

Promoting conceptual development in prospective mathematics teacher online course

Promovendo o desenvolvimento conceitual em um curso on-line na formação inicial em matemática

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RESUMO

O desenho e a implementação de avaliações interativas e práticas de melhoria da aprendizagem constituem um desafio para os ambientes on-line. A pandemia de Covid-19 evidenciou esta complexidade e dificuldade, mesmo para os investigadores. Este artigo ilustra parte de um processo de avaliação realizado em ambiente online com futuros professores de matemática implementado durante o período de pandemia. O instrumento de avaliação é denominado desenvolvimento conceitual. A experiência integra um projeto de pesquisa sobre interações, instrução e aprendizagem em ambiente virtual de aprendizagem com futuros professores de matemática. O estudo aborda questões sobre avaliação on-line em sala de aula, sugerindo um instrumento de autoavaliação que pode promover o compartilhamento de ideias, autonomia e análise conceitual. A ferramenta também tenta superar o momento de busca de informações on-line para promover a reflexão analítica e o aprendizado no meta-nível.

PALAVRAS-CHAVE: Formação de Professores, Ambiente virtual, Integração de tecnologia, Isometria.

ABSTRACT

The design and the implementation of interactive assessment and learning improvement practices is a challenge for online environments. The Covid-19 pandemic highlighted this complexity and difficulty, even for researchers. This paper illustrates part of an assessment process provided in an online environment with prospective mathematics teachers implemented during the pandemic time. The assessment instrument is called conceptual development. The experience integrates a research project concerning interactions, instruction, and learning in a virtual learning environment with prospective mathematics teachers. The study addresses issues on online classroom-based assessment suggesting a self-evaluation instrument which can promote sharing ideas, autonomy, and conceptual analysis. The

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tool also tries to overcome the moment of searching online information to promote analytical reflection and learning on the meta-level.

KEYWORDS: Teacher Education, Assessment, Virtual environment, Technology integration, Isometry.

INTRODUCTION

In online teachers training programs one challenge is to critically engage the prospective mathematics teachers (PMT) in practices to revise, reflect and deeper understand their conceptual development (Lager; Lavonen, 2023). Researchers in mathematics education using virtual learning environment (VLE) emphasize that the assessment of learning through synchronous or asynchronous interactions needs to consider the discursive realm and the particularities of each communicative space (chat, forum, WhatsApp etc.) of the environment (Bairral, 2023). Considering this utterance singularity, the assessment process should provide more analytical than descriptive reflection (Brandes; Boskic, 2008). What strategies can be used to assess the improvement of this analytical process?

When technology is used as a teaching tool without adapting it to human cognition, the learning becomes focused on the tool rather than on the learner (Ovadiya; Segal, 2017). The integration² of digital technologies (DT) into tasks and the careful analysis of content on the internet are also other challenges in mathematical assessment in VLE. Integration can take place in the design of the task and/or throughout the assessment process. In this article I³ illustrate one possibility for the second case, i.e., in which different technologies are integrated into the evaluative production of the PMT when reflecting on their conceptual development (CD) of isometries⁴ in an online course⁵. The analysis is geared towards the question: What particularities does a recording and shared communication tool bring to the prospective mathematics teacher's learning process? This study contributes indicating the importance of the teacher's role and the design of VLE to promote learning through different DT and interactive strategies⁶.

² In the same sense of Bittar (2011) and, Ovadiya and Segal (2017).

³ Since I'm sharing my experience teaching the course under analysis presented here, I decided to write in the first person.

⁴ This study integrates a research project granted by CNPq and Faperj.

⁵ I dedicate this article to Paulo Abrantes, one of my greatest inspiring changes in my assessment practice.

⁶ A previous version of this paper was discussed and published in ICME-15 (2024) Proceedings on TSG-3.14 (Research and Development in Assessment in Mathematics Education).

ASSESSMENT PRACTICE IN VIRTUAL LEARNING ENVIRONMENT

Evaluation cannot be reduced as a measurement practice. Assuming mathematics learning as changing discourse (Sfard, 2008) that takes place in a continuum of interactive processes managed by the teacher, seven principles are guiding my online classroom-based evaluation (Abrantes, 1995; Bairral, 2023): **(1)** assessment must be seen as a continuous process and closely related to motivation, interaction, the utilization of different instruments and teaching strategies that enable the improvement of learning; **(2)** assessment have to itself generate new learning situations; **(3)** assessment may allow subjects to make explicit their conceptual or procedural understandings; **(4)** assessment should take place in an environment of transparency and confidence, in which the criticisms and suggestions are seen as natural and productive for the learning of all (students and even the teacher); **(5)** evaluation of conceptual understanding must be considered as personal processes potentialized by collective interactions integrating different DT and considering the situated context, it must be focused on interaction and complicity established among the classroom subjects. Although writing is the technology that temporarily stabilizes a certain conceptual development, **(6)** other forms of expression of thought (drawings, constructions in geometric dynamic environment, graphical, pictorials, gestures, touches on screen, etc.) should also be encouraged and considered in assessment practice. Finally, **(7)** assessment practice can also be a factor to be considered when redesigning the VLE and not changing only the technology itself.

If learning mathematics is a change of discourse within a **continuum of** interactive processes managed by the teacher, but in an environment in which the teacher is also involved, according to Sfard (2008, p. 255-257), it is possible to distinguish between two types of learning:

Object-level learning that expresses itself in the expansion of the existing discourse attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives; this learning, therefore, results in endogenous expansion of the discourse; and

Meta-level learning, which involves changes in meta-rules of the discourse and is usually related to exogenous change in discourse. This change means that some familiar tasks, such as, say, defining a word or identifying geometric figures, will now be done in a different, unfamiliar way and that certain familiar words will change their uses.

Chart 01 - Conceptual Development instrument details

Conceptual Development instrument = chart + diary + oral presentation										
	Chart with different versions (V)								Diary	Sort oral presentation
Subject	Concept (no more than 4 by semester)	V1 (day x)	V2 (day y)	Obs. between 1st and 2nd	V3 (day z)	Obs. between 1st, 2nd, and 3rd	V4 (day ?)	Obs. between 2nd, 3rd, and 4th etc.		
	-Suggest by the teacher on the first-class using Google Form. -You can decide for a broader idea (isometry, for instance) or a specific topic (rotation etc.).	-share		-In the first two versions the ideas tend to be freer. They are emerging aspects from their mind, previous experience		-From this moment is expected PMT to add some ideas from readings suggested by the teacher or by the colleagues	-The number of the version is a teacher decision. -In the analysis presented here we applied.		-Is a personal writing reflection about the role process, including ideas from some provided reading on the course.	-Presentation around 10 minutes. - There is no template. The PMT decide the format (video, power point, conceptual map etc.) to share and explain their conceptual development. -Depending on the complexity of the concept under improvement or the number of PMT on the

	<p>-The file is shared by the teacher as soon as it is filled by all PMT.</p> <p>-Each PMT creates their own file and saves it with their name. This file will be used and shared (on the environment) through the course.</p>			s about the topic etc.		on their exchanging.	only three versions.			classroom the teacher could suggest focusing on some specific concept or split the group according to some concept.
Moment	Diagnostic	Formative, processual							Summative	
Formative process	Previous knowing, invitation	Implication, contrast, and refining ideas							Summary, self-reflection	Socialization, commitment

Source: Author

During the moment of presenting each version, I randomly select some PMT to comment on their processes and explain their intentions for the next step. Theoretical aspects of the readings and studies carried out during the course are expected to be included from the third version. As a summative moment, each PMT presents (using different technological resources) their development process in each concept under analysis. This moment can be self-assessment or done by another PMT (peer-assessment), previously agreed in the classroom. The focus on this moment is not on the presentation format, but on the reviewed ideas and conceptual understanding and self-criticism.

The subjects were prospective mathematics teachers in the process of completing their degree. The content of isometries is absent from the curriculum, and they had only studied one subject (Group Theory) which dealt with this topic, but with an algebraic focus. The course was not focused only on this content. It was one of the concepts discussed on the course and suggested on CD tool⁷. The course was not compulsory; it had 60 hours and took place 100% online in the second semester of 2021⁸. I chose to analyze this concept to check if they observe the geometric and functional aspects (Hollebrands, 2003) and to have a better conceptual understanding about isometries. The data were analyzed through written records, files generated and shared, videos and other resources used for presentations, and recordings of the presentations. The semantic analysis was prospective, i.e., what the prospective mathematics teachers produced and reflected on their learning.

⁷ The other ones were technology, learning environment and virtual environment. Plane transformations are one of the concepts being studied in our research group using mobile devices with touchscreen (Assis; Bairral, 2022) or virtual learning environment (Bairral; Silvano, 2023).

⁸ Still pandemic time in Brazil and only online courses at my university.

RESULTS

I will summarize and present ideas from two PMT – Matheus (MG) and Pricy (PR) – concerning isometry and two PMT – Elisa (ELI) and Viviane (VIVI) – focused on translation. MG begins by saying that isometry is the characteristic of muscle contraction, something related to what is practiced in the gym.

Chart 02 - MG's three versions

Version 1 (MG)	Version 2	Obs.1	Version 3	Obs.2
Isometry is the characteristic of muscle contraction , exercising at a standstill.	It's a concept in mathematics where we can rotate and translate geometric shapes without changing their measurements	In version two I was able to understand what isometries all are about. <u>The lessons</u> contributed to this.	It's a concept in mathematics where we can rotate and translate geometric shapes without changing their measurements.	From version 2 to 3, my view of isometries has not changed much .

Source: Author

In the second and third versions, he places the concept in mathematical perception, particularly by exemplifying two isometries, that of rotating and translating. He pointed out that we can rotate and translate geometric shapes without changing their measurements, but he did not fixate on the idea of position, which is an important aspect in the study of isometry. According to him, in the third version, his idea has not changed much from the second. His discourse moves from a non-mathematical context (V1) to a mathematical one (V2 and V3), but he is unable to explain the differences between the three versions. Moreover, has not changed much (Obs.2) indicates a certain insecurity. On the other hand, the underlines in all the versions and observations show (on chart 3) how important the readings and tasks suggested in the course were in Pricy's conceptual development.

Chart 03 - PR's three versions

Version 1 (PR)	Version 2	Obs.1	Version 3	Obs.2
I remember studying isometries in the Group <u>Theory subject</u> , and I remember having some concepts such as reflection, symmetry and translation related to this term. I believe it is directly linked to questions of the position of an object, parallel to its geometry in the plane.	I studied⁹ isometries <u>by reading a text</u> in the subject and I found the different ways of working isometries through dynamic activities very interesting. What surprised me most was using isometries to teach functions.	I was able to understand in practice what isometries actually were by reading a <u>suggested article</u> on the concept of symmetry in relation to a line. As a result, my answers changed , broadening my view of knowledge. I realized that this knowledge can be transmitted <u>in a playful way</u> .	We also had <u>the opportunity</u> to study isometry using GeoGebra, in an exercise list on VMTcG ¹⁰ . The teacher worked with us on translations and rotations. I believe that isometry is a geometric transformation of a figure, which does not change its angles or the measurements of its sides, but only changes its position in the plane.	In version 2 of my attempt to explain isometries, I brought an experience <u>I had in this subject</u> with reading an article that gave some examples of activities that related isometries. In version 3, I brought another experience I had in the same subject, <u>with VMTcG</u> , in which we learned the concepts of isometries by manipulating <u>constructions on GeoGebra</u> .

Source: Author

The idea of isometry as a transformation that maintains the measurements and only changes the position of the figure was evident. Moreover, she found it surprising to study isometries in the teaching of functions. In part of her diary, she reflects on a reading done in the subject: "The notion of isometry, a type of geometric transformation, has a great importance, both from a theoretical and practical point of view. As well as allowing a better understanding of some mathematical objects, isometries bring mathematics closer to other areas of knowledge, favoring interdisciplinarity". Based on this quote, she adds: "**I remember that** when I was studying Group Theory, the teacher, when giving a list of exercises on isometries, gave some examples of floors and coverings in which it was possible to see types of isometries. So, I agree with the author because I was able to see isometries in some

⁹ I use **bold** to highlight some personal reflection.


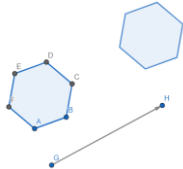
¹⁰ Virtual Math Team with GeoGebra (<https://vmt.mathematicalthinking.org/>).

buildings” (PR). In her reflection, she presents possibilities for working with isometries in floor and wall coverings and points out that it is a subject that makes interdisciplinary work possible.

I also selected these two examples to show that MG learning is still at the object-level, as it expresses the endogenous change of an existing discourse, in a ratified ($V2=V3$) and insecure way (Obs.2). PR learning signals a change at the meta-level, as it involves changes in the meta-rules of the discourse and is related to exogenous transformations in the discourse. Observe that she has changed her V1 (**I believe this** is directly linked to **questions of the position of an object**) to V3 (**I believe that** isometry is a geometric transformation of a figure, which does not change its angles and measurements of its sides, but only changes its position in the plane). Although I believe that learning at the object-level is crucial, I think that in the initial training of mathematics teachers we need to enhance learning at the meta-level (Sfard, 2008).

Changes from endogenous to exogenous utterance are not easy, demand time, persistence and creativity for all subjects involved in the assessment process. Most of the time, our discourses remain fully consistent with our experience of reality. We need a discursive change to become aware of new possibilities and arrive at a new vision. We thus often need a change in how we talk before we can experience a change in what we see (Sfard, 2008, p. 257). Throughout the examples shown above, we observe the idea of isometry as a transformation on PRs’ reflections. In her ideas, and on the following two PMT, reflecting specifically about the concept of translation - ELI (using as an example a picture downloaded on Internet) and VIVI (based on a task designed with GeoGebra) - geometric figures can be transformed through manipulation of their properties, taking into consideration their elements (Silva; Almouloud, 2021).

Chart 04 - ELI's and VIVI's three versions

Version 1	Version 2	Obs.1	Version 3	Obs.2
<p>(ELI) It's taking a picture and moving it to another place. It can be a copy and paste or a cut and paste. In other words, it's a change of location, a change of coordinates of the figure's position.</p> 	<p>A displacement is made, the image remains, but it is taken to another place.</p>	<p>I have no observations</p>	<p>It is a geometric transformation <u>which</u> moves all the points of a figure in the same distance and direction.</p>	<p>I think my last update was just more formal and shorter.</p>
<p>(VIVI) Translation is when we move an object in the plane without changing its size.</p> 	<p>Translation is when we move an object in the plane given a certain distance and direction, without changing the size of the shape.</p>	<p>I added the idea of moving given a distance and direction, thinking about using GeoGebra and the idea of tasks that use translation.</p>	<p>[...] size, shape and without rotating the figure, regardless of direction.</p>	<p>In V2 I had only commented on the size, but the figure does not undergo any kind of change other than translation.</p>

Source: Author

In both answers, the idea of displacement, moving an object or changing position seems to be more associated with the transformation considered as a punctual application of the plane over itself (functional object), for instance, as we can see in ELI-V3 (which moves all the points) and in VIVI-Obs.1 (**I added the** idea of moving given a distance and direction). Although the observations of these two PMT are more endogenous, they are also thought-provoking so that the teacher can come up with strategies to discuss and analyze another possibility for the figure and its position, i.e., the image of the figure through a certain transformation rather than its actual

displacement. In other words, to explore the transformation as the image being interpreted as transformation, identifying the image of the figure by the transformation T of the plane, that is, determining an isometry that transforms a figure into another, i.e., the transformation considered as a functional tool (Veloso, 2012).

In this emergent movement it is interesting to see how small adding on the versions (words, pictures, observations etc.) can contribute to change and enrich the statement and inspire for designing a new assessment source. In this process, the teacher also learns from their PMT, and they should not be interested in passing only the curriculum prescribed concepts (Bairral; Silvano, 2023).

Chart 05 - Possibility of teacher interaction with PMT

PMT common sentences	Teacher's possibility of reaction
It was possible to broaden my initial idea of the concepts covered and better organize my conceptions.	What initial idea? Name and comment on a conception.
I was able to understand much more than I thought I knew at the beginning of the course, in a process of evolution of my training.	Give an example of your evolution.
Another interesting point was the possibility of linking classroom issues with our experiences at school.	Name and comment on one of these issues and how it has enriched your experience.
At the end of the course, I believe I have broadened my view of the concept and better understood the difference between the concepts of symmetry and rotation.	What do you think you've learned? Name something that is still unclear to you and comment on possible reasons. How did you feel when you did this activity?
I was in doubt about translation.	What are your doubts about? Comment a bit more and use an example if it's easier for you.

Source: Author

SOME PEDAGOGICAL POTENTIALITIES, CONSTRAINTS AND CHALLENGES

The CD resource, with all the teaching support, is helpful for this. The CD tool is a culminating moment and does not just involve an evaluative activity of doing and submitting a certain piece of work. It is a resource that stimulates creativity, develops autonomy, and enhances various forms of recording, including writing (Bairral, 2023). The PMT need to improve their ways of communicating and expressing their ideas analytically (Brandes; Boskic, 2008).

Using this tool is not a simple activity and it is a lot of work for the teacher, as they need to read and monitor everyone's development. However, although it is an individual resource, by sharing it with the group, the teacher can request shared analysis from the PMT and thus generate more complicity and collective commitment.

In the CD analytical process, the teacher always needs to act on improving the PMT's ideas, providing them suggestions throughout new readings, tasks etc., and not giving them only their conceptions or telling them that they are right or wrong. In other words, in this assessment resource the idea of error and success should not be the focus. During the comparison among the versions, some sentences are recurrent, and the teacher needs to be careful and elaborate discursive strategies for improvements on such statements, for instance:

Moreover, this type of resource gives the PMT the opportunity to question information available on the internet and develop their critical sense and scientific attitude (Lager; Lavonen, 2023), which is not easy, especially with the increasingly frequent practices of generating content via ChatGPT etc. Nowadays, since the use of ChatGPT and other generative artificial intelligence tools seems to be even more frequent by individuals, resources like CD could be helpful in online assessment practices. Besides the new designed assessment resource, it will be also relevant to consider the type of question to ask on the ChatGPT, the way to save the given information and to check it using other sources (articles, books, websites etc.), and the way of interaction. Chart 6 summarizes some contributions, constraints and challenges regarding the use of CD tool.

Chart 06 – Potentialities, constraints and challenges of the CD tool

Potentialities	Constraints	Challenges
<ul style="list-style-type: none"> -Autonomy -Individual and collective commitment -Creativity -Improvements of writing hypertextually integrating different modes (pictorial, graphical, table, drawing, picture etc.) -Integrate different sources of information 	<ul style="list-style-type: none"> -Lack of experience in writings and reflect collectively about your ideas -Student autonomy and their continuous demand for teacher suggestion -Necessity of questioning the gathered information 	<ul style="list-style-type: none"> -Elaborate more analytical reflections -Promote changes on the meta-level learning -Provide continuous moments for discussion and refining the gathered information

Source: Author

FINAL REMARKS

In this article, I shared part of my assessment practice on VLE and suggested an example of an assessment tool that seeks to break away from just searching for information on the internet or other computer devices. The proposed tool was constructed considering the diagnostic (previous knowing, invitation to reflection), formative (implication, contrast, and refining ideas) and summative (socialization, commitment) moments. In all three moments, all forms of language and interaction were valued. The diagnostic moment is important for the teacher to get to know the PMT's prior knowledge. In the formative moment, the teacher identifies the sources of study and research and offers new ones. This moment is extremely important because the PMT need to know the reliability of their sources and they need to learn to contrast the information obtained. In the summative moment, synthesizing, socializing, and collective commitment are highlighted. The summative moment should not be focused on presentation format. It should be a moment in which the PMT also develop their creativity and autonomy.

As well as allowing information to be refined and improved, the tool enables prospective mathematics teachers to improve their communication skills by building different strategies and integrating the technologies available to them. As for learning, meta-level learning (Sfard, 2008) requires time, more analytical experiences (Brandes; Boskic, 2008) and multiple forms of recording (Bairral, 2023). In this case, it is up to the teacher to systematically monitor the PMT's productions and continually promote analysis (individual and collective) of the concepts and relationships involved. Technology enables this sharing, which should be continually enhanced by the teacher in a variety of interactive dynamics. Therefore, in addition to overcoming the moment

of searching online, this tool also adds value to teacher's planning and monitoring, which should be prioritized over fashionable approaches such as the flipped classroom. In the virtual learning environment, the analysis of knowledge construction should go beyond the assessment in isolated instruments, and the teacher always takes an important role.

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