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Mathematical tasks in mathematics teacher education

Tarefas matemáticas na formação de professores que ensinam matemática

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ABSTRACT

Based on a diachronic movement of research and educational experiences, this article presents a text of a reflective and interpretative nature on educational actions and evidence resulting from them that allow understanding the role of work with mathematical tasks in/for Mathematics Teachers (MT) education. In this movement, three actions are highlighted in MT formative contexts, based on work with mathematical tasks: resolution and analysis of mathematical tasks; selection, adaptation, design and exploration of mathematical tasks; and reflections and discussions about working with tasks in the classroom. The elements that surround these actions show a dense and articulated framework that points to the potential of working with mathematical tasks for professional learning of MT. The problematized framework can guide the planning, implementation and evaluations of MT education programs and actions based on working with mathematical tasks, both in the field of research and practice.

KEYWORDS: Mathematical Tasks. Mathematics Teacher Education. Teachers' Education Actions. Teachers' Professional Learning.

RESUMO

Com base em um movimento diacrônico de pesquisas e de experiências formativas, este artigo apresenta um texto de natureza reflexiva e interpretativa sobre ações de formação e indícios decorrentes delas que permitem compreender o papel do trabalho com tarefas matemáticas na/para a formação de Professores que Ensinam Matemática (PEM). Nesse movimento, são sublinhadas três ações em contextos formativos de PEM, alicerçadas no trabalho com tarefas matemáticas: resolução e análise de tarefas matemáticas; seleção, adequação, elaboração e exploração de tarefas matemáticas; e reflexões e discussões a respeito do trabalho com tarefas na sala de aula. Os elementos que circunstanciam essas ações evidenciam um quadro denso e articulado que aponta o

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potencial do trabalho com tarefas matemáticas para a aprendizagem profissional de PEM. O quadro problematizado pode orientar o planejamento, a implementação e avaliações de programas e ações de formação de PEM alicerçados no trabalho com tarefas matemáticas, tanto no campo da pesquisa quanto da prática.

PALAVRAS-CHAVE: Tarefas Matemáticas. Formação de Professores que ensinam Matemática. Ações Formativas de Professores. Aprendizagem Profissional Docente.

Introduction

Investigations indicate that mathematical tasks, as they are part of everyday life in the classroom, play a relevant role in student learning and in the professional practice of Mathematics Teachers - MT[3] (ARBAUGH; BROWN, 2005; DOYLE, 1983; SHIMIZU et al., 2010; SIMON; TZUR, 2004; STEIN; GROVER; HENNINGSEN, 1996; STEIN; SMITH, 1998; STEIN et al., 2009; WATSON; SULLIVAN, 2008).

Mathematical tasks determine the reasoning that students develop when solving them (STEIN; SMITH, 1998). In this sense, tasks that require routinely carrying out a memorized procedure lead to a type of opportunity for the student to think. In contrast, those that require engagement with concepts and that encourage making connections lead to a different set of opportunities. Therefore, different tasks constitute diversified learning opportunities. Some have the potential to foster complex ways of thinking in students, and some do not.

Through mathematical tasks, the MT can articulate the contents to be developed and achieve their teaching objectives. Thus, it is important that the teacher chooses, adapts or designs tasks that meet their intentions and that allow the creation of a classroom environment that encourages the student to engage in the resolution of these tasks, since "tasks with which students engage constitute, to a great extent, the domain of students' opportunities to learn mathematics" (STEIN *et al.*, 2009, p. 131).

According to Chapman (2013), working with mathematical tasks demands knowledge from teachers that need to be considered in their education process. Focusing the attention of future teachers and MT on studying and working with mathematical tasks can be a powerful path for their professional learning.

Given this scenario, we began to study investigations developed in the QUASAR[4] project, whose main representatives are Edward A. Silver, Mary Kay

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[3] The acronym MT will be used in the text invariably to refer to the singular (Mathematics Teacher) and the plural (Mathematics Teachers), according to the context in which it is used.
[4] The *Quasar Project* (*Quantitative Understanding: Amplifying Student Achievement and Reasoning*) was developed in the United States and was based at the Center for Research in Learning and

Stein, Margareth Schawan Smith, Suzanne Lane, Barbara Grover and Marjorie Henningsen. These studies raised reflections and several questions, such as: *What is the role of working with mathematical tasks in education geared towards Mathematics Teachers (MT)? What actions can be developed in MT education programs that prepare them for the exploration of mathematical tasks in the classroom? How is it possible to exploit mathematical tasks to promote professional learning with MT?*

In the search for possible answers to these questions, the Studies and Research Group on Mathematics Teacher Education - Gepefopem and, more recently, the Group of Studies on Practice and Technology in Mathematics and Statistics Education - GEPTEMatE established educational contexts based on study groups. In these groups, the MT can study, share experiences and repertoires, discuss and reflect on their pedagogical practice, having as one of their actions the work with mathematical tasks. Some of these groups formed a Community of Practice - CoP (WENGER, 1998).

In these CoPs, teachers in training assumed mutual commitment/engagement around articulated undertakings, such as the role of working with mathematical tasks in the classroom and negotiated meanings that revealed their professional[5] learning. The participating teachers assumed the role of protagonists in the process of building their professional knowledge, whose formative dimension was guided by the problematization of knowledge, beliefs, understandings and feelings that the teacher has, to the detriment of models based on the presentation of (new) knowledge that the teacher does not possess, which they lack. Thus, in the actions developed by the groups, the singularities, emotions (frustration, fear, desire to succeed, insecurity), experiences and knowledge of the teachers involved were respected and legitimized, constituting an environment of trust and mutual respect.

Taking into account the investigations carried out by Gepefopem and GEPTEMatE in these educational contexts (CYRINO; JESUS, 2014; ESTEVAM; CYRINO; OLIVEIRA, 2018; JESUS, CYRINO; OLIVEIRA, 2018; NAGY; CYRINO, 2014; MAGGIONI; ESTEVAM, 2021; BRANDELERO; ESTEVAM, 2023), in a diachronic movement with our studies and our experiences as instructors, in this

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Development at the University of Pittsburg. This project aimed to promote the teaching of mathematics to students attending *middle schools* in economically disadvantaged communities, with an emphasis on thinking, reasoning, problem solving and communicating mathematical ideas.

[5]According to Wenger (1998), the process of negotiation of meanings is a mechanism for learning that occurs through the interaction between two inseparable processes: participation and reification.

article we present a reflective and interpretive text on educational actions and evidence arising from them that allow understanding the role of work with mathematical tasks in/for the MT, without any intention of presenting conclusive answers to the questions listed above.

To this end, in the next sections we present discussions about educational actions and evidence of the relevance of working with mathematical tasks in/for MT education, namely: resolution and analysis of mathematical tasks; selection, adaptation, design and exploration of mathematical tasks; and reflections and discussions about working with tasks in the classroom. Finally, we make considerations about the challenges facing MT education programs and research on the subject, as well as some final reflections.

Resolution and analysis of mathematical tasks

Considering the complexity involved in MT's knowledge for working with mathematical tasks in the classroom, one of the actions developed in the study groups involved the *resolution and analysis of mathematical tasks* proposed by the instructors and by the MT who participated in the programs. Research suggests that the resolution and analysis of mathematical tasks are promising practices in promoting experiences and reflections with a view to developing professional knowledge with MT (STEIN; SMITH, 1998; LILJEDAHL; CHERNOFF; ZAZKIS, 2007; GUBERMAN; LEIKIN, 2013; CYRINO; JESUS, 2014; ESTEVAM; CYRINO; OLIVEIRA, 2018; MAGGIONI; ESTEVAM, 2021).

In this action, teachers in training had the opportunity to discuss the difference between task and activity, build and mobilize mathematical knowledge, (re)think the role of mathematical tasks and the relevance they have for the teaching and learning processes and, therefore, for their pedagogical practice.

Christiansen and Walther (1986) clearly distinguish between task and activity. The activity is realized through a system of actions that are processes directed towards the objective originated by the reason for the activity. According to Leontiev (1978), activity arises after the manifestation of a need. This need is linked not only to a material object, but also to an ideal object that, in order to be achieved, requires different actions to be performed. Each of these actions is oriented towards a concrete objective, which must, as a whole, agree with the general reason for the activity. The objective of the action does not always coincide with the objective of the activity. However, it is the concrete situation, that is, the conditions in which this activity is carried out that will ultimately determine through which structures of operations these actions will be carried out.

In this sense, we assume a task as a proposition made by the teacher in the classroom, whose objective is to focus the students' attention on a certain mathematical idea (STEIN *et al.*, 2009). Thus, a task, when proposed by the teacher, becomes the object for the student's activity. However, it is necessary to distinguish the task that the teacher presents to the students and the activity that they develop (or not), depending on their adherence to this task and the ability to carry it out, in a context that is also mediated by the teacher. It should be noted, therefore, that “the setting of the tasks together with related actions performed by the teacher constitute the major method by which mathematics is expected to be conveyed to the students” (CHRISTIANSEN; WALTHER, 1986, p. 244).

Thus, it is essential that the teacher understands that mathematical tasks alone are not enough to generate meaningful mathematical activity, neither for teachers nor for students. We do not have the naive view that, in order to transform teacher education and mathematics teaching, it is enough to propose *good tasks*. However, we recognize the need for teachers to resolve and reflect on the types of tasks, their cognitive demands, so that they can make intentional and appropriate choices and propositions for the activity they intend to trigger and their students' learning.

When solving tasks, teachers in training have the opportunity to build their own strategies, recognize their difficulties, explain their vulnerabilities and negotiate meanings, when sharing these strategies with their peers. In this process of negotiation of meanings, they explain their reasoning, arguments, beliefs, hypotheses and ways of validating the results, presenting reifications. The process of negotiation of meanings is a mechanism for learning, and this process occurs through the interaction between two inseparable processes: participation and reification. The participation process means “[...] a complex process that combines doing, talking, thinking, feeling and belonging. It involves our whole person, including our bodies, minds, emotions and social relations” (WENGER, 1998, p. 56). The process of reification is based on converting abstract aspects into real *things*. It encompasses processes such as “[...] making, designing, representing, naming, encoding, and describing, as well as perceiving, interpreting, using, reusing, decoding, and recasting” (WENGER, 1998, p. 59). Whereas in the process of participation we recognize each other from relationships with other individuals and experiences of meaning, in the process of reification we project our meanings onto the world, so that this projection assumes an independent existence (we do not need to recognize ourselves in it), which takes on a reality of its own in the context of social groups, recognized as focal points of CoP negotiation and related to its accrual regime.

By negotiating the meanings of their resolutions, teachers share their repertoires and can seek a sense of agency for their vulnerabilities and, with this, develop self-confidence that encourages them to find ways to reframe their practice. The dynamics established by the members of the study groups and the experiences shared by the teachers expand the possibility of planning work with mathematical tasks with their students and implementing this dynamic in their classrooms. Teachers share and build a communicative repertoire in order to understand that different tasks can provide different opportunities for students to develop different ways of thinking.

In the meetings, in addition to solving tasks, the MT also had the opportunity to analyze the solved tasks**.** Mathematical tasks can be analyzed from several perspectives**:** didactic possibilities they offer, types of representations involved, variety of ways in which they can be solved, levels of cognitive demand or evoked mathematical aspects.

When working with teachers in training, we chose to treat mathematical tasks according to their cognitive demand because, according to Stein *et al*. (2009, p. 17), “[...] the cognitive demands of mathematics instructional tasks are related to the level and kind of students learning”. The level of cognitive demand of a task is related to the types of mathematical reasoning that are required of students to perform it, as well as the level and type of learning it provides to students.

In Table 1, we present the four categories or levels of cognitive demand for mathematical tasks discussed by these investigators.

Table 1 - Levels of cognitive demand of mathematical tasks

Low-level cognitive demand	High-level cognitive demand
▪ Memorization. ▪ Procedure without connection (with understanding, meaning, or involving concepts).	▪ Procedure with connections (with understanding, meaning, or involving concepts). ▪ Doing mathematics.

Source: Adapted from Stein *et al.* (2009).

Stein and Smith (1998) developed a tool named Task Analysis Guide (Table 2), which consists of a list of task characteristics in each of the four levels of cognitive demand, aiming to provide support to teachers in task analysis, according to cognitive demand.

Table 2 - Characteristics of mathematical tasks according to cognitive demand

<table>
<tr><th colspan="2">Tasks that involve a low level of cognitive demand</th></tr>
<tr><th>Tasks that only require memorization</th><th>Tasks involving procedures without connection to meanings</th></tr>
<tr><td>▪ involve either the reproduction of previously learned facts, rules, formulas, or the memorizing of facts, rules, formulas or definitions.
▪ cannot be solved using procedures because they are not required or because the time in which the task will be completed is too short for using a procedure.
▪ are unambiguous: the issue involves both an exact reproduction of previously viewed material and what is to be reproduced is clearly and directly presented.
▪ have no connection with the concepts or meanings underlying facts, rules, formulas or definitions being learned or reproduced.</td><td>▪ are algorithmic, such that use of the procedure is either specifically requested, or is evident from prior instruction, experience, or location of the issue.
▪ require limited cognitive demand for successful completion and there is little ambiguity about what needs to be done and how to do it.
▪ have no connection with concepts or meanings behind the procedures initially employed.
▪ are focused on producing correct answers rather than developing mathematical understanding.
▪ do not require an explanation or, when they do, they are explanations that focus solely on the description of the procedure that was used.</td></tr>
<tr><th colspan="2">Tasks that involve a high level of cognitive demand</th></tr>
<tr><th>Tasks involving procedures with connection to meanings</th><th>Tasks that involve doing mathematics</th></tr>
<tr><td>▪ focus students' attention on using procedures to develop deeper levels of understanding of mathematical concepts and ideas.
▪ explicitly or implicitly suggest paths to be followed, which are broad and general procedures that have a close connection with conceptual ideas.
▪ usually allow representation in multiple ways, with visual diagrams, manipulative materials, symbols, and problem situations, making connections between multiple representations that help to develop the meanings.
▪ require cognitive effort. Although general procedures can be followed, they cannot be followed without understanding. Students need to engage with conceptual ideas that underlie the procedures to be followed to successfully complete the task and develop understanding.</td><td>▪ require complex and non-algorithmic thinking, and the task does not explicitly suggest a predictable path, instructions for its execution, or an example to be followed, which, when well trained, lead to its resolution.
▪ require students to explore and understand the nature of mathematical concepts, procedures, or relationships.
▪ require high monitoring or high regulation of their own cognitive process.
▪ require students to mobilize relevant knowledge and experiences, and make appropriate use of them on the job during task resolution.
▪ require students to analyze the task and actively examine whether it may have limited possibilities for resolution strategies and solutions.
▪ require considerable cognitive effort and may involve some levels of anxiety for the student, as they do not have an advance list of processes required for the solution.</td></tr>
</table>

Source: Adapted from Stein and Smith (1998).

In the resolution and analysis of mathematical tasks, the MT had the opportunity to recognize, particularly, that cognitively challenging tasks: do not limit and do not explain how to solve them; allow students to build their own strategies based on their previous knowledge; promote the establishment of meanings and relationships between mathematical ideas, situations and concepts; and foster students' creativity and reasoning.

We assume that cognitively challenging tasks are those in which the individual who solves them has no knowledge of procedural or algorithmic tools that determine how to solve the situation and, therefore, will have to build or invent a subset of mathematical actions to solve them (POWELL *et al.,* 2009). This aspect emphasizes, therefore, that a task cannot be classified in absolute terms, but is influenced by the context in which it is explored, by the conditions of the student who solves it, and by the teacher's actions in directing the dynamics in the classroom.

Cognitively challenging tasks allow students to build their own strategies based on the knowledge they already have. In this way, this type of task has the potential to develop students' self-confidence - also teachers in training -, as they can build *their* own paths and develop strategies according to what they interpret (STEIN *et al.*, 2009).

In his studies on mathematical tasks, Ponte (2005) argues that the main point of a task lies in the degree of challenge and the degree of structure (closed or open). In closed tasks, “it is clearly stated what is given and what is requested, and the open task involves some indeterminacy at least in one of these aspects” (PONTE, 2005, p. 20). Open mathematics tasks are considered the most challenging, but to be successful, they must pose good questions (SULLIVAN; CLARKE, 1992), encourage students to work collaboratively, encourage multiple solving strategies, and develop mathematical knowledge, making it more articulate and richer (GUBERMAN; LEIKIN, 2013).

However, especially when talking about cognitively challenging tasks, it is important to take into account the influences that other constraints have on the task, particularly the teacher's action. Corresponding actions are essential and, while encouraging student engagement, collaboration and explanation of reasoning, they should not offer clues, directly or indirectly, orally, gesturally or implicitly, that compromise the students' autonomy and authorship in resolution processes (STEIN; SMITH, 1998; STEIN *et al.,* 2009).

In this sense, Stein and Smith (1998) present a table of factors that can influence the maintenance or decline of the level of cognitive demand (Table 3).

Table 3 - Factors associated with the maintenance and decline of high-level cognitive demands

Factors associated with the maintenance of high-level cognitive demands
1.Scaffolding of student thinking and reasoning is provided.
2.Students are given the means to monitor their own progress.
3.Teacher or capable students model high-level performance.
4.Teacher presses for justifications, explanations, and meaning through questions, comments and feedback.
5.Tasks build on students' prior knowledge.
6.Teacher draws frequent conceptual connections.
7.Sufficient time is allowed for exploration - not too little, not too much.
Factors associated with the decline of high-level cognitive demands
1.Problematic aspects of the task become routinezed (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem).
2.The teacher shifts emphasis from meanings, concepts, or understanding to correctness or completeness of the answer.
3.Not enough time is provided to wrestle with demanding aspects of the task, or too much time is allowed and students drift into off-task behavior.
4.Classroom-management problems prevent sustained engagement in high-level cognitive activities.
5.Task is inappropriate for a given group of students (e.g., students do not engage in high-level cognitive activities because of lack of interest, motivation, or prior knowledge needed to perform; task expectations are not clear enough to put students in the right cognitive space).
6.Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect explanations are accepted; students are given the impression that their work will not "count" toward grade).

Source: Stein and Smith (1998, p. 274).

Thus, articulating the results of our research and educational practices with theoretical aspects, we realized that analyzing the characteristics of a mathematical task (or mathematical tasks), taking into account their cognitive demand, allowed teachers in training to discuss the role they have in working with these tasks in the classroom, namely the importance of:

- choose assignments that suit your teaching objectives;
- initiate a teaching process that prioritizes challenging tasks, in which students can make meaningful connections, or relate mathematical ideas and concepts;
- recognize that tasks can express more than content;
- understand how the tasks influence their teaching and, consequently, the students' learning;
- provide a learning environment during mathematics lessons; and

- realize the impact of their actions on the students' teaching and learning processes.

In this way, when solving and analyzing mathematical tasks in formative actions, the MT were able to reflect, particularly, on the characteristics of cognitively challenging tasks; the potential of this type of task for student learning; and their role as a teacher in establishing dynamics consistent with the desired activity in exploring the task in the classroom.

Selection, adaptation, design and exploration of mathematical tasks

In the interactions that took place in the groups, the MT produced new meanings regarding the *selection, adaptation and design of tasks and their role in the process of exploring these tasks in the classroom*.

Tasks assume an important role in the teaching and learning processes, as they influence students' learning as they direct their attention to particular aspects of mathematical content and point to ways of processing information **(**DOYLE, 1983**).** Therefore, when selecting, adapting, preparing and organizing work with tasks in the classroom, the teacher needs to be clear that the aspects to be considered go beyond the contents to be mobilized for their realization. These aspects encompass cognitive processes related to understanding, the definition of strategies and procedures, and the validation of the resolutions presented by the students.

The type of thinking mobilized by students in solving a mathematical task is closely related to the nature of that task (STEIN; SMITH, 1998; STEIN; LANE, 1996; STEIN *et al.*, 2009; SULLIVAN *et al.*, 2011). According to Steele (2001, p. 42), “no other decision that the teacher takes has such a great impact on the students' opportunities to learn and on their perception of what Mathematics is, as the selection or design of tasks”.

Some MT in the programs reported that, when planning their classes, teachers choose tasks based on the contents worked or the presence of these tasks in textbooks. In this context, tasks can become synonymous with lists of exercises, in which the students' work is limited to solving them mechanically and, in some cases, having as a starting point a *model exercise* previously explained by the teacher. In this sense, problematizing with teachers the characteristics of different types of tasks and their potential for activities in the classroom, from their selection, adaptation and design processes, assumes a central role in MT education contexts based on mathematical tasks.

Chapman (2013), based on the guidelines of the NCTM (1994) and other research, points out that selecting, adapting and design cognitively challenging mathematical tasks, in order to optimize their potential for student learning in the classroom, demands knowledge from the MT that need to be considered in their education process.

Each task must be chosen, adapted or design in order to guide the student to develop forms of reasoning and strategies that allow them to go beyond the simple memorization of facts or procedures. According to Stein and Smith (1998, p. 269),

> tasks that ask the students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student to think.

In the process of selecting, adapting and designing mathematical tasks, the MT needs to: understand the nature of *advantageous tasks* in terms of mathematical content and learning opportunities; to know the levels of cognitive demand of tasks and their relation with the class objectives; apreciating students' understandings, interests, and experiences and relate these to the diverse ways in which they learn; and knowing what aspects of a task to highlight to promote student learning in the classroom.

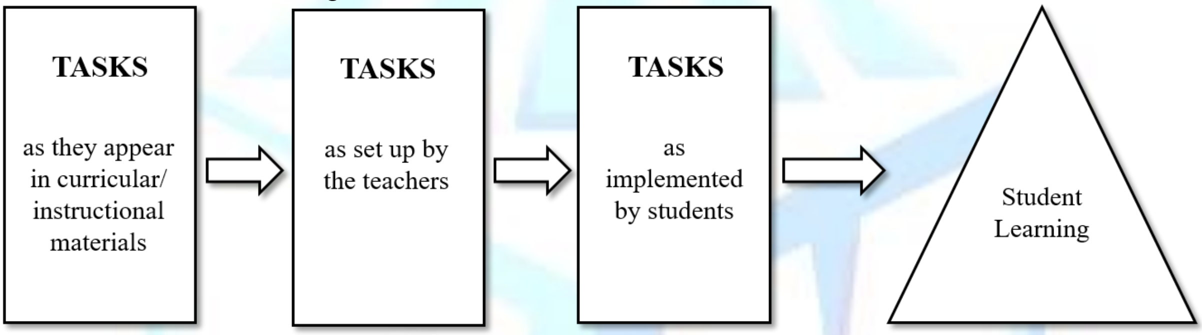
When determining the level of cognitive demand of tasks, Stein *et al.* (2009) state that it is important for the teacher to be clear about *which students* they are intended for without distorting the focus to superficial characteristics (requirement to use manipulative material, use of real-world context, involving several steps, actions, or judgments, using diagrams, being a word problem, etc.).

In addition to selecting, adapting and designing cognitively challenging mathematical tasks, it is important for MT to reflect on how to organize and manage students' work in exploring these mathematical tasks in the classroom, what to ask and how to support them, as the way the teacher explores these tasks can influence the way students make sense of Mathematics.

When negotiating meanings regarding the different types of questions that can be formulated during the exploration of mathematical tasks, the MT expressed the existence of difficulties in elaborating and proposing questions that help to challenge and keep students involved in complex forms of thinking and reasoning (STEIN; SMITH, 1998). At the same time, they had the opportunity to express their understanding regarding the successes and limitations for maintaining the level of cognitive demand of mathematical tasks.

When proposing tasks to students, the MT has certain expectations that may not be fulfilled. Depending on how they direct their pedagogical action, changes may occur in the cognitive demand of a task during their own development in the classroom. Thus, a task classified as challenging may not provoke high-level thinking and reasoning as intended, due to the way, for example, that students work on that task. According to Stein *et al.* (2009), tasks take on a *life of their own* after being introduced into the classroom setting, being influenced by the actions of the teacher (who proposes them) and the students (who carry them out). According to these authors, tasks go through steps, named by *mathematical task phases* (Figure 1).

Figure 1 - The Mathematical Tasks Framework



Source: Stein et al. (2009, p. xviii).

The first phase is related to how the tasks are proposed in the curricular materials or how they are adapted or design by the teacher. This phase involves cognitive demands required of the solver. The second phase is related to the tasks proposed by the teacher in the classroom.

> The *setup phase* includes the teacher's communication to students regarding what they are expected to do, how they are expected to do it, and with what resources. The teacher's setup of a task can be as brief as directing students' attention to a task that appears on the blackboard and telling them to start working on it. Or it can be as long and involved as discussing how students should work on a problem in small groups, working through a sample problem, and discussing the forms of solutions that will be acceptable” (STEIN *et al.*, 2009, p. 15).

According to the authors, in this phase, it is common for teachers to change the cognitive demand of the task in relation to how it was initially thought. This modification may occur on purpose or unintentionally.

The third is the phase when the students carry out the task, as they actually carry it out. This phase begins as soon as the students start working on a task and continues until, together with the teacher, they start working on a new mathematical task. In this phase, the behavior of the teacher and the students is considered essential for the development of the task. Both in the second and third phases, it is the moment when “[...] tasks leave the printed page and become entangled with the thoughts and actions of the teachers and students who give them life during classroom lessons” (STEIN *et al.*, 2009, p. 13).

During the phase of carrying out the task, the cognitive demand can be easily modified, especially the high-level ones, which can assume fewer demanding ways of thinking from the student. According to Stein *et al.* (2009), there are several factors that can collaborate with the maintenance or decline of the high level of cognitive demand of tasks in the classroom and cause them to undergo changes (STEIN; SMITH, 1998; STEIN *et al.*, 2009).

Furthermore, it must be considered that the context as a whole is also a factor that influences the effectiveness of a task in promoting the intended type of thinking and activity. Thus, it is also necessary for the teacher to take into account the individual and material conditions; established practices and forms of work; students' expectations of themselves and each other; as well as students' sense of self-confidence, agency (mathematics and social) and identity (WATSON; MASON, 2007).

In this way, knowing and discussing the levels of cognitive demand of the tasks and their development phases can enable the teacher to direct their gaze towards the choice, adaptation and design of tasks that are connected with their learning objectives, with the characteristics of their students and with the material resources available for their practice. Reflecting on their actions in the classroom, bearing in mind their consequences and influences, can help the teacher to identify factors that affect the proposition and exploration of tasks in the classroom, consciously or unconsciously. The role of these reflections will be discussed in the next section.

The choice, adaptation, preparation of tasks and identification of factors that affect their proposition and performance in the classroom involve knowledge inherent to the teaching profession that is relevant for decision-making related to their teaching practice. This knowledge pervades mathematical, pedagogical aspects (general and specific to Mathematics), as well as beliefs, conceptions and images of the MT about themselves, their teaching profession, Mathematics and the teaching of Mathematics.

Based on the complexity that involves this knowledge, therefore, research suggests that the choice, adaptation, design and exploration of mathematical tasks express promising practices in the promotion of experiences and reflections with a view to the development of professional knowledge of Mathematics teachers (STEIN; SMITH, 1998; LILJEDAHL; CHERNOFF; ZAZKIS, 2007; GUBERMAN; LEIKIN, 2013; CYRINO; JESUS, 2014; ESTEVAM; CYRINO; OLIVEIRA, 2018; MAGGIONI; ESTEVAM, 2021). In summary, these studies, associated with our research and educational experience, show that this practice allows the MT to:

- understand the direct relationship between the characteristics of mathematical tasks and the establishment of well-defined teaching objectives;
- select, adapt and design (different) tasks in line with your teaching objectives, students' characteristics and available conditions, so that these actions are intentional and not arbitrary;
- critically articulate curricular guidelines, support manuals and the desired learning objectives;
- evaluate mathematical tasks based on the learning opportunities offered, focusing on the development of meaningful mathematics, to the detriment of superficial and uncontributive characteristics;
- develop the habit of listening to students and developing their sensitivity to understand students' thinking and possible emerging obstacles in the teaching process.
- challenge approaches dominated by procedures oriented towards mechanical memorization and the mechanical use of routines and algorithms, in order to challenge you to make sustained choices based on your students' needs and your purposes; and
- reflect on mathematical and pedagogical knowledge (general and specific to Mathematics) associated with the practice they carry out and, possibly, the one they intend to carry out.

Reflections and discussions about working with mathematical tasks in the classroom

The reflections and discussions that took place in the study groups, regarding the experience that the MT had in *exploring tasks in the classroom*, allow identifying signs of changes in the perception of these teachers regarding: working with students, confronting the beliefs that permeate their pedagogical practice, and their professional view on the processes of teaching and learning mathematics.

Discussing the work with students in solving mathematical tasks with a high level of cognitive demand allowed the MT to reify the image they had regarding student performance, at the same time that they modified their modes of participation in the study group, as in which they felt safe to express, argue and defend their ideas.

Reflecting on the tasks that are proposed to students can be a way for teachers to be aware of the teaching and learning processes and assess the impact that their decisions have on these processes. This is because, as Christiansen and Walther (1986, p. 264) point out, “the problem is to identify means by which the teacher can promote a unified conception – within the learner – of the role of task-and-activity, of learning, of mathematics and of his personal and conscious control of his own learning process”.

The MT's pedagogical practice is not limited to their actions. It is a product of the intersection of different contexts and is directly influenced by competing practices (society, educational policy, curriculum, school culture, availability of access to information, school supervision, among others). The classroom, as a social learning fabric, constitutes an environment in which the interactions of all partners, teachers and students, are based on knowledge and beliefs that explain the culture and social contexts to which they belong.

Upon arriving in the classroom, the teacher brings with them a series of information and knowledge already constituted in the process of design the mathematical tasks, which allows them to constitute schemes that articulate, through dialogue, the students' knowledge and school mathematics. In this dialogue, the student reveals their beliefs and prior knowledge elaborated in everyday life or in a school context, constituted by internal and intuitive representations.

The teacher, as directly responsible for organizing teaching, plays a crucial role in social interactions in the classroom. When planning mathematical tasks and their exploration, the MT must propose situations so that they and the students can expand, modify and construct meanings. Similarly, you need to be open to modifying your professional view of the processes of teaching and learning mathematics.

The action of reporting and analyzing the report of other teachers regarding the development of tasks in the classroom led MT to: evaluate the management of the time allocated to students to solve tasks; understand that the level of thinking at which the student works can determine what they will learn; value the production of students; think about their actions when monitoring work with tasks in the classroom; recognize the importance of mathematical communication in the classroom and the role of the teacher's communicative actions. This action made some teachers feel challenged to rethink and modify their practices.

The management of time allocated to work with mathematical tasks was the subject of wide debate among teachers in training. Stein and Smith (1998) argue that one of the factors that can cause a decline in the high-level cognitive demand of a task is the lack or excess of time for students to deal with its challenging aspects.

Similarly, when problematizing sustained practices in the exploration of tasks, teachers presented reports and reflections that show perceptions that the level of thought at which the student works can determine what they will learn. In this sense, MT had the opportunity to discuss practices that highlight the students' thinking, according to the summary in Table 4.

Table 4 - Teaching practices that give visibility to students' thinking.

Making space for student thinking
▪ Elicit students' ideas. ▪ Give students time to think. ▪ Recognize and publicly associate an idea with a student.
Attending to and taking up student ideas
▪ Pause during instruction to consider an unsolicited idea, question, or response. ▪ Re-voice or rephrase a student's idea for the class to consider.
Pursuing student thinking
▪ Ask students to explain how they got an answer. ▪ Ask students to explain their reasoning. ▪ Press for further explanations. ▪ Pose alternative examples/questions for students to think about as a way to check and clarify student understanding.

Source: Sun and Van Es (2015, p. 205).

At the same time, the MT were able to understand the importance of organizing a teaching process in which the path chosen by the student to solve a task is more valued than the correct answer, since only the final solution can say little or nothing in relation to the student's effective knowledge, and wrong solutions/answers may indicate promising strategies for the pedagogical process (ESTEVAM *et al.*, 2019). Teachers who engaged in working with cognitively challenging tasks report that the focus of discussions should be the ideas elaborated by the students, and not just the correct answers, which should also be considered, but not as a priority. The MT emphasize that what should be valued in these tasks are the ideas, how the students did and thought.

When reflecting on the work of monitoring the development of tasks in the classroom, the teachers reported several actions, including observing, listening and interpreting the students' mathematical thinking and their resolution strategies, in order to understand how the whole process involved occurred and verify whether the solutions presented are consistent with what is expected from the task.

According to Sullivan *et al.* (2011), a task has the potential to engage the student in an intellectual activity when it can be solved in different ways. This engagement can develop the student's autonomy and self-confidence, so that they are not afraid to expose their ideas in front of the teacher and their peers, as they feel that their ideas will be respected and their thinking will be valued (STEIN; GROVER; HENNINGSEN, 1996). According to Smith and Stein (2013, p. 2), “students learn when they are encouraged to be authors of their own ideas and when they are held responsible for reasoning and understanding key ideas”. They are often used to waiting for the teacher's correction, the right way to solve the task, as they do not trust their strategies (SCHOENFELD, 1992). In other situations, a question, a word or a gesture from the teacher is enough for them to erase their resolutions, without considering what they did, and start to adopt what was presented by the teacher. In the classroom, both the teacher and the students need to value the different solution strategies for a task, because in this way they will be expanding their learning opportunities (SMITH; STEIN, 2013).

When discussing the established dynamics and the resources used in classroom, the MT were able to understand the importance of the intentionality that motivates each decision. Teachers realized that learning opportunities are not simply related to the fact of putting students in groups, or in front of manipulative materials, or computational resources, but with the reflections that are promoted in the process of solving tasks, as well as in the discussion and systematization of the ideas involved in these resolutions. Working with mathematical tasks will be powerful insofar as the dynamics and resources promote reflective inquiry (ARTIGUE; BLOMHØJ, 2013), which offer opportunities to draw students' attention to aspects of mathematics and to mobilize mathematical thinking.

Recognizing the importance of mathematical communication in the classroom was also discussed in the study groups. In these discussions, the MT highlighted that promoting mathematical communication in classroom environments is a fertile strategy for developing students' mathematical thinking. Although teachers use communication strategies in the classroom, such as conversations, discussions and records of mathematical activities, the relationship they establish between communication and learning is still fragile in their practices (KAYA; AYDIN, 2016).

According to Ferreira *et al.* (2014), communicative processes can be understood as persuasion (when communication is assumed as transmission of information) or negotiation of meanings (when social interaction occurs). In the transmission of information, there is a communicator whose objective is that the listener reacts in the way they expect, acting in a similar way to the way they were communicated. As a social interaction, communication is a social process in which subjects interact, exchanging information, influencing each other in the construction of shared meanings. In this way, communication assumes the function of creating and maintaining consensus and understanding between individuals, through the interpretation of the other, in an action of complementarity and mutual recognition.

The perspective of communication assumed in the study groups was that of social interaction. According to Rodrigues, Cyrino and Oliveira (2018), when discussing work with tasks in the classroom, teachers in training defended the importance of the teacher recognizing that the student learns mathematics from a set of actions that go beyond the listening, but also involve talking, conjecturing, doing, asking, answering, arguing, recording, etc. In a classroom environment where this mathematical communication perspective is employed, students are expected to listen, comment and reflect on their own mathematical thinking and that of their peers (PAPE; BELL; YETKIN, 2003). In this way, the teaching process and the way the teacher communicates with the students need to be thought out, in order to provoke and guide these actions, with intentionality and organization. Teacher feedback is an important instrument in the context of these actions.

In classroom communication as a social interaction, the teacher can promote negotiation of meanings through specific actions, such as explaining, questioning, listening and answering. These “communication processes between individuals, through discursive acts, include silences, gestures and behaviors, looks and postures, actions and omissions” (RODRIGUES, 1990 apud GUERREIRO *et al.*, 2015, p. 281).

According to Wenger (1998), it is through the process of negotiation of meanings that learning occurs. We consider learning to be both an individual and a collective process, resulting, respectively, from students' interaction with mathematical knowledge, in the context of a certain mathematical activity, and also from interaction with others (classmates and teacher), focusing on negotiation processes of meanings. Kaya and Aydin (2016) point out that mathematical communication is mainly related to the mutual mathematical understanding of participants in a class.

Based on this understanding, the MT were able to discuss teachers' communicative strategies to improve students' mathematical thinking, based on those problematized by Cooke and Buchholz (2005), namely: offering opportunities for self-expression; serve as a facilitator in the expression of ideas and language; provide opportunities for students to connect new understandings to prior knowledge; connecting routine classroom tasks to mathematics; ask a variety of questions; and encourage the use of appropriate mathematical terms.

In this way, mathematical communication can promote the production of meanings and the dynamics of a mathematics class (RODRIGUES; CYRINO; OLIVEIRA, 2018). In other words, mathematical communication can be seen as a transversal ability to learning and as a methodological guideline (SERRAZINA, 2018).

The development of students' mathematical communication capacity as a transversal capacity involves not only the action of expressing their ideas, but also interpreting and understanding the ideas presented to them - by colleagues, by the teacher, by teaching materials, etc. - as well as the action of actively participating in discussions about mathematical ideas, processes and results (SERRAZINA, 2018). Sfard (2001) emphasizes the importance of mathematical communication when describing thinking as a case of communication. In this way, thinking constitutes a dialogic effort, in which questions are asked, possible solutions are investigated, and one reflects on them. This supports the perception of teachers in training that asking students to explain how they thought is an essential aspect of working with cognitively challenging tasks. When explaining how they solved the task, the student needs to reflect on its resolution, organize their ways of thinking, their records, so that later they can orally communicate their ideas and, therefore, their reasoning. By demanding a justification for the strategy used by the student, the teacher can support them in explaining their mathematical thinking and collaborate so that they (re)structures themselves cognitively.

It is important to remember that communicating is not just talking or writing – gestures or even objects can be important vehicles for communicating ideas. In mathematics class, therefore, oral, visual, gestural, iconic, object or written communication can be promoted.

Oral communication can promote interaction between students and between students and the teacher, so that students can mobilize their own ideas. For example: when explaining to a colleague or to the teacher what they did, the student needs to establish connections between ideas that they want to explain and between their ideas with others that are shared by the interlocutors (student or teacher). An explanation starts from a question, whether explicit or implicit, proposed by another student or by the teacher. In the search for these connections with other subjects, the student mobilizes concepts or procedures that they have already constituted, even if partially, to find meanings for what they are learning (new knowledge). By explaining their procedures and strategies to an interlocutor, the student can identify their mistakes, correct them and gain confidence in themselves.

Mathematical communication allows for active, not merely reactive, involvement on the part of students in critical-reflective listening and in expressing their own thinking. It is necessary to consider that there are limitations in oral communication, as well as in any other form of communication, such as writing.

Through written communication, the student can develop different types of representations (for example, natural native language, numerical, symbolic, graphic, tabular, pictorial, algebraic, geometric, analogies, diagrams, figures, schemes, etc.). In this form of communication, the student needs to select the most appropriate linguistic forms for different situations and explain their reasoning with coherence, logic, clarity and vocabulary appropriate to the content. In addition to being a means of communication, writing can promote learning and discovery (SABRIO; SABRIO; TINTERA, 1993).

> To build a text, students need to examine their ideas and reflect on what they already know, becoming aware of their difficulties. Thus, students write to learn and learn by writing mathematics. It should be noted, however, that mathematical writing does not only cover the action of writing a response to a task, but it is also about explaining the reasoning that led to the response. [...] The process of explaining ideas to others, with the aim of being understood, promotes the evolution of your own understandings. The act of writing, forcing the explanation of conjectures and conclusions, constitutes an opportunity to elucidate, organize and consolidate the student's thinking, and to develop mathematical knowledge, the ability to solve problems, the power of abstraction as well as the ability to reason and confidence in yourself [...] (MARTINHO; ROCHA, 2018, p. 34).

Associating these different perspectives, the process of explaining ideas to others, with the aim of being understood, promotes the evolution of their own understanding of mathematical concepts and fundamentals. These aspects emerged in a similar way in the reflections of the teachers participating in the study groups.

The different mathematical representations (such as numerical, symbolic, graphic and verbal) require frequent use so that students can give meaning to them and use them in their practice. Formal and rigorous mathematical language does not need to be imposed but can arise naturally and become commonplace due to the necessity of its use.

Communication as a methodological guideline, therefore, is associated with the teacher's practice of favoring a meaningful mathematical discourse, asking pertinent questions, using and relating mathematical representations, among other aspects. In this process, the MT were able to understand that it is the role of the teacher:

i) promote social interactions with dialogic approaches in the development of the task by the students.

ii) give *feedback* based on students' responses to the development of mathematical activity.

iii) take into account students' learning experiences, their willingness to question, discuss and reflect on different ideas in the classroom;

iv) favor a meaningful mathematical discourse, so that students can build a shared understanding of mathematical ideas, intentionally exchange ideas, articulate and justify their ideas, reason based on their own ideas and those of others (classmates and teachers), and develop a deep understanding of mathematics;

v) make emerge, simultaneously, the individual logic and the collective logic (in the negotiation of shared meanings);

vi) consider different types of questions for different purposes, such as recognizing information, exploring thought, focusing, inquiry, making mathematics visible, encouraging reflection and justification;

vii) not answer directly to a question asked by the student, but create a new question, provide an explanation or additional information, in order to avoid the immediate validation of the answer presented by the student;

(viii) maintaining the cognitive level of the initial task, dealing with incorrect or incomplete answers, making them the subject of discussion;

(ix) support the instructional process, in decision-making, participating in the exploration and negotiation of meanings with students, thinking with them and not for them.

In summary, the action of reporting and analyzing the reports of other teachers regarding the development of tasks in the classroom allowed the MT to: share classroom experiences in a critical and respectful way, issuing and defending ideas; indicate that they could have made choices similar to those reported; express their understandings regarding the successes and limitations of those involved in the development of tasks; offer suggestions for class management; demonstrate a positive reaction to the suggestions and provocations of others to incorporate them into their teaching practice; reveal changes in their way of acting in the classroom; recognize their difficulties and the need for new learning; valuing the work of others (students and teachers); and recognize the role of communication and established dynamics as essential to the intended learning.

Closing remarks

The Mathematics Teachers in training were challenged and encouraged to critically analyze their practices; to reflect on knowledge, beliefs and conceptions; and to study strategies for working with cognitively challenging tasks. In this process, there is evidence of attribution of meanings to elements that constitute the practice of teaching and learning Mathematics, particularly those involving work with mathematical tasks in the classroom. In this way, professional learning is evidenced that manifests itself in changes in the patterns of teacher participation in the practices they carry out (VILAS BOAS; BARBOSA, 2016). Based on the dual processes that support learning in social terms - participation and reification - we consider that both the changes in practice carried out in the classroom and the meanings attributed to sustain them are indicative of learning, even reverberating in the identity of these professionals. In this way, we admit that changes in the patterns of teacher participation in the practices they carry out can manifest themselves, in addition to what they do, in what they say (and what they do not say), especially in the meanings and interpretations that support their sayings and doings, which make use of knowledge, beliefs, conceptions and images. However, we have no way of assessing whether the knowledge produced by the teachers participating in the program was incorporated into the classroom, or how long it will take for this to happen. We only know that the reflections, reports and discussions regarding the development of tasks in the classroom show evidence that they consider this perspective of working with mathematical tasks plausible.

In these reports and discussions, we identified evidence of the development of self-confidence by teachers participating in the program in/for work with cognitively challenging mathematical tasks. At the same time, the educational actions enabled those involved to think about their image as teachers, how learning changes who we are and creates personal stories of transformation in the educational context (CYRINO, 2017).

The way in which a mathematical task is presented, developed, worked on and brought to a conclusion influences the MT's worldview, including beliefs and conceptions regarding mathematics, its learning, how it can be taught, and what the role of its pedagogical practice in the constitution of future generations (CYRINO, 2017). In this sense, Watson and Mason (2007) emphasize that teacher education must consider that, for teachers, learning and action are the same thing: their professional choices of actions are the manifestation of what they have learned or are learning and, therefore, the type of task and the dynamics explored in educational contexts significantly influence the practice that MT bring to the classroom, their teaching, learning and, therefore, the movement towards the constitution of their professional identity (CYRINO, 2016; 2017; 2021).

However, it must be confirmed that exploring tasks in teacher education does not consist only in proposing and solving tasks, similar to the activity expected of students in the classroom. The work with mathematical tasks in educational contexts raises other purposes that aim at the movement towards the constitution of the participants' professional identity. To this end, it should encourage reflection on the pedagogical practice and the impact of the teacher's decisions on the teaching and learning processes, involving: intentional, sustained and situated planning, with the outline of the task(s) to be explored; questions to be associated to trigger reflective processes that articulate the discussions to the actions to be undertaken by the teacher and (future) MT during the educational program; opening for negotiation and flexibility for articulations to the demands manifested by the participating teachers; consideration of teachers' knowledge, beliefs, feelings and conceptions as a starting point for the practices developed; and commitment to the learning of those involved and the development of their autonomy.

Educational actions involving mathematical tasks can involve different practices, and Table 5 summarizes those identified in our reflections and interpretations, based on the established approaches.

Table 5 - Potential practices associated with educational actions involving mathematical tasks

Solve and analyze math tasks
▪ analyze the solved tasks, and discuss types of tasks, characteristics and potentialities. ▪ feature cognitively challenging tasks. ▪ discuss the role of the teacher in working with mathematical tasks in the classroom.

▪ putting oneself in the role of a student and constructing meanings for strategies, procedures, reasoning and ways of supporting students' activity in the classroom. ▪ establish dynamics consistent with the intended activity.
Select, adapt, design, and explore mathematical tasks
▪ relate types of tasks and student thinking. ▪ know the levels of cognitive demand of tasks and their relationship with the lesson objectives. ▪ reflect on the organization and management of student work. ▪ discuss about crafting questions that keep students engaged in complex forms of thinking. ▪ understand teacher actions that can influence the cognitive demand of the task. ▪ situate mathematical tasks in the context of practice.
Reflect and discuss working with mathematical tasks in the classroom
▪ reflect on the work of and with students, based on (different types of) mathematical tasks. ▪ establish practices to encourage mathematical thinking in students. ▪ opposing beliefs and conceptions about teaching and learning mathematics. ▪ (re)thinking their actions and their influence on student activity. ▪ recognize the importance of mathematical communication in the classroom. ▪ share experiences emitting, defending and confronting ideas and (re)elaborating. mathematical understandings, as well as teaching and learning mathematics.

Source: the authors.

Finally, it is important to point out some dilemmas associated with educational activities based on work with tasks that require further studies, such as: dealing with the immediacy of teachers who seek directly replicable situations; evaluate the suitability of the proposed tasks for the material conditions, classrooms, current curricula and teachers' knowledge; and consider particularities in working with tasks in the different fields of Mathematics, as well as in the different levels of education.

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