a technological environment. The investigation was carried out with sixteen first-year undergraduate Mathematics students through a hypothetical learning trajectory with elements of Design-Based Research and GeoGebra software applets. The authors used the SOLO Taxonomy (Structure of the Observed Learning Outcomes) developed by Biggs and Collins (1982) to analyze the progression of conceptual understanding, the construction and consolidation of the concept of periodicity of a trigonometric function, as it provides a model for evaluating of the student's cognitive development from mathematical thinking. In their conclusions, the authors point out that the tasks of the experiment provided a coordination of two semiotic systems: the analytical and the geometric, and emphasize that "[...] the construction of a concept is a progressive process of extension to new situations and of coordination between different semiotic systems" (COSTA; FIGUEIREDO; LLINARES, 2019, p. 18 – our translation). From the analysis, the authors found that the activity carried out in a technological context provided an interaction and a dynamism in the students' actions so that the simultaneous coordination between the interrelated analytical and geometric representations helps them to advance not only in the construction of the concept periodicity of trigonometric functions, but also in understanding other mathematical concepts.

The research by Costa and Allevato (2019), using the constructs of the Theory of Meaningful Learning by David Ausubel (1980), aimed to deepen the knowledge that 40 students in the 2nd year of high school already had about trigonometric functions, in a way that those students could evaluate the effects of parameters on the graph of sine and cosine functions. Using activities extracted from Caderno do Aluno, material prepared by the São Paulo State Department of Education (SEE-SP, the initials in Portuguese), and graphics created in GeoGebra software as a cognitive bridge between students' prior knowledge and new content to be learned, the authors concluded that the software enhanced the learning situation, as "[...] it made it possible to explore the parameters and how they acted directly on the variation of period, amplitude, translation, and image set of a trigonometric function" (COSTA; ALLEVATO, 2019, p. 137 – our translation). During the analysis of the results, the authors found that, despite the lack of mathematical formalization, the students were able to answer the questions satisfactorily, however, the research could not identify whether they would be able to construct the graphs manually. It is important to note that, due to a limitation in the material provided by the SEE-SP, the authors were unable to address the parameter responsible for the horizontal translation of the graphs of trigonometric functions.

For being quoted in the literature as a great facilitator in the study of functions, the GeoGebra software was used with an approach based on mathematical investigation in the research by Braz, Castro, and Oliveira (2019). In this work, the authors aimed to introduce the study of trigonometric functions sine, cosine and tangent through a mini course offered to students of the 2nd year of high school at a public educational institution. The theoretical framework of the research is the mathematical investigation (PONTE; BROCARDO; OLIVEIRA, 2009). The students recorded the results of their investigations about the properties of trigonometric functions, and the authors were able to conclude that, in relation to the sine function most of the students presented correct conjectures, but in relation to the cosine and tangent function, some only repeated the answers given in the investigation on the sine function, evidencing a difficulty in conceptualizing these mathematical objects; however, during the process of socialization of responses and formalization of the content, the wrong conjectures were corrected. In their conclusions, the authors highlight that "[...] the development of activities in GeoGebra generated satisfactory results. [...] However, we believe that just developing activities in GeoGebra is not enough" (BRAZ; CASTRO; OLIVEIRA, 2019, p. 82 – our translation), discussing the importance of promoting a moment of socialization and discussion of the conjectures raised by students.

With a methodology based on Duval's theory of Semiotic Representation Records (1993) and using the GeoGebra software as a teaching resource, Neto (2010) conducted a didactic sequence in the form of a workshop for teaching the trigonometric functions cosine and sine. The research subjects were nine students from a public teaching institution in Santa Catarina who already had knowledge about the topics covered in the workshop. The author concluded that "the application of a computational tool must be combined with a proposed methodology; since, by itself, it does not guarantee the effectiveness of the teaching process" (NETO, 2010, p.100 – our translation). He points out that the advantages of using computerized environments that allow manipulation of abstract objects become evident because they allow a series of manipulations in a short time, which would not be possible in a learning environment with only pencil and paper. Despite the satisfactory results obtained in his research, the author emphasizes that the idea is not to replace traditional teaching methods with the use of educational software, but to take advantage of and extract the best characteristics of both environments, so that one supports the other.

Understanding that Mathematics teachers' initial training on the use of digital technologies is the first step to collaborate with the improvement of teaching and learning processes of trigonometric functions, in this research, in the light of the Three Worlds of Mathematics (TALL, 2013) , we seek to analyze the contributions of the use of GeoGebra, and the manipulation of embodied mathematical objects represented in the software, in the creation and validation of conjectures, established by future Mathematics teachers, about the influence of the parameter responsible for the horizontal translation of the graph of the cosine function.

## Theoretical Reference

David Tall (2013) developed a theory about how an individual's mathematical thinking develops across all ages. Mathematical thinking involves understanding mathematical structures through concepts that are connected and combined with knowledge structures, becoming crystalline and sophisticated, endowed with a mathematical structure (TALL, 2013).

According to the author, the different types of cognitive development allow mathematical thinking to develop in three distinct, but not disjoint, ways. Tall (2013) also states that the concepts inhabit Three Worlds of Mathematics: the embodied conceptual, the symbolic operational, and the formal axiomatic.

The Three Worlds of Mathematics thought by Tall (2013) are briefly described below.

The embodied conceptual world is based on human perceptions, sensory experiences and actions on mathematical objects, and it results in the creation and development of mental images of such objects. The verbalizations of mental images are increasingly sophisticated so that they become a perfect mental entity of the mathematical object (TALL, 2013). We use characteristics of this World, for example, when we physically or mentally represent graphs of trigonometric functions, so that their characteristics (period, amplitude, domain, image) can be identified and understood.

The symbolic operational world is primarily based on actions in the embodied world and "develops from human actions embedded in symbolic procedures of calculation and manipulation that can be compressed into procedures to allow flexible operational thinking" (TALL, 2013, p. 133 – our translation). In this world, mathematical symbols are used in calculations, arithmetic manipulations and in algebra, through procedures that can aim to obtain an answer or can be understood as the object to be operated on, focusing on its properties and on the concepts that these symbols represent. Characteristics of this World are evidenced in algebraic calculations of trigonometric identities, in the numerical representation of metric relations of the right triangle, in the representation of trigonometric functions through tables and algebraic expressions of their formation laws.

The formal axiomatic world arises from the combination of embodied conceptions and symbolic manipulations, and concerns the construction of formal knowledge through axiomatic systems, specified according to definitions from Set Theory and Topology, that is, theorems, definitions and axioms, so that the properties of mathematical objects can be deduced through mathematical demonstrations (TALL, 2013).

In the study of trigonometric functions, we find characteristics of the Formal World in definitions such as the study of the period $P$ of the function $f(x) = \cos(a \cdot x)$, which is given by $P = \frac{2\pi}{|a|}$.

## Materials e Methods

In this section, we describe the methodological procedures used in the research on the contributions and limitations of the GeoGebra software for the study of trigonometric functions by a group of undergraduate Mathematics students.

Adopting field research as a methodological basis, a workshop entitled *The role of parameters in the graph of trigonometric functions with the assistance of GeoGebra*, whose objective was to rescue some concepts and ideas related to trigonometry and enable participants to investigate the impacts of parameters on the behavior of the graph of the cosine function, as well as presenting some features and tools of the software.

The research participants are undergraduate Mathematics students at a public teaching institution in the State of São Paulo, who are studying to become teachers. The workshop was held remotely in two meetings, through the Google Meet and GeoGebra Classroom platforms, and was attended by thirteen students on the first day and, on the second meeting, eleven of the thirteen students attended. The meetings were held on July 1st and 2nd, 2021, lasting two and a half hours each.

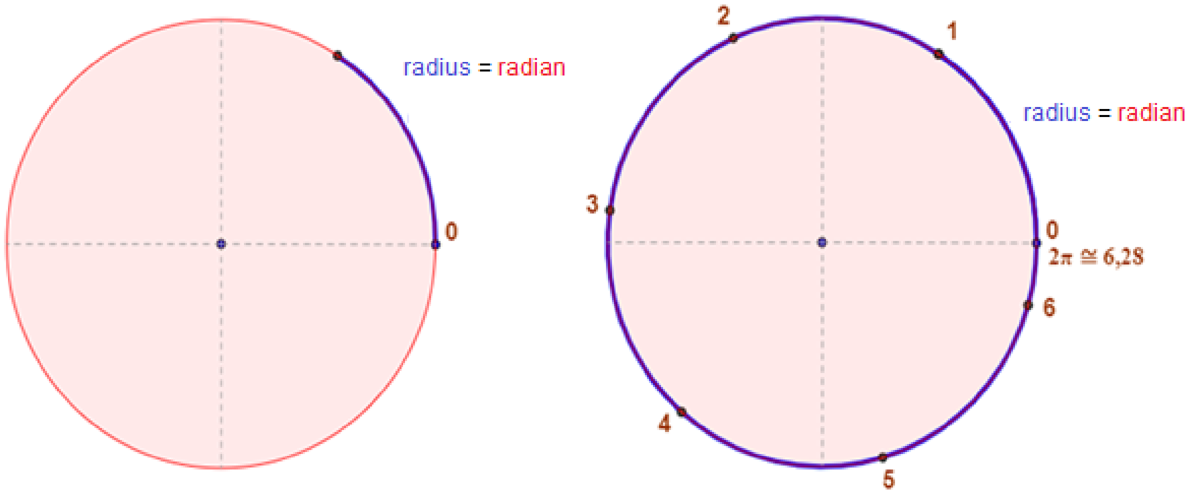
The undergraduates who participated in the workshop were on third semester, that is, they had already completed the discipline Fundamentals of Mathematics II, offered in the second semester of the course, which addresses trigonometry contents, along with complex numbers and polynomials.

Initially, after all participants accessed the virtual room on the Google Meet platform, a reading of the Free and Informed Consent Term (FICT) was made available through a link on the Google Forms platform so that students could sign it, if they agreed to participate in the research.

For everyone to had access to the concepts necessary to develop all the activities carried out in the workshop, in the first meeting, some trigonometry concepts were discussed and revised through interactive applications (applets) of the GeoGebra Classroom. The first concept discussed was the radian.

With the assistance of an applet (Figure 01) it was possible to dynamically demonstrate that a radian is a unit of measurement for arcs, that is, an arc of a circle whose length is the same as the radius measures one radian. It was also possible to illustrate that approximately 6.28 radians fit in the perimeter of a circle.

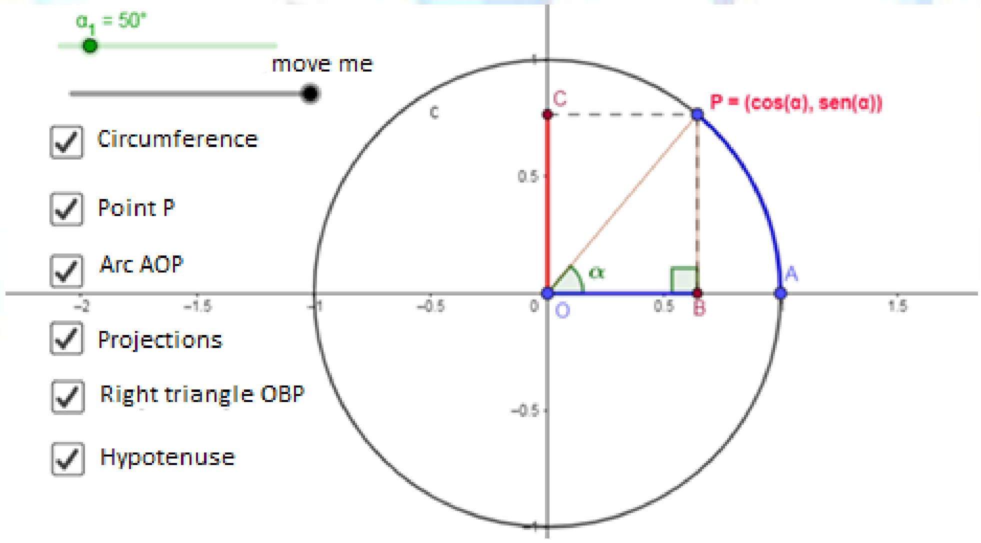
Figure 01 – first applet: radian.




Source: Adaptation of Marco A. Manetta's applet on the geogebra.org platform (2020).

In the second applet (Figure 02), the unit circle was presented (name given to a circle whose radius length is equal to a unit of measure), also called trigonometric circle, presenting the orientations of the arcs. By relating the elements of a right triangle with a point on the circumference, it was demonstrated how the values of the cosine and sine ratios of an arc of a circumference can be found using coordinates of the Cartesian plane.

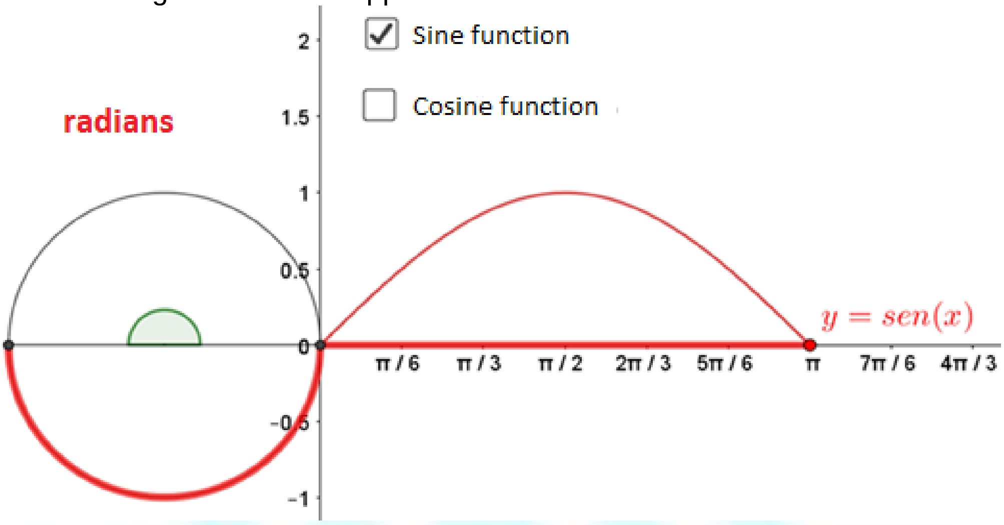
Figure 02 – second applet: cosine and sine in the trigonometric cycle




Source: Prepared by the authors.

Then, using another GeoGebra applet that allowed stretching the arc of a circle along the ordinate axis, some characteristics of the trigonometric functions cosine and sine were presented and discussed, such as: domain, counter domain, image set, period, and amplitude. The use of this applet, illustrated in Figure 03, allowed the students, in a visual way, to perceive the trigonometric relations as functions that have the measure of the arcs in radian as their domain.

Figure 03 – third applet: construction of the sine function.




Source: Prepared by the authors.

For the participants to know how to use the necessary tools for the development of the workshop activity, some features and tools of the GeoGebra software were presented, especially the slider, emphasizing how this resource makes the study of the behavior of function graphs dynamic. The slider is a parameter whose values are easily changed in GeoGebra, which makes it possible to cause variations in objects.

Using linear functions, first- and second-degree polynomial functions as example, it was illustrated how to associate a slider to a coefficient of a function and how to manipulate it, so that the impacts of the coefficients on the behavior of the graphs of the mentioned functions could be dynamically and visually observed.

After the stage of discussion and presentation of the main concepts and tools necessary for the development of the activities, the stage of carrying out the investigations began. The activities were developed on the GeoGebra Classroom platform and were designed so that participants could perform them on multiple platforms, such as desktop computers, notebooks, and smartphones. The choice for GeoGebra Classroom is also because the environment serves as a data collection tool, as student responses are recorded in real time and activities can be paused at any time so that responses are not changed after the activity ends.

The undergraduates investigated the influence of parameters on the behavior of the graph of the cosine function throughout the four activities that contained two questions each. In the first question of each activity, the parameters were explored individually so that the participants should build in GeoGebra a slider, the function $f(x) = \cos(x)$ and the function $g(x)$ with the parameter to be explored.

In the first activity, students explored the function $g(x) = cos(a \cdot x)$. In the second, they explored the function $g(x) = b \cdot cos(x)$, in the third, the function $g(x) = c + cos(x)$ and, finally, in the fourth activity, they explored the function $g(x) = cos(x + d)$. In each step, they should, through the manipulation of the slider, investigate the transformations that the coefficients a, b, c and d provoke in the behavior of the graph, also observing the period, image, and domain of these functions.

Each activity lasted an average of fifteen minutes and, after each one of them, it was held a discussion about the participants' observations and a formalization of the content that validated, or not, the conjectures raised by them.

After completing all activities, students were asked to answer a questionnaire so that they could evaluate the workshop, the proposed activities and the strategies used by the researchers in their development. The questionnaire was made available on the Google Forms platform and contained questions that seek to understand whether future teachers would use activities of this type in their classes, and what are the advantages and difficulties observed by them in implementing these in High School.

## Results e Discussion

In this article, we only analyze question seven, which addresses the function $g: \mathbb{R} \to \mathbb{R}$ defined by $g(x) = cos(x + d)$. Next, we present the proposed question and highlight its objective.

*Question 7 - Construct the graph of the function $g(x) = cos(x + d)$ and freely move the selector d. What is the influence of the parameter d for the graph of the function? Talk about the image and period of this function.*

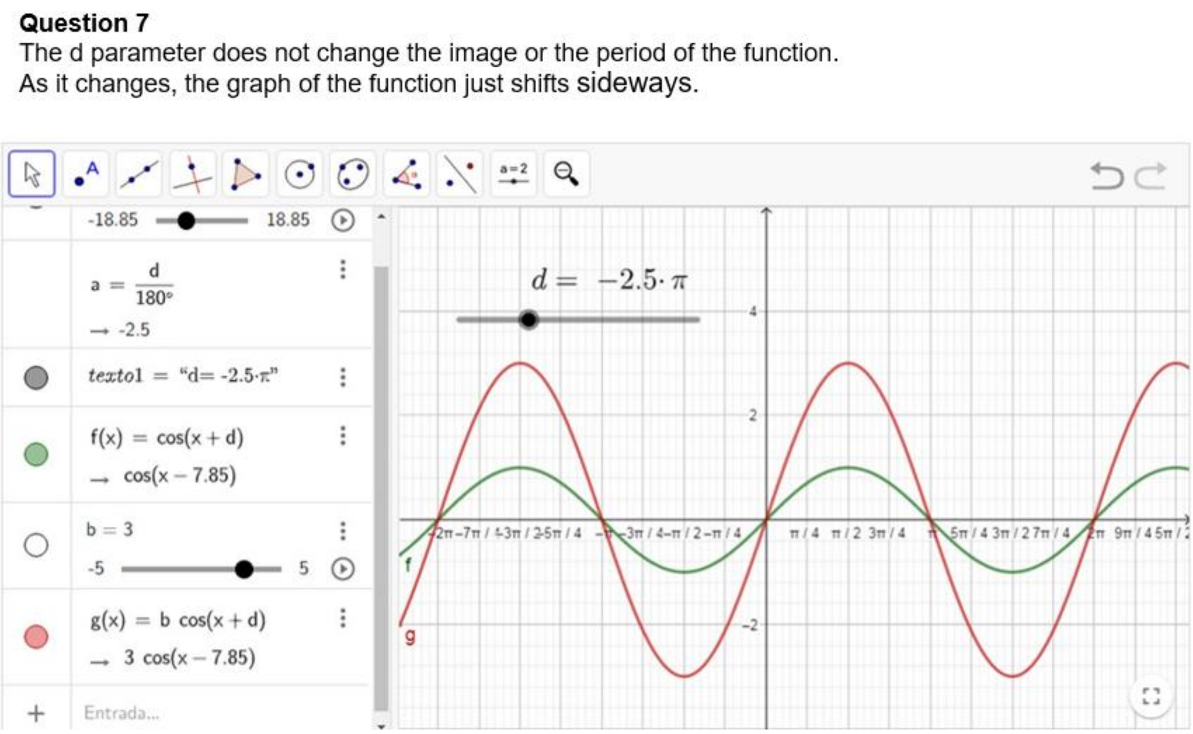
The objective of this question was to verify if the manipulation of the slider helps the students in the perception of the impacts of the parameter d on the behavior of the graph of the function $g(x) = cos(x + d)$. The students should realize that the parameter d causes the graph to shift along the abscissa axis, that is, the graph undergoes a horizontal translation. It was expected that, in their conjectures, students would show that when the parameter d is greater than zero, the graph of the function shifts to the left, and when the parameter d is less than zero, the curve shifts to the right, while the image and the period of the function do not change.

We emphasize that the elaboration of the workshop activities and the analysis of the collected data were carried out in the light of the theory of the Three Worlds of Mathematics proposed by Tall (2013) and that the participants who had their answers analyzed below will be treated by codenames, so that anonymity is guaranteed.

Next, we present the analysis of the participants' responses to question 7 of the activity. It is worth mentioning that when the participants performed this activity, previously in the workshop, it was studied and formalized the impacts that the parameter a (responsible for changing the period of the function $g(x) = cos(a \cdot x)$), the parameter b (responsible for changing the amplitude of the function $g(x) = b \cdot cos(x)$), and parameter c (responsible for the vertical displacement of the graph of the function $g(x) = c + cos(x)$) in relation to the axis of the ordinates) cause in the behavior of the graph of the cosine function. Besides that, in the GeoGebra window used by the students in this question, there was a slider previously created to facilitate the work of exploring the function. This slider was incremented by $0{,}5\pi$.

When analyzing the answers, we found that the participants were able to identify the influence of the parameter d on the behavior of the graph of the function, but they did not present justifications for their conjectures, highlighting only the horizontal translation of the graph without relating the direction of this translation (right or left) with the parameter sign. Petrúcio's answer (Figure 04) exemplifies some of the answers given to question seven.

Figure 04 – Petrúcio's answer to question 7.

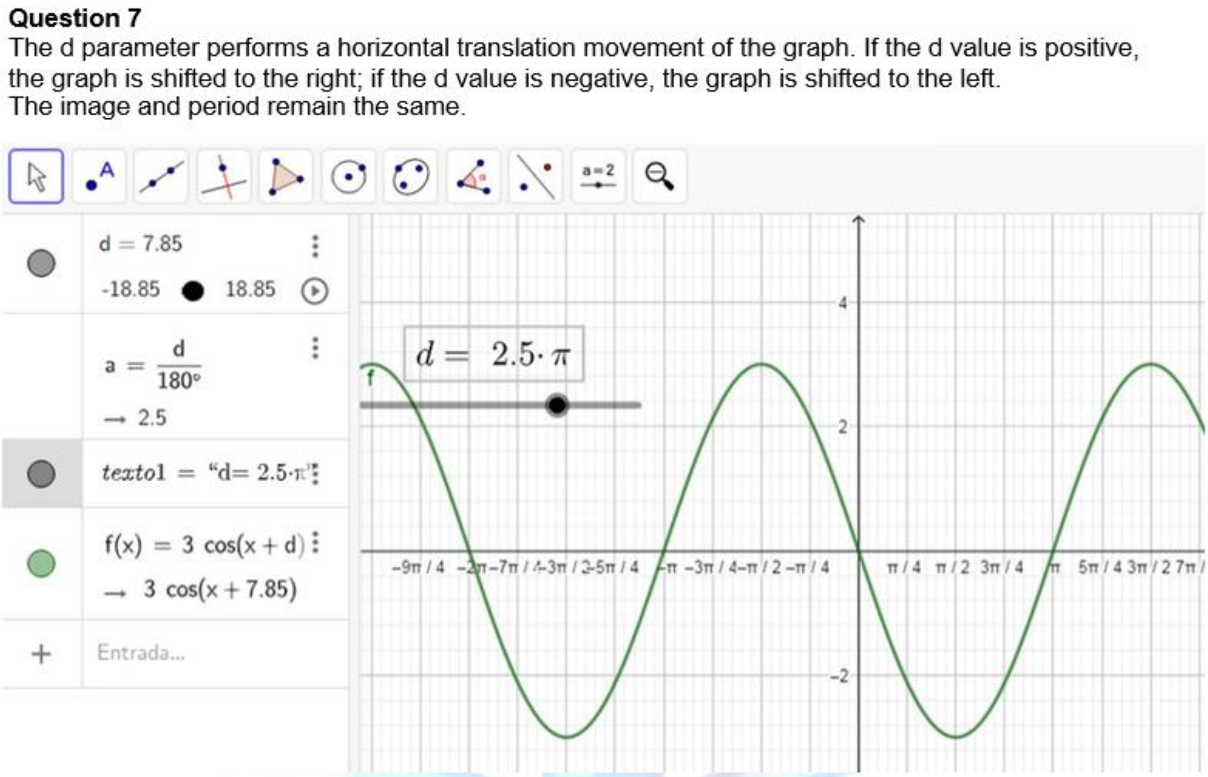


Source: research data.

Based on the answers presented, we identified that the students presented characteristics of the Embodied World, evoked through the manipulation of the slider associated with the parameter d, and the observations of the graph of the function.

Pedro (Figure 05), in addition to correctly observing that the studied parameter causes a horizontal translation in the graph, was able to perceive the relationship between the parameter sign and the translation direction. However, the meanings observed by Pedro in relation to the sign of the parameter would only be correct if the parameter responsible for changes in the period had a negative value, which was not the case.

Figure 05 – Pedro's answer to question 7.

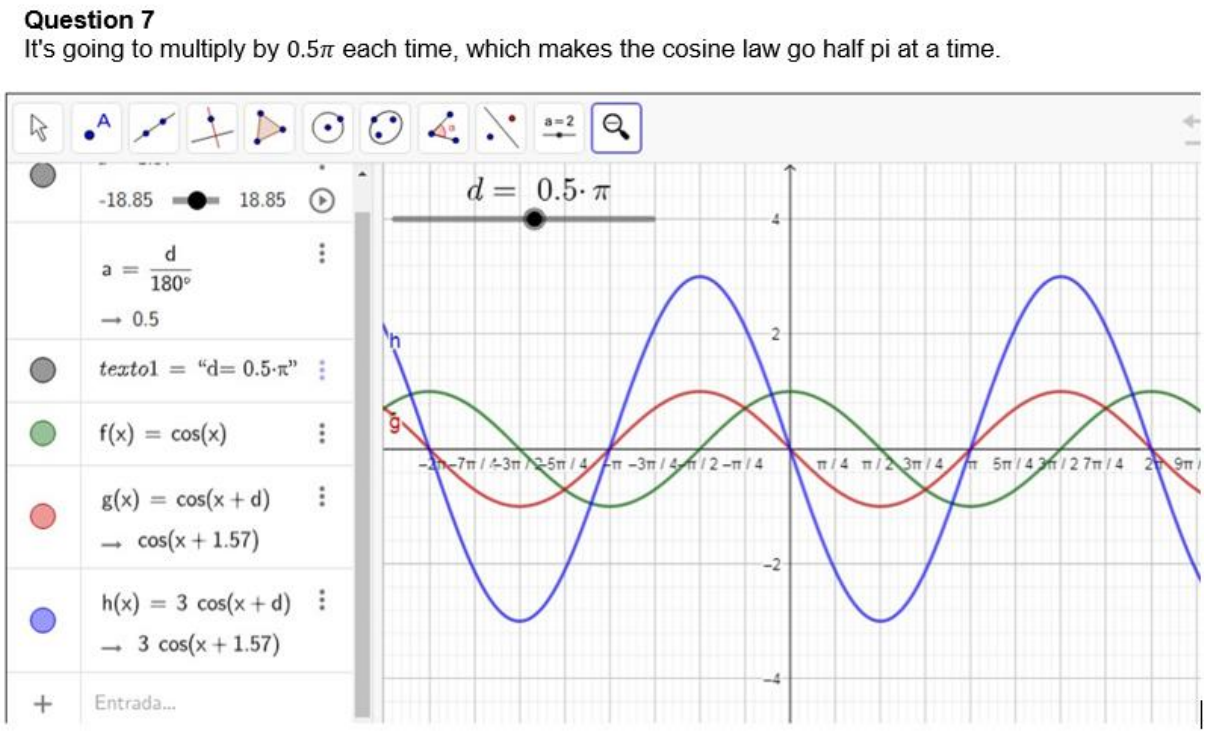



Source: research data.

Through the manipulation of embodied characteristics, the student was able to establish a conjecture and, even making a mistake, when taking the parameter sign into account in his analysis of the displacement of the graph, relating these characteristics to the symbolic characteristics, he demonstrated to be in transition from the stage of practical mathematics, of the manipulation and observation of objects to theoretical mathematics.

The slider previously created to facilitate the work of exploring the function was incremented by $0{,}5\pi$ and may have influenced Diego's conjecture (Figure 06). The student made a mistake when saying that the parameter will multiply something when it is just added to the domain values, and when referring to the curve as the law of cosines.

Figure 06 – Diego's answer to question 7.




Source: research data.

The student did not notice the horizontal translation caused by the variation of the parameter, writing only that the function “walks”, not actually managing to register the influence of the parameter on the behavior of the graph and, despite presenting characteristics of the Symbolic World, it is confused in relation to the mentioned arithmetic operation. Observation and perception, characteristics of the Embodied World, were not enough for the participant to establish relationships with the objects of the Symbolic World and did not allow him to make correct conjectures, that is, they did not lead him to the development of formal characteristics.

We continue with the discussion of participants' assessments and perceptions of the activities discussed in the workshop.

1 - As a math teacher, would you use these activities in high school? Why?

In general, the participants were very enthusiastic and interested with the workshop and activities applied. Unanimously, the future Mathematics teachers declared themselves willing to apply this type of activity in the classroom. The participant João (Figure 07) highlights the intuitive aspects of the activities carried out in stages, which develop one concept at a time, so that knowledge is acquired progressively, being a great option to introduce mathematical content in the classroom.

Figure 07 – answers of João, Júlia and Diego.

Yes, because they were very intuitive and built the concept little by little, in each question, what would be an interesting way to introduce these concepts in the class.

I would use it, because it is a progressive series of exploration of the relationships of parameters with the cosine function, being useful for teaching in high school.

I believe so, it is not something very advanced, and I believe that students, like me, would like to explore.

Source: research data.

The manipulation of the embodied mathematical objects represented in the software, so that the explorations of the relationships of the parameters with the cosine function were carried out progressively, is highlighted by the participants Júlia and Diego (Figure 07) as a justification for the use of these activities in High School, arguing that it is much likely that students enjoy exploring these relationships.

2 - What difficulties do you identify for the application of this type of activity in High School?

Among the possible difficulties indicated by the participants Severino, Getúlio and João (Figure 08) for the application of this type of activity in High School are the students' lack of prior knowledge and the unavailability of technological resources in schools. Regarding the students' lack of knowledge, it should be noted that the study of trigonometric functions must be linked with a previous study of trigonometry, therefore, when carrying out these activities, students should have already studied other topics related to the content, observing the question of the curricular horizon. Once the teacher plans his classes properly, anticipating all the prerequisites, this is an easy problem to solve.

Figure 08 – answers of Severino, Getúlio and João.

Lack of students' knowledge and availability of technology.

Schools that do not have technological resources.

In my view, the difficulties would be in the infrastructure of schools in relation to the possibility of using computers, if available. However, cell phone use may be viable; perhaps the problem would be in the size of the screens or the concentration of the students.

Source: research data.

Regarding the availability of technology, like João (Figure 08) we mention the cell phone as an alternative to overcome the difficulties of schools in meeting this demand, since the cell phone is a technological resource present in the lives of most people today, including High School students.

As we highlighted earlier, Chinellato (2014) argues that teachers do not implement technological resources in their classes because they have difficulty using an educational approach that they themselves have not experienced. And, in response to this difficulty, he states that it is necessary that, in the initial formation, future teachers have contact with subjects that promote an interaction with this equipment, in order to be able to work with digital technologies that they can use in the classroom in the future.

In this sense, the workshop and set of activities elaborated and discussed in this article constitute a response to the considerations raised by Chinellato (20’4), as they seek to train a new generation of future teachers to be more familiar with the use of technology in the classroom, since they are experiencing it in their initial formation.

3 – What advantages do you identify in the application of this type of activity in High School?

In his answer, José (Figure 09) highlights that the insertion of this type of activity avoids mechanical mathematics, which values only algorithms, as it allows the student to think and reflect on the concept he is learning when elaborating conjectures, having more action in the process of building his knowledge.

Figure 09 – answers of José and João.

The student's understanding goes beyond calculating a function, he understands why and I think this helps a lot to absorb the content.

Advantages, mainly, in the interactivity of the students in the conjecture of knowledge and the dynamics that can be given to the class.

Source: research data.

Neto (2010) emphasizes that the student's interaction with the software is fundamental, since he "needs to be as involved as possible with the proposed activities, otherwise he will answer anything and not observe all the details available on the screen" (NETO, 2010, p. 101 – our translation).

We understand that the fact of being able to interact with dynamic constructions in GeoGebra can arouse the interest of the learners, because from the manipulation of the mathematical objects represented in the software, they can create conjectures and test them in a short space of time, contributing to a dynamic class, advantage raised by João (Figure 09) for the use of these activities in class.

The process of mathematical thinking is to make conjectures and try to justify or disprove them. The interactivity of the students and the dynamics of the class made possible by the software open possibilities for teachers to develop and teach students the process of justification in Mathematics in their classes.

## Final Remarks

The obtained results indicate that the GeoGebra software can provide a very useful learning environment in the study of trigonometric functions, as its tools allow an easy and dynamic manipulation of characteristics and properties of activities involving these mathematical objects. It was evident in our analysis that the activities carried out in this technological context promoted the simultaneous coordination of representations of the cosine function, favoring the construction of concepts through interactive and dynamic actions of the research participants, a perspective that corroborates the positions highlighted by Costa, Figueiredo, and Llinares (2019).

The Geogebra Classroom proved to be an interesting learning environment and a great data collection tool, as it made it possible to observe student responses in real time and archive them for analysis. Due to these features, the workshop could be carried out remotely and students could explore trigonometric functions on multiple platforms, such as desktop computer, tablet, and smartphone.

In the same way as in Costa and Allevato (2019), in our activities, manual construction of graphics was not carried out, considering that the use of GeoGebra dynamizes these constructions. This allowed future Mathematics teachers to advance in the creation of meanings and concepts about the influence of parameters on the behavior of the graph of trigonometric functions, working with objects from the embodied, symbolic, and formal worlds. In this way, the participants could only worry about analyzing the embodied characteristics of the graphs instead of drawing them, relating them to the symbolic characteristics present in the algebra window, establishing and validating, or not, conjectures with formal characteristics, in a way that that the transition process between the Three Worlds of Mathematics was fluid during the activities.

We have observed, like Braz, Castro, and Oliveira (2019), that the moment of socialization and discussion of the conjectures raised by the students was as important as the phase of exploration of the functions since, in the discussions, we were able to identify lags and obstacles not only related to the trigonometric functions, but also to the knowledge about trigonometry, both in the right triangle and in the trigonometric circle. In addition, at this stage, we formalized the content so that the participants had contact with elements of formal Mathematics, such as theoretical definitions and mathematical proofs, promoting contact with the objects of the Formal Axiomatic World.

We observed that the undergraduate students indicated, in the evaluation questionnaire of the activities, that they intend to use GeoGebra in their future classes in High School, as they considered the activities of exploration of the proposed trigonometric functions a way to introduce the content in a progressive and attractive way for students. And, despite seeing the lack of technological resources in schools as a barrier that makes it impossible to carry out these activities in High School, future teachers indicated the possibility of using the cell phone of their future students as an alternative for technology to be present in Math classes.

Even though it is a tool that enhances the study of functions, it is worth mentioning that the activities developed in GeoGebra did not work completely, so that it was not enough for some students to elaborate conjectures with the detail expected for future Mathematics teachers, a perspective that was evidenced in the analysis of question 7. This highlights the importance of combining other teaching methods with the use of educational software, so that better results are obtained when extracting characteristics of the two environments, as observed by Neto (2010).

For future study, we suggest the exploitation of the parameters by taking them, at least, two by two, so that students can perceive the relationships between them, and how these interactions affect the behavior of the graph of a trigonometric function. In addition, we understand that the proposed activities should be applied to High School students, so that their potential and limitations are studied and improved with the audience they are originally intended for.

## References

BORBA, M. C.; LACERDA, H. D. G. Políticas públicas e tecnologias digitais: um celular por aluno. **Educação Matemática Pesquisa**, São Paulo, v. 17, n. 3, p. 490-507, 2015.

BRASIL. Ministério da Educação. **Base Nacional Comum Curricular**. Brasília, 2018.

BRASIL. **PCN+ Ensino Médio**: orientações curriculares complementares aos Parâmetros Curriculares Nacionais. Ciências da Natureza, Matemática e suas tecnologias. Brasília: MEC/SEMTEC, 2002.

BRAZ, L. H. C.; CASTRO, G. T.; OLIVEIRA, P. M. O GeoGebra no estudo das funções trigonométricas: uma experiência em um minicurso com alunos do 2° ano do ensino médio. **Revista do Instituto GeoGebra de São Paulo**, v. 8, n. 1, p. 70-84, 2019. Disponível em:

<https://revistas.pucsp.br/index.php/IGISP/article/view/42089/29079>. Acesso em: 28 mar. 2021.

CHINELLATO, T. G. **O uso do computador em escolas públicas estaduais da cidade de Limeira/SP**. 2014. 104 f. Dissertação - (mestrado) - Universidade Estadual Paulista, Instituto de Geociências e Ciências Exatas, 2014. Disponível em: <http://hdl.handle.net/11449/91021>. Acesso em: 28 mar. 2021.

COSTA, F. A.; ALLEVATO, N. S. G. Estudo das funções trigonométricas a partir da teoria da aprendizagem significativa. **REVEMOP**, v. 1, n. 1, p. 126-142, jan./abr. 2019. Disponível em: <https://periodicos.ufop.br/revemop/article/view/1721/1461>. Acesso em: 29 mar. 2021.

COSTA, N. M. L.; FIGUEIREDO, S. A.; LLINARES, S. Um experimento de ensino sobre periodicidade: fatores relevantes para a aprendizagem. **REVEMAT**, v. 14, n. 1, p.1-21, 2019. Disponível em: <https://periodicos.ufsc.br/index.php/revemat/article/view/1981-1322.2019.e61915>. Acesso em: 28 mar. 2021.

MORENO-ARMELLA, L.; HEGEDUS, S. Co-action with digital technologies. **ZDM Mathematics Education**, [s. l.] v. 41, n. 4, p. 505–519, ago. 2009.

NETO, J. R. D. **Registros de representação semiótica e o GeoGebra:** um ensaio para o ensino de funções trigonométricas. 2010. Dissertação (Mestrado em Educação Científica e Tecnológica) – UFSC, Florianópolis, 2010.

SÃO PAULO (Estado). Secretaria da Educação. **Currículo do Estado de São Paulo**: Matemática e suas tecnologias. 1. ed. São Paulo: SE, 2011.

TALL, D. O. **How humans learn to think mathematically**: exploring the three worlds of mathematics. 1. ed. New York: Cambridge University Press, 2013.

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