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Arithmetic questions, algebraic questions, and choice of operation: interrelationships in a literature review

Questões aritméticas, questões algébricas e escolha da operação: interrelações em uma revisão bibliográfica

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ABSTRACT

In Mathematics classes, it is common to ask questions such as: “Is this math operation addition or subtraction?”, “It is written 'earned', so the question is about addition?”, and the answer given by the teacher does not always help the student in the understanding of the utterance. Therefore, this research aimed to establish an overview in relation to what was studied about the choice of arithmetic operation in mathematical questions and the interrelationships with the ideas of arithmetic question and algebraic question. It is bibliographic research with a qualitative approach, carried out through a Google Scholar search in Portuguese and English. It became evident, after analyzing 35 publications, that since the 1970s there has been mention of the choice of operation in mathematical matters and that the first research found were in English. It was also found that these surveys in English, to a large extent, supported studies in Portuguese.

KEYWORDS: Mathematics. Teaching. Operation choice. Arithmetic Questions. Algebraic Questions.

RESUMO

Em aulas de Matemáticas, são comuns questionamentos como: “Essa continha é de mais ou de menos?”, “Está escrito ‘ganhou’, então a questão é de mais?”, e nem sempre a resposta dada pelo professor auxilia o estudante na compreensão do enunciado. Diante disso, essa pesquisa teve como objetivo estabelecer um panorama em relação ao que foi estudado sobre a escolha da operação

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aritmética em questões matemáticas e as interrelações com as ideias de questão aritmética e questão algébrica. É uma pesquisa bibliográfica de abordagem qualitativa, realizada por meio de busca no Google Acadêmico em idioma português e em inglês. Ficou evidenciado, após a análise de 35 publicações, que desde a década de 1970 há menção à escolha da operação em questões matemáticas e que as primeiras pesquisas encontradas foram em idioma inglês. Também foi constatado que essas pesquisas em idioma inglês, em grande parte, embasaram estudos em idioma português.
PALAVRAS-CHAVE: Matemática. Ensino. Escolha da operação. Questões Aritméticas. Questões Algébricas.

Introduction

When there are activities to solve questions in mathematics classes, especially in the early years of elementary school, questions may be asked regarding the type of operation to be used in the calculation. “Is this math operation addition or subtraction?”, “Does this problem use multiplication or division?”, “It is written earned, so the question is about addition?”, “It is written that she 'divided', so in this question uses division?” All those are few examples among several other questions that are asked by students that doesn't always have a satisfactory answer. It turns out that there are questions that have the terms ‘earned’ or ‘divided’ in which the addition or division operation directly does not fit.

In this way, diffident questions that appear simple with obvious answers, cause discomfort when answered in an obscure way for the student. So, it is likely that for the teacher it is not always easy to explain that some questions with the term “earned” are solved with addition operation and others, with the same term, are solved with subtraction operation. If such a situation is not easy to explain, it should not be understood either.

The choice of an operation is frequently addressed in academic research that investigates solving mathematical questions, but it is not common to see it as the main object of study. Usually, the choice of an operation is approached adjacently or in a way that complements those type of research.

The classification of questions into arithmetic or algebraic was investigated by Sá (2003), in her doctoral thesis in education at the Federal University of Rio Grande do Norte (UFRN). The ideas of these 2 types of questions are based on the fact that the operation is chosen directly from the semantic connotation, arithmetic questions, and the choice of the operation can be made based on the feature of the inverse operation, algebraic questions.

In view of this finding and knowing that there are studies that have the choice of operation as the main object of study, the guiding question of this research is established: What is researched about the choice of operation in arithmetic questions

and what is the relationship with the ideas of arithmetic questions and algebraic questions?

To answer this question, this research aimed to establish an overview of what was studied about the choice of arithmetic operation in mathematical questions and the interrelationships with the ideas of arithmetic question and algebraic question. To this end, a bibliographic review was carried out on the Google Scholar indexing platform, in English and Portuguese, in which searches were carried out to map foreign and national research that addressed the choice of math operation in some way in their studies. It should be noted that the choice of operation in mathematical questions is a very hard subject and that it is not the purpose of this paper to exhaust research on this topic.

Research Procedures

With the development of scientific publications on digital platforms, several metadata indexing databases that are completely free to access such as Scielo, DOAJ, Redib, Latindex, Scilit, OpenAIRE, BASE, among others, Google Scholar stands out for being a project of a huge corporation in the internet area and aggregates metadata of articles with free access directly from journals or index bases. There is also an indication of the location of paid articles in large indexing bases, such as SCOPUS, Web of Science, Elsevier, JSTOR, among others. It should be noted that many indexing bases that were previously fully paid access, such as Web of Science, are putting part of their repository with free access articles. So, this means that when searching on Google Scholar, you are doing a survey of publications in a large corpus worldwide, both free and paid access.

About the paid articles indicated in the research, the access was made through the Federated Academic Community (CAFe) platform, which is linked to the CAPES Periodicals Portal. This remote access system is provided by the federated institutions that have joined this service provided by the National Education and Research Network (RNP) (BRASIL, 2020).

The research was carried out with a qualitative approach through bibliographic research, which according to Fachin (2005), is a method that assists in intellectual activity and contributes to cultural knowledge in all forms of comprehension. According to Trigueiro et al (2014), it is a model used in theoretical research on a given object.

Köche (2011) highlights that the purpose of bibliographic research is to map, know and analyze the most relevant theoretical contributions in relation to a particular research object. According to Lakatos and Marconi (2003), the procedure for carrying

out this research is carried out in eight distinct phases: a) choice of theme; b) elaboration of the work plan; c) identification; d) location; e) compilation; f) registration; g) analysis and interpretation; and h) writing. Thus, the research was carried out based on these phases.

To obtain a parameter regarding what was studied about the choice of operation, a survey was carried out in January 2020 on Google Scholar with the keywords indicated in Table 1. There was also a search in the references of the publications found.

Table 1 - Numbers of hit per search

Language	Term	Frequency
Portuguese	(ensino OR educação) AND “escolha da operação”	218
	(ensino OR educação) AND “escolher a operação”	134
English	(teaching OR education) AND “choice of operation”	996
	(teaching OR education) AND “choose the operation”	479
Total		**1.824**

Source: Research data.

After reading the titles and, if necessary, the abstracts, 35 publications were found that covered the scope of the research. These publications ranged from articles, book chapters, books, works at events and research from training courses. From this research corpus, 20 publications were in Portuguese and 14 in English.

To present the results, whenever possible, an attempt was made to maintain the chronological order of the research. However, at times it was necessary to interrupt this order due to the convergence of authors’ thoughts in different periods.

Choice of math operation in arithmetic and algebraic questions

Based on studies by Vergnaud (2009) on his Theory of Conceptual Fields, Sá and Fossa (2008) developed a study on issues involving the four fundamental arithmetic operations and their plurality of meanings, which directly interfere in the formalization of concepts.

When dealing with questions that involve fundamental arithmetic operations, the study by Sá (2003) points out relationships in two aspects: the semantic aspect, related to the question that the operation answers, and the symbolic aspect, concerning to the result of the handling of the symbols involved in the performing of each operation and

can only be done by consulting the table of the operation, that is, without any interpretation.

Given this, Sá and Fossa (2012) present the existence of two categories of verbal questions: arithmetic and algebraic questions. These are related to its modeling, that is, the conversion of semantic data to mathematical language. In arithmetic questions, the unknown is isolated in one of the members of the equality, being used to indicate the result of the operation. Also, according to Sá and Fossa (2008, p. 269), arithmetic questions are those "[...] that, in their operational resolution, the additive or multiplicative properties of equality are not used implicitly or explicitly".

Such statements lead to the modeling of an arithmetic problem, in which the unknown value is isolated in one of the members of the equality and the choice of operation is made from the context of the question, as shown in Table 2.

Table 2 – Modeling arithmetic questions

Example of question	Operation	General Modeling	Modeling of question
I had R$50,00 and I earned R$20,00 in a contest. How much I have now?	Addition	a + b = ?	50 + 20 = ?
A seller, having 150 meters of phone wire, sold 80 meters of this wire. How many meters of phone wire is left for selling?	Subtraction	a - b = ?	150 - 80 = ?
A cinema has 15 rows with 18 seats each. If you are not allowed to watch a movie standing up, what is the maximum number of people who can watch a movie per session in this theater?	Multiplication	a x b = ?	15 x 18 = ?
I have 1200 candies to distribute equally in 5 boxes. How many candies should I put in each box?	Division	a : b = ?	1200 : 5 = ?

Search: Adapted from Sá (2003, p. 76).

In algebraic questions, the inquiry is not isolated in one of the equality members, and this is used to indicate the required equilibrium relationship between the data. According to Sá and Fossa (2008, p. 270), they are those "[...] in which, in their operational resolution, the additive or multiplicative properties of equality are used explicitly or implicitly".

While the modeling of an algebraic problem always results in an expression in which the unknown value is not isolated on one side of the equality, as shown in Table 3.

Table 3 - Modeling algebraic questions

Example of question	Operation	General Modeling	Modeling of question
My father had R$25.00 in his safe. After saving a certain amount, he had R$78.00. How much did my father have in the beginning?	Addition	a + ? = b	25 + ? = 78

My father had a certain amount in his safe. After saving the amount of R$25.00, he now has R$78.00. How much did my father have in the beginning?		? + a = b	? + 25 = 78
A tradesman owned 2000m of wire. After selling a few meters, he found that he still had 1890m of wire. How many meters of wire did the tradesman sell?	Subtraction	a - ? = b	2000 - ? = 1890
I went to a store with a certain amount. After spending R$156.00, I found that I still had R$95.00 left. How much did I have when I got to the store?		? - a = b	? - 156 = 95
Triple a certain amount is 120. What is the amount?	Multiplication	a x ? = b	3 x ? = 120
A quantity multiplied by 3 is 120. What is that quantity?		? x a = b	? x 3 = 120
I Distribute 28 toys among some children. Each child received 4 toys. How many children participated in the distribution?	Division	a : ? = b	28 : ? = 4
A certain number of toys was distributed equally among 9 children. Each child received 5 toys. How many toys were distributed?		? : a = b	? : 9 = 5

Search: Adapted from Sá (2003, p. 77-78).

In this type of math problem, differently to what happens with arithmetic questions, Sá (2003) shows that the choice of operation can be made based on the property of the inverse operation. In collaboration, Sá and Fossa (2008, p. 269) state that “[...] the use of the inverse operation, to maintain the validity of equality, is the essence of the method of solving equations and one of the characteristics of algebra is the solving equations”.

With this, by the previous modeling, it is possible to see that the identification of the operation is determined by its enunciation, while algebraic questions use an operation, since, although the modeling indicates the existence of an operation, it will not necessarily be used in the resolution, that is, it uses the operation, but is not of that operation.

Because of Carpenter's ideas et al (1993) and Sá (2003, p. 78), the strata were classified into Additive Questions or Multiplicative Questions, Arithmetic Questions or Algebraic Questions. The studies by Silva (2015) and Santos (2017) indicated a hierarchy of greater difficulty to solve, in which the Arithmetic Additive Questions and the Algebraic Multiplicative Questions are, respectively, of lesser and greater difficulty of resolution.

An overview of research that addressed the choice of operation in arithmetic questions in Mathematics

The first research found in chronological order was a reference that Zweng (1979) made to Burch (1953) when highlighting that the author found that answering

questions such as “what is given?” or “what to find?” can help performance in solving math questions, but it is also harmful. Burch (1953) hypothesized that, when answering questions such as “what is given?”, the data are removed from the context of the question and, therefore, the relationships that provide the information for the choice of the operation are lost.

Nesher and Teubal (1975) conducted an experiment on the use of "key words" for teaching question solving. The researchers indicate that this experiment makes it apparent that specific keywords have a great deal of influence in determining the choice of operation. However, they state that while the choice influenced by the specific verbal wording of the question may not meet the actual demands of the task, there is no reason to ignore these keywords.

Also, according to Nesher and Teubal (1975), regarding the solution of mathematical questions, it is necessary to consider that the use of a limited specific vocabulary and the use of keywords in many cases result in an artificial model of establishing mathematical questions and may derive questions that are not related to everyday language. In addition, keywords can also influence the choice of the mathematical operation to be used to solve the question, an operation that may not correctly solve the question. This happens because choosing the operation based only on the keywords may lead to a conversion mistake between the verbal formulation of the problem and its mathematical expression (NESHER; TEUBAL, 1975).

Bell, Swan, and Taylor (1981), in their study, point out that it is evident that there is a need to work directly on the choice of operation, as well as on the underlying subconcepts that are needed from understanding the question. They also indicated that there was a need for further experiments to research the general strategies of using numbers and arithmetic problems of a single operation, as well as to try to separate the effects of various aspects of teaching, because these gaps in learning can perpetuate throughout an individual’s school career, i.e., the "persistence of the same difficulty hierarchies into adulthood shows how, even when more advanced notions are available, students try methods that cause less cognitive impairment before adopting the more difficult ones" (BELL et al., 1989, p. 447).

Thus, it is possible to draw a completeness between the inadequacy of solving questions only by means of keywords, as advocated by Nesher and Teubal (1975) and which may have the same consequences as argued by Burch (1953), and the inadequacy of forming concepts and subconcepts of operations for understanding mathematical questions, by not being able to work separately on each operation, as

supported by Bell, Swan, and Taylor (1981). This is because the comprehension of the problem that is not only by observing keywords is closely related to the concepts and subconcepts formed about the operations.

In their research, Schwartz and Budd (1981) report that after students thoroughly understand what the mathematical question is asking, choosing which mathematical operation to use should be a slightly less difficult step. But before this, students need to know what operations there are, what their symbols are, and what the operations perform. More specifically, students should know that addition exists and that it joins items or groups of items. The truthfulness of this example extends to the operations of subtraction, multiplication, and division.

Additionally, the researchers mentioned in the immediately preceding paragraph point out the need for the student to be able to estimate the value of the result of the question so that he/she can logically determine whether his/her choice of operation is correct. This estimate also helps the student to verify his/her answer when the resolution is complete. Similarly, also according to the survey respondents, if a student is unable to at least provide estimated answers that are in the direction of the expected answer in the question, they may not adequately understand the operation. Difficulty in estimation may also indicate a misunderstanding of the problem or that, even though the choice of operation may have been correct, the wrong numbers were selected for that operation (SCHWARTZ; BUDD, 1981).

Afekenstam and Greger (1983) claim that the result of their evaluation indicated that the focus of studies on solving mathematical questions should not be on questions with one operation that can be solved directly, but on questions that the authors called non-routine (which are not questions like those solved as examples), demanding from the student more than just the application of an arithmetic operation. Also, according to the authors, the difficulties with non-routine problems appeared to be rooted in many students' misconception that "solving mathematical questions" meant just choosing the correct arithmetic operation and applying it to the numbers given by the question.

The researchers Bell, Fischbein, and Greer (1984) have carried out a study that investigated the type of number used in mathematical questions. In referring to the effects of the types of numbers involved in calculations, a consistent trend was that students had difficulty performing multiplication and division by numbers between 0 and 1. In addition, as noted by the authors, the operation chosen by students while solving questions with an operation can be influenced by several factors that was not the focus of the research conducted. Bell, Swan, and Taylor (1981) indicate that some

factors are due to the misconceptions, such as multiplication always making the initial value larger or division always making an initial value smaller and are taken to higher stages of study if not corrected at an early stage. These considerations are also made by Prediger (2009).

The idea of the necessity of students to be able to estimate the result, defended by Schwartz and Budd (1981), can generate difficulties in the situations described by Bell, Fischbein, and Greer (1984), in conceptual misconceptions such as the hypothesis that multiplication "always increases" and division "always decreases". However, both ideas can be complementary, understanding that the student needs to perform estimates already with the concept that when multiplying by values between 0 and 1, the increase or decrease will be inverse in respect to numbers larger than 1. These conceptions are easily assimilated for arithmetic type questions, however there is a substantial possibility that making estimates are not efficient in algebraic type questions.

Regarding the solving of questions with numbers with commas, Fishbein et al (1985) further adds that a multiplication by 0.22 or 5/3 having no intuitive meaning is not to say that it has no mathematical meaning, because students know well that $1{,}20 \times 0{,}22$ e $9 \times 5/3$ are legitimate mathematical expressions. But when the question data involves these types of numbers, these students may not be able to comprehend the question to figure out which operation to perform.

In this context of types of numbers in the question, Greer (1987) states that it has already been consistently established that a student's choice of operation for the verbal question solution of an operation is often affected by the types of numbers being used. A more punctual observation is that the choice of different operations takes place even when two problems differing only in the numbers involved are juxtaposed and attention is drawn to the similarity between them. In other words, in questions with identical writing where only the types of numbers used differ, the students may choose different operations for solving them.

From an emotional perspective with respect to the choice of operation, Marshall (1989) points out that there was some evidence of affective connections to the planning component, which is embedded within the processes for solving the question. The author mentions the example of a student who frequently ended his/her comments about the solution of each question with negative statements, such as "It's probably wrong" and "I'm doing very badly". Most of those comments appeared to refer on

his/her choice of operation and were made after he/she had described why he/she had chosen to use a specific arithmetic operation.

The situation of the size of the quantity that the number represents was one of the study results obtained by Martinez (1995), who countered Hart's (1981) study that claimed there was an influence of the size of the quantity that the number represents on the choice of the appropriate operation to solve questions with an operation, thus questions with numbers representing smaller quantities were easier to recognize. In contrast to this, Martinez (1995) indicated that in the final test, the differences between parallel items containing small and large numbers, respectively, were on the order of 1.5%.

A study carried out in Brazil by the researchers Souza, Chaquiam and Sá (2002), who conducted a survey with a group of teachers, indicates that there are signs of a depreciation of memorization of the fundamental relations of each operation. According to the authors, the opinions of 187 teachers surveyed indicate that memorization of multiplication tables is always linked to mental calculation and, in a lower percentage of cases, to the identification of operations in problem situations. In this same study, also based on the answers given by the teachers consulted, 70.59% reported that the domain of the four arithmetic operations is held by those students who have difficulty in forming and performing calculations, but who correctly identify the operations involved in mathematical questions. For the remaining 29.41% of the teachers, the domain of the four operations is held by those students who perform calculations but have difficulty in identifying the operations involved in mathematical questions.

The researcher Justo (2004) claims to find that while the student deals with the mathematical situation presented in the question without having domain of the connections between the operations of addition and subtraction, for example, he/she tries to solve by the operation that characterizes the question. That is, if the situation is an additive one, he/she tries to solve by addition, if the situation is subtractive, he/she tries by the subtraction one. Or, sometimes, the student establishes relations between keywords such as “plus”, “earned”, “got”, “together”, among others, when making the choice of the mathematical operation. So, the researcher highlights that, to understand the additive conceptual field, the student needs to “progressively dominate the additive operational structure, understood by the operations of addition and subtraction, by the relations between the concepts, meanings, invariants and signifiers that define the understanding of the additive structure” (JUSTO, 2004, p. 114).

Regardless of what strategy a student uses to solve a mathematical question, Valentin and San (2004) indicate that the task of solving these questions is the most critical part and represents the edge for a successful solution. The authors mention that the process of constructing a representation of the situation presented in the question involves mapping the written structure onto a mathematical schema already understood by the student. In their research results, it is indicated that students have more difficulty solving multiplicative field questions than additive field questions. Two hypotheses are also established for this fact. The first indicates that this is because students of this school age deal more frequently with additive questions in the environments in which they are living and socializing. The second hypothesis indicates that additive questions involve only extensive quantities (which are represented directly), while multiplicative questions can involve extensive and intensive quantities[3] (which are derived from other quantities). Martins (2011) also found it more difficult to solve multiplicative questions than additive questions, but the researcher did not present any factors contributing to this scenario.

As for the teacher's approach to the teaching of mathematical questions in the additive or multiplicative fields, Panizza (2006) points out that when the student, while attempting to select the appropriate operation to solve the problem, asks the teacher if "Is this a plus account?" and the teacher replies affirmatively to the student's question, it can be said that the student has solved the account, but has not solved the question. In this way, by answering affirmatively to the student's question, the teacher has turned a solution to a mathematical question into a resolution to do a mathematical calculation. In this type of scenario, the student arrives at an answer to the question, but no longer has the need to put into practice the concepts necessary to choose the operation. More briefly, Pereira (2010, p. 26) states that in this case the students "solved the account but did not need to put into practice all the necessary knowledge to deal with the problem situation, the interpretation of the situation was not done". Borges (2015), in an identical situation, explores the fact that a student participating in her research was insecure when making the choice of the operation and asked the question "Is it a plus?" After this student's interrogation, some reflections were made on the situation proposed by the question and then she concluded that she would need to choose subtraction to resolve the question.

When expanding the context of the students' questions during the resolution of a mathematical question, Oliveira (2007) indicates that questions such as "this account

[3] The study of extensive quantities and intensive quantities are expanded in detail in the next section.

is too much" reflect the students' insecurity in selecting the correct operation. Moreover, Oliveira (2007), supported by Guimarães (2009), states that the difficulties in choosing the operation can be based on two, but not the only factors. The first factor implies that the pedagogical practice based on the introduction of a concept followed by questions to the replication procedure aiming to "fix the content", often causes uncertainty in the choice of the operation. The second factor is due to the way teachers deal with operations, as if they were opposite and unconnected to each other, when these operations are embedded in something more complex, which are the conceptual fields.

In the research results of Guimarães (2009) appear examples of the use of keywords to solve questions. When being asked about the choice of the operation, one of his surveyed students replied that "simple, he is asking how much the two together, it means it must add up. Absolutely." However, it has been seen in Nesher and Teubal (1975) and Bell, Swan, and Taylor (1981) that solving questions by simply using keywords can lead to difficulties in some types of questions. If the questions were of algebraic type, this method could bring more difficulty to the resolution, because it requires wider knowledge than associating the keywords with the operation to be performed, because, as Guimarães himself (2009, p. 12) says in his considerations, "the presence of keywords most likely influenced the choice of operation to be used".

Justo and Dorneles (2010) state that when solving diversified questions, students put into practice the knowledge they already have, improving and adapting such knowledge to each new situation proposed by new questions. The authors also argue that to solve a mathematical question, the student needs to choose the operation and perform the calculation, which demands knowledge that exceeds the act of just performing mathematics properly. Then, still according to the researchers, it is necessary for the student to have a network of established concepts about mathematical operations, building meanings linked to the various situations to which they belong. In cooperation with this thought, Espindola and Moura (2017), who have considered the choice of operations to solve the questions used for their research, confirm the importance that the situations used in each question cannot be scrutinized with only the allowance of a single concept. Which means that a question, as simple as it may be, involves more than one concept.

In this discussion about the importance of the context in which the operations are inserted in a question, it is worth noting what Comério (2012) indicated, that the resolution of a mathematical question is not reduced to numerical calculation, however

this is not the same as saying that the algorithms development of the addition, subtraction, multiplication, and division operations are not essential to the development of the student's mathematical thoughts.

In reference to the knowledge of operations and mathematical symbols, Weber (2012) highlights that, although students know them, they are unable to associate them to the situation presented by the question and set up a strategy to choose the operation required for its resolution. In fact, the author brings to the discussion that there seems to be limited practice in the reading and writing of the national language in mathematics classes, which seems to have a predominance of mechanical resolution using fixed formulas and repetition of analogous questions.

Regarding learning about relational calculus, Nunes et al (2012) emphasizes that often students may have the knowledge to perform a specific calculation, however they have difficulties to determine if this calculation is the most suitable to solve the question. In agreement with Pereira (2017), the necessity then arises to reason in a logical-mathematical way about the question to subsequently decide on their resolution strategy.

Furthermore, on relational calculus, Dorneles (2013) makes it clear that relational calculus supports the understanding of the relationship between addition and subtraction operations, and that they are interdependent. Dorneles and Dorneles (2017) suggest that by comprehending the inverse relation, the student is more likely to reason about the question statement and correctly select the correct operation and calculation to solve it. In his master’s studies, Rocha (2019) observed that in solving the questions, mainly two types of errors were made, errors in numerical calculation (32.4 %) and errors in relational calculation (48.5 %). "This shows that the difficulties are not only in setting up and solving the math, but mainly in comprehending the relationships that are set up between the data in the problem statements" (ROCHA, 2019, p. 137).

In explaining about the choice of operation, Moretti and Brandt (2014) point out that in the resolution of a question the choice of operation is something complex that can cause problems of incompatibility between the question and the operation selected. Miranda (2014) has encountered this situation in his studies, because when citing the keyword “plus” as an integral in a question, he could observe that this keyword is inconsistent with the operation that will be performed.

On the other hand, in line with Beck and Silva (2016), the difficulty in grasping the expression "how many are missing" may be due to the translation of this expression

to mathematical language, which directly affects the choice of operation to be utilized in the resolution. It is worth initially noting that the expression can create an enigma for the student when making the choice of operation, because "how many are missing" taken as keywords can denote the need for addition by needing to add to an already established value. Yet, there is a high possibility that, in the context of this question, the expression "how many are missing" is embedded in an algebraic type of question that requires subtraction to be resolved. These discussions about using keywords in the question for operation choice are also addressed by Xin (2019), in which he describes some words that students often associate with only one specific operation. Fischer et al. (2019), in their research, says it is evident that the presence of the verb "to gain" in the question wording can explain the mistaken choice of the operation, which they termed a misleading verbal inducer.

Finally, at the end of this study, it can be seen that many national and foreign research have investigated various dimensions about question solving. More specifically, those presented here addressed the choice of operation, starting from the same object, but with some different perspectives. A perspective presented is transposed during the study, which is the resolution of questions based on keywords, either as a resolution strategy adopted by the student or as a teaching method performed by the teacher. It should be noted that the practice of studying using keywords may be convenient for some types of questions, but if the student follows as a rule to solve all the questions, they may select operations that do not help in the resolution.

Final Considerations

According to Lima and Mioto (2007), well-done bibliographic research can help establish hypotheses or interpretations that can serve as a starting point for future investigations, especially on themes that have been little explored.

To map studies that had operation choice as a proposition, this investigation was developed to answer the question: What is researched on operation choice in arithmetic questions? Based on the results presented, it can be established that it has been answered. For it was clear the establishment of a panorama of the research found and their interconnections, even if they are publications in different languages with realization in different countries.

In a way that is implicit in most of the research explored, except for research that relates the choice of operation to the type of number, the discussions converge on the same difficulties encountered in studies of arithmetic and algebraic type questions.

It should be noted that the research conducted does not cover the entire existing corpus of publications that investigated the choice of operation in solving mathematical questions, and neither would it have the audacity to do so. However, it was successful in sketching a panorama of research publications on operation choice.

With the publication of this research, it is hoped that it can serve as an initial reference for researchers who are interested in researching the choice of arithmetic operation in mathematical questions. Also, a reflection question should be made: Since 1970, research indicates that solving questions based on keywords is not the best strategy. In view of this, what are the impacts on teaching practice in and for teacher education?

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