REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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Pedagogical narratives as (self) education practices for teachers who teach mathematics in the primary school

Narrativas pedagógicas como práticas de (auto)formação e professoras que ensinam matemática nos anos iniciais do Ensino Fundamental

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RESUMO

Este trabalho refere-se a uma pesquisa realizada em um grupo colaborativo de professores que ensinam matemática na educação básica[3]. Tem como objetivo evidenciar o papel da colaboração – existente no grupo e mediada pela produção, discussão, análise e reflexão de narrativas pedagógicas. Apoia-se teoricamente na perspectiva histórico-cultural, com aportes metodológicos de design research. O recorte aqui apresentado toma duas narrativas pedagógicas de professoras pedagogas – e no processo analítico foram consideradas três unidades temáticas: a intencionalidade pedagógica da professora; a análise dos raciocínios dos estudantes; e as aprendizagens compartilhadas. O grupo se constitui em espaço de (auto)formação docente, pois tem possibilitado aos participantes aprendizagens de práticas a partir de narrativas pedagógicas, bem como modos de analisar os raciocínios matemáticos dos estudantes.

PALAVRAS-CHAVE: Perspectiva Histórico-Cultural. Design Research. Colaboração. Aprendizagens Docentes.

ABSTRACT

This work is based on a study conducted in a collaborative group composed of teachers who teach mathematics in K-12 education. It aims to show the role of group collaboration mediated by the production, discussion, analysis, and reflection of pedagogical narratives. We are theoretically grounded in the cultural-historical perspective with methodological contributions from design research. The work presented here encompasses two pedagogical narratives from pedagogue teachers. The analytical process considered three theme units: teacher's pedagogical intention, the analysis of students'

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reasoning, and shared learning. The group establishes itself as a space for teacher (self)education by enabling participants to reach practice learning from pedagogical narratives and the analysis of students' mathematical reasoning.
KEYWORDS: Cultural-Historical Perspective. Design Research. Collaboration. Teachers' Learning.

Research context

This work results from a longitudinal research developed in a collaborative work group, certified in the Group Directory of *Conselho Nacional de Desenvolvimento Científico e Tecnológico* (CNPq) and connected to the University's Education Graduate Program. It is composed of teachers who teach mathematics and work in early childhood education, elementary, middle, and high school, and higher education. Our biweekly meetings take place hybridly on Thursdays from 6:30 pm to 8:30 pm, allowing the participation from teachers in other Brazilian regions and a Colombian teacher. The meetings occur via *Google Meet* enabling video recording. The participation in the group is voluntary, and not all participants are present in every meeting, even due to the current conditions of work intensification – sometimes the exhaustion does not allow participation in the group; or, in other moments, the teachers are called for activities in the school at the same time as the meetings.

We focus on the study of themes that compose the mathematics curricula and seek to understand teachers' learning processes, as well as to evaluate the potential of tasks created within these themes to contribute to teaching practices and students' learning.

When creating the tasks, we divided ourselves into three subgroups: early childhood education; elementary school; and middle and high school. All tasks are available in a shared drive and collectively analyzed by the group.

Our work methodology is close to *design research* (Matta; Silva; Boaventura, 2014; Molina; Castro; Castro, 2007; Powell; Ali, 2018), which consists on a iterative spiral involving: study sessions about a given theme; creation of tasks that can be developed in a K-12 classroom; development of the task by a group participant alone or in partnership with another teacher; records of classroom events, collecting students' productions, and the elaboration of a pedagogical narrative by the teacher; discussion and analysis of classroom records; rewriting of the task, when necessary; followed by a new development in the classroom. From then on, a new iterative cycle begins. At the end of the study period, the group systematizes the results and each teacher, aiming to publish the work, selects one of their narratives to be discussed. These narratives are broadly and collectively discussed in the group. At this point, the

texts are revised and expanded into a dynamic collaboration, in which all participants contribute to analyzing the author's narrative context. It is a learning moment, as the pedagogue-teachers bring to the group the classroom practices with the children in early childhood education and elementary school, marked by didactic-pedagogic knowledge, which often have conceptual gaps in mathematics that can hinder the analysis of students' reasoning, and need the collaboration of teachers specialized in mathematics. In turn, the mathematics teachers have the collaboration of the pedagogue-teachers in discussions about pedagogical work in the classroom.

This approach has been discussed by other Brazilian researchers, sometimes under new names due to the translation of the expression. For instance, Schons and Bisognin (2023) use the expression "research based on design" – as we do, the authors understand this methodology as an interventionist research, with cycles that occur in an iterative way.

Besides our approximation to *design research* as a methodology for group work, we are supported by a cultural-historical perspective on the organization of the work context in the classroom and on the analysis of teachers' and students' learning processes.

For this work, we selected two pedagogical narratives of pedagogue teachers working in elementary school and analyzed the group moments in which, seeking publication, these narratives were discussed. To do so, we analyzed excerpts from the narratives and the discussions held in the groups on April 4 and 8 of 2024, which were videorecorded and transcribed. The analysis is based on a comprehensive and interpretative approach (Souza, 2014) seeking to interpret and attribute meanings to the oral and written narratives of the teachers and group participants.

This text aims to highlight the role of collaboration – which exists within the group and is mediated by the production, discussion, analysis, and reflection on pedagogical narratives – in the processes of teacher (self) education. We stress that the use of parentheses aims to show duplicity of meanings: they refer to the self-education of the narrative's author and the education of the colleagues in the group. We chose three analytical themes present in both narratives portrayed in this text: teachers' pedagogical intentionality; the analysis of students' reasoning; and the shared learning.

Besides this introduction, the text is organized into four sections in which we discuss: the theoretical principles that support the collaborative relationships in the group; the presentation of tasks that support the work in the classroom and the

production of pedagogical narratives; the movement of analysis, discussion, and reflection of these narratives; and our final remarks.

In the next section, we discuss the principles of the cultural-historical perspective that guide our actions and our work, as well as our understanding of the production and sharing of pedagogical narratives.

Theoretical principles that support the collaborative relationships in the group

The cultural-historical perspective of Lev S. Vygotsky and his collaborators has been guiding our work. We understand collaboration, professional learning, teaching and learning relationships, and teachers' roles through these theoretical lenses, with pedagogical intentionality when mediating these processes. Thus, we opted to start the discussion delineating the place from which we speak about human development.

Grounded in this theory's assumptions, we understand development through two fundamental theoretical categories in Vygotsky's work: the social and the cultural. We agree that the forms of social organization go beyond the field of natural phenomena that, in itself, are unable to explain human sociability. Therefore, "human social existence presupposes the passage from the natural order to the cultural order" (Pino, 2000, p. 47).

In his "general genetic law of cultural development". Vygotsky (2000) argues that the psychological role is first a social event, a relationship between people, as cultural development passes through three stages: internal, external, and egocentric. In this perspective, we build ourselves through the other and from the other; and the intersubjective relationships we establish with it and with our surroundings. Hence, the nature of higher psychic functions lies in the process of internalizing what was once external.

When considering the nature of the relationships established in a group with collaborative characteristics that, throughout time, creates shared practices and ideas, we can identify this process of internalizing what was external before. Throughout the years, participants have been socializing their practices, which dialogue with their colleagues' practices, and – during the process of studying, discussing, producing, and collectively reflecting on this production – new practices are produced, while others are (re)signified.

In the Vygotskyian theory, thought, a higher psychological function, is also a social relation and the "essence of intellect is in the tools" (Vigotski, 2000, p. 24). Therefore, the development of thought and culture occurs through the production of tools and signs – mediators of human action. The first mediates the action over nature

(materiality); the latter over man himself (signification). Signs constitute themselves as subsets of productions of human mental activity over symbolic objects and are exteriorized through the language expressed in speech, writing, sound, graphic, and aesthetic forms, etc. (Pino, 2000).

An intersubjective relationship can only take place through sign mediation and from it, which designates "[...] the role that the systems of signs play in the relationships between individuals and in their relationships with the environment" (Pino, 1991, p. 33). The centrality of these processes lies in the word, understood as a sign that represents the unit of thought and language (Vigotski, 2009).

In our group's context, the work developed in the classroom needs to be recorded by the students and the teachers. Thus, participants in the group practice writing pedagogical narratives of the classes. According to Frauendorf et al. (2016), pedagogical narratives are texts written to share lessons learned from experience, reflect about the experience, the observation of peer practice, collective discussion, reading, study, and research. These narratives can enhance teachers' learning and professional development through the act of writing and sharing them with peers.

We understand that signification comes from interlocution and is not at the word, which carries, exactly through the dynamic guaranteed by interlocution, an ideological content (Volóchinov, 2017). In this perspective, the word acquires meaning through interaction and by it. Therefore, the discourses are processes in which significations are produced in interpersonal relationships.

In the Vygotskyian perspective, the sign activity acts in the consciousness dimension, when talking to someone, it mobilizes and produces alterations also in the speaker. In another perspective, but in consonance with it, in the sign activity, the sign is interpreted and assumes, in this way, a mobile and dynamic character that depends on the enunciation context (Bakhtin, 2011). Hence, writing of narratives helps teachers to appropriate themselves of the practices of analyzing students' records and the discourses raised, so as to broaden their gaze to perceive the clues about mathematical reasoning, based on the observation of relationships established, the strategies used, the hypotheses raised, and the conclusions reached – when selecting the records and analyzing them. The appropriation of the practice of deeply analyzing the classroom and students' mathematical discourses establishes itself as a self-education process for the narrator-teacher and a collective education for the group discussing the narratives. Therefore, there is evidence of teachers' learning and professional development of all participants. In this text, we expand our discussions

about professional development regarding the work presented at the *VIII Seminário Internacional de Pesquisa em Educação Matemática* (Sipem), in which we analyzed the importance of partnership for this development (Custódio e Nacarato, 2021).

We defend that "it is not what the individual is, a priori, that explains their ways of relating with others, but the social relationships in which they are involved that can explain their ways of being, acting, thinking, and relating" (Smolka, 2000, p. 30). In these relationships and context, teachers appropriate themselves on how to act in their teaching practices and position themselves, in a political commitment, in favor of their students' learning, be it on how they manage and intentionally plan their practice to create a class culture that prioritizes teaching and learning relationships grounded on mediation and dialogue, be it on recording and sharing their practices to be discussed, analyzed, reflected, and validated by their peers in the group.

For this work, we selected narratives from two pedagogue-teachers produced from the development of some tasks created by the group in the classroom. To help readers understand, before presenting the narrative, we briefly described each task in the next section.

Presentation of the tasks that ground the classroom work and the production of pedagogical narratives

We selected two narratives from pedagogue teachers who developed the tasks in the classrooms. The first narrative, written by teacher Beatriz, refers to a task we called "Popcorn game" in which students are organized into two teams and each student makes a popcorn with crumbled paper. The game takes place outdoors, with a line separating both teams. When hearing the start signal, children have to throw the popcorn into the field of the opposing team and, simultaneously, grab and return those thrown by the competitors in their field. When the game is over, the children count the amount of popcorns in the field. The winning team is the one with the least popcorns.

Five teachers developed this task in school. It was first conceived to be used in Early Childhood education, but it was also developed in Years 1, 2, and 3 classrooms. It aims to identify the relationships children establish between magnitudes, such as the number of children and the number of popcorns; greater/lesser speed and greater/lesser number of popcorns thrown; popcorn size and distance to fall in the opposite field, among others.

We selected Beatriz's task, who chose the narrative of this activity to be published in the group's ebook. She works in Year 1 of a municipal public school in the city where the University campus is located and attends meetings in person. She

developed the task with her class in 2023 with the collaboration of two colleagues in the group to whom she asked for help, as she believed the class was a difficult one – there were 17 children, two with them with disabilities (Down Syndrome and Autistic Spectrum Disorder). The narrative was previously made available to the group, and its analysis and discussion happened on April 4, 2024.

The second narrative is from teacher Suzi, who works in Year 2 at a laboratory school connected to a federal public university. It was based on a task called "Exploring the *Cuisenaire* bars", created by the subgroup that acts in elementary education. The *Cuisenaire* bars consist of a material composed of ten colorful parallelepiped-shaped bars, whose length varies from one to ten units. The smallest one (the white bar) is taken as a measurement unit. The task foresaw children's initial exploration of the material, expecting that they would establish relationships between the bars' measures. The items proposed in sequence explore the relationships between direct and inverse proportionality between these measures.

Suzi was one of the last teachers to join the group in 2023, and, as she is very active, she has been doing several tasks for her students. She chose the narrative for this task to be shared and analyzed by the group during the April 18, 2024 meeting.

In the next section, we present our group's movement as we discuss and analyze the narratives.

The movement of analysis, discussion, and reflection of the pedagogical narratives

Our aim was not to analyze the pedagogical narratives separately, but to show how the existing collaboration in the group enables discussion, reflection, and analysis of them and enhances formative processes. To do so, we elected three theme units: teachers' pedagogical intentionality; analysis of students' reasoning, and shared learning.

Teacher's pedagogical intentionality

A strong characteristic of elementary teachers is the pedagogical intentionality when taking on classroom tasks. Whenever possible, we discussed during work meetings the classroom as a space for teacher and student learning and how they needed to be organized to guarantee an environment conducive to learning. In both narratives we discuss in this text, we identified teachers' intentionality in choosing tasks based on students' profiles.

For instance, when questioned by her colleague Suzi about her concern with handicapped students in her narrative, Beatriz says:

> *When I chose this activity, the fact of throwing the popcorn was what made me think about the student with Down syndrome, because that was his best ability. He kept this movement of throwing and catching, throwing to another. I thought it was a game he would enjoy playing. Even if he didn't participate in the whole fame, I thought that, at some moment, he would stand up and throw the paper ball with the other kids. This didn't happen. He was sitting down the whole time watching it. At one time, he stood up to participate. But later, when reflecting, whenever we presented him with something, he wouldn't accept doing it at first. I think that if I played popcorn every day, every week, he would participate because everything is very hard with him. [...] The autist [student] participated. He was the child who would throw it on purpose at the other.*

We observe that Suzi, when questioning Beatriz, also talked about her students with handicaps and reflected on how teachers should perceive the work development in the classroom:

> *the first thing that caught my attention was that you had a student with Down syndrome and an autistic one. Because I do too, in my classroom, I had a [student with] Down [syndrome]. And he also had autism. […] because when you're thinking about the activity, you have to think about these children. On how you'll develop these activities considering these children, how they'll answer these activities, which is different.*

Both teachers show signs that the task choice was not random but aimed at reaching all children and their needs. In her narrative, Beatriz wrote*: "I chose the early childhood activity to work with my class because playing was in my everyday life, as I have students who could not register and were nonverbal. Thus, through games, we created a communication, a routine, and an intentionality".*

Serena questioned her, highlighting the importance of play and the fact that the task was not exclusive to early childhood, as shown in the excerpt of their dialogue:

> Serena*: the popcorn task is an activity to be developed with children in early childhood, Year 1, and Year 2. Children will seek strategies to register, to solve problems […] because the Year 1 and 2 children still have the right to play, playing is still part of this pedagogical work.*
>
> Beatriz*: just as you said. We enter Year 1 and it seems that the parents tell the children: you won't play any longer, nobody plays, and that we can't play anymore. I am one of the few who still play and this doesn't affect my children's development, they don't learn less than in other classrooms, on the contrary. They have a broad development with this play.*

The movement of reflexive listening is part of the formative process created in the group. The participants feel safe to share their practices and pedagogical choices

because they understand the importance of shared reflection, mediated by different lenses, be they theoretical or emerged from their teaching trajectories.

Teacher Serena, based on her experience as an early childhood coordinator, questioned Beatriz about an excerpt from her narrative, in which she recounts how she organized the children into two groups to play the game. Here is an excerpt from their dialogue:

> Serena*: I was thinking, when you say you divided the teams, and that you wouldn't count the child with Down syndrome. And then, in this part, I was concerned. I mean, the child will participate but, at the same time, I'll tell the group he doesn't count. So, how does that work with the whole group?*
>
> Beatriz*: This is something I regret doing, later on, when I wrote it, I was thinking that I shouldn't have done this, I should have worked with a number and divided it. However, I took this activity to the class thinking about his participation, but a part of me knew that maybe he wouldn't participate, he wouldn't be in any group, he would move between groups, he would want to participate in the group he wanted with the child he felt most at ease with.*

When problematizing the fact that Beatriz affirmed that the task choice aimed to include the student with Down syndrome, but that when organizing the team, the child was not counted as a member of any group, Serena raises a reflection that allows Beatriz to try to attribute a meaning to her choices and analyze if they contributed to the student's lack of participation. As teachers, we are always involved in decision-making, these decisions often lead to the development (or not) of a learning process. Reflecting on them and how they affect the learning environment is extremely powerful for teachers' formative process and allows the (re)signification of the pedagogical practice.

Contrary to Beatriz, Suzi did not justify her choice to develop the task with the *Cuisenaire* bars in her narrative, what Ana highlighted when reading the text and helped her think about the intentionality of this choice, as she recognized Suzi's practices in other narratives. The excerpts of the dialogue between both show Ana's collaboration role to expand the narrative, which was reflected and accepted by Suzi:

> Ana*: So the task had already been thought, reflected…but considering the culture of mathematics class you have in your class, you evaluated that this task had some potential for your students. So, it was not any choice, because you have a task repertoire, why did you choose this one? Because it is a task that allows for investigation, experimentation. Thus, the choice of task is because you have a classroom that will allow you to work with it. Because if you have a class that doesn't have this movement that your class has, the task that will not raise the same effect that Ivy did [referring to Ivy's previous intervention]. Does that make sense?*

> Suzi: *What Ana said is very nice because that was exactly it, when I see it, I noticed, "wow! This sequence of activities matches what I'd like to show my students because I have the material there and my students will…and I'm working with numbers and this ideal of proportion matches what I want, this concept of proportionality will be very interesting to work with my students and that was great". So that is really it.*

In the cultural-historical perspective, pedagogical intentionality is central; the teacher needs to know the students from where they are so that there are advancements in their development. There is also the participants' intentionality in discussing and collaborating on the analysis of the narratives; these are moments of collective reflection that boost individual and/or collective teacher learning. The way these interactions take place enable teachers' theoretical, methodological, and experimental knowledge, resulting from several formative and professional spaces, which can anchor processes of resignifying mathematical concepts and teacher practices.

Analysis of the students' reasoning

In both narratives, the participants in the group collaborated with the two teachers to broaden the analysis of children's reasoning – recorded in the texts through transcriptions of the dialogue between teachers and students- and the drawings they produced. There is a group practice of incentivizing teachers to explore different types of students' records. The narratives present oral dialogues (which were transcribed) between the teacher and students and among themselves. They also show written records of the resolution strategies, how the task took place, schemes, tables, and drawings.

In the case of Beatriz's narrative, Carlos analyzed in detail all the mathematical concepts and ideas presented in the text. As it is a long analysis, we present here excerpts from it, with suppressions, to avoid hindering understanding. First, based on a previous comment by Serena, Carlos reflects on what mathematics is in early schooling:

> *I was reading from a certain perspective, trying to look at everything that had mathematics. As Serena said, other teachers don't see that there is mathematics in things; sometimes it seems we have to search for it in things, but, in fact, you don't, it has always been there, when it is built with an intentionality, something that is there.*

This type of interpretation is quite common among teachers, especially early childhood teachers, who, due to conceptual gaps in mathematics, cannot identify the presence of mathematics in games and plays with children. Later, Carlos began to

highlight mathematical ideas that emerged from Beatriz's narrative. The first of them refers to the strategies that Beatriz and the children used to divide the class into two teams.

> *The first [highlight] that I wanted to share was the division of the class into two teams. Just this would raise a great discussion, as it appears here, because when we talk about dividing, first we think about dividing into equal parts, the way division is taught, in which you divide into equal parts. And, in fact, I don't know, there could be a group with more children than the other, it depends on what is agreed in the rules beforehand. [...] Something else, though you talked about dividing into two teams, João started to divide them into several teams. Instead of dividing into two teams, when dividing into teams of two people, we could think about the relationship that could occur there.*

We believe that Carlos identified the two ideas of division, that is, when the teacher and part of the children thought about dividing with the idea of separating the 16 students into equal parts, João operates with the division by quotas, i.e., he thought about groups of 2 students in each and not 2 groups of students. This was an important intervention because no other participant in the discussion had noticed that.

Still on the organization of the groups, Carlos tries to provoke Beatriz with the possibilities to advance mathematical ideas in the children:

> *You could take one more step, instead of asking eight plus eight, ask the double of eight or two times eight to check if the answer would be as fast. I don't know, maybe not yet... sometimes, yes, because they can already know this part of double and will relate to this thing of adding it to itself. But this is something we should be watchful of this in the future, because maybe that is where the confusion starts, which we'll see in other Years, between the addition and the multiplication reasoning. Of not having a clear distinction between both. It doesn't matter in the case of the calculations, you can add eight plus eight or multiply by two. But, perhaps, in the future, these two concepts can be mixed up.*

In this intervention, Carlos calls our attention to the importance of gradually working on children's multiplication reasoning, considering that addition would not help to establish proportional thinking.

Finally, what caught our attention in Beatriz's narrative was her discussions with the children about time. We opted to present an excerpt of Beatriz's narrative when talking with the children:

> Teacher*: How long do you think the game should last?*
>
> João*: 6 hours.*
>
> Teacher*: But 6 hours is more time than we spend in school.*
>
> Students*: 6 hours is a lot!*
>
> Someone says*: 1 minute.*

> Teacher*: Is 1 minute good?*
>
> João*: No. It's too little, you can't do anything.*
>
> Someone says*: 1 hour.*
>
> Teacher*: 1 hour is a long time, we have to define minutes to give you time to play and then go to the break.*
>
> *Students shout: 1 minute, 3, 4...*
>
> Teacher*: let's vote, who thinks 2 minutes is good? And 3?*
>
> *Some wanted 4. We decided to play for 3 minutes, for which the majority voted.*
>
> Teacher*: let's play for 3. If you think that 3 was too long, we can reduce the time to…[some students talked] 2 and if you think it's little we increase to…4.*

This part of the narrative raised interesting discussions and reflections among the participants in the group. Carlos, for example, highlighted: *"when you stop to think, one minute and one hour use the same algorithm, but that one value is much bigger than the other. So, once again, you have mathematics in the comparison of units of measure".* Ana continued:

> *João says six hours to which Beatriz intervenes: "but 6 hours is more time than we spend in school". Because you can see that they don't yet have this dimension of time but see how Beatriz's words mediate it and the students reflect, "6 hours is a lot!". So she was quite fast to provoke them.*

Carlos highlighted the strategies that could be used to count the popcorns and stressed that there are different ways to do it, counting one by one, two by two, or three by three. He emphasized the richness of this game, as winning means having fewer popcorns, which goes against most games involving counting.

Initially, in the case of Suzi's narrative, Ciça made a very broad analysis and highlighted different ideas and mathematical tools presented in the text and that could be better explored in the following version of the narrative, considering that they are rewritten after the group discussion:

> *So I highlighted how the issue of children's manipulation was important, going towards the cultural-history theory…so we perceive this quite strongly, regarding the question of photos, but also the dialogue that sometimes emerges. Children's experience, how much they experience with the Cuisenaire movement, is important, the question of observation, the way children observe, they test, remove, place again, and how important it is for the development they build during the narrative. The comparison, the whole time they are comparing, trying to place them, removing from the place again, so all this movement together with the manipulable material, how important it is, but through mediation. So, these are all languages that the child will raise through the narrative, during the task, and how important it is. And then, when you look at the record – it's so nice how well they clearly write what they are thinking, but the additive field emerges, but*

> *sometimes I notice they do: 4 + 4 + 4 + 4 = 16, or 4 x 4 = 16. I think this discussion needs to emerge in the narrative for proportional thought. What the elements in the additive field allowed, and what this multiplication field also allows. The issue of the table as well, it needs to be quite present, the issue of organizing the record for proportional thinking, so these elements need to appear more in your narrative. Because they are fundamental to proportional thinking. I think it is important to make this clear in the narrative.*

Ciça is a scholar of the cultural-historical perspective, and her words reveal her concern with issues of language, the technical-semiotic tools, such as the materials, and the process of conceptual elaboration itself. In the Vygotskyan perspective, collaboration among partners, with a more experienced partner, is a necessary condition for development. As mentioned at the beginning of the text, the group members have different educational backgrounds, act at different educational levels and in different roles, which makes the space for discussion and reflection a very fruitful one. Thus, we advocate that group collaboration arises from the interactions established among the different knowledge of each participant.

On his turn, Carlos highlighted the role of interactions and mediations that Suzi did with the children when exploring the relationships among the *Cuisenaire* bars:

> *I think it was a cool interaction here, in the sense of the questions, which is something that we've already talked about in other moments, how much a good question is worth in this mediation. "What do you think about using the red bar?" "I think it's nice" "how many will you use?" To provoke really. And after a student explains how he will place it and later you ask: "did it work?", "it worked", it seemed to me a very productive interaction as it is a question that provokes the student to see what he will do. See how he, himself, tried to see if it was right. This is also something important. Because sometimes the student will ask if it is right or wrong. And not there, it was an interaction in which the student could, through the construction, say if it was right or wrong, depending on what he had planned.*

In synch with the perspective we adopted, Carlos reinforces the role of Suzi's mediations. Later, Ivy resumed Carlos' words and added:

> *When they are there, trying it, and you ask them to reflect on whether it worked and what they tried to do, you're inciting this validation. You're showing that it's not enough to say: it's like this. You have to try to see if this will work. Within mathematical thinking, this is extremely important: the validation that your strategy is actually powerful, that it solves the problem, that it is valid, and that it opens paths to other possibilities. This also needs to appear.*

The practice of testing and validating the hypotheses is part of the investigation classes, a practice present in Suzi's classes; thus, her intentionality when choosing the task because it would allow an investigative work, in which the students are active agents of their learning process, the teacher is a mediator, who orchestrates the class

plan: from choosing the task, the group organization and division; the ways to record; the socialization of these records; up to a synthesis of the main ideas and strategies.

Ana and Ivy stressed the movement of the measure concept in students' manipulation of the bars. Ana says:

> *Which concept is there? It's the concept of measurement, so how much does the student already know about the concept of measurement is fantastic for a Year 2 [student], because he knows he has a little bar and wants to know how many times the red fits in the bar. So, I have to overlap it, so I keep overlapping it, it will fit three times. So, who is the green bar? Three times the red. Then it is very nice, and I think this is the evidence we will observe when we analyze.*

Ivy continues: *"What you said Ana, I was going to say the same, about the issue of measurement. This has to appear [when rewriting the narrative] because they worked with the question of measurement, intuitively, they worked this throughout the dialogue".*

The suggestions presented by the colleagues contributed not only to expand the pedagogical narratives – due to the incorporation of ideas and reflections – but also for the education process of all group members, when questioning their colleagues and commenting on the practices and records from the narratives, mobilizing individual and/or collective reflections that trigger (self)education processes, in which language is a sign that mediated the whole movement.

Shared Learning

The discussions about the teachers' narratives raised reflections and enabled the identification of evidence of how much we learn from each other. In the discussions arising from the analysis of the narratives, we identified evidence of learning from the colleague's words, the narrative reading, and participation in the group.

Throughout the collective discussions or in the narrative itself, several excerpts emerge that show evidence of learning, for instance, when Suzi states:

> *Your words were very pertinent, Ivy, as I was writing, building my text, my writing, I've noticed myself as well. I was building myself in the group, with the words of every colleague, because we build ourselves in the group, through reading the text of each one, the authors…you base yourself in the words, and you build yourself.*

Suzi highlights the group role in her professional development and how much the words, reflections, questions, and readings – whether theoretical or from colleagues' narratives – ground her own words, her discourse, and her pedagogical practice.

Beatriz also undertakes this movement of identifying the role of the group and the words of every colleague:

> *Then it's that thing that if I were going to do, I'd do it differently. I think we can discover new things every time, even due to what they say. You won't expect the answers they'll give amidst the questions. And then these talks will prepare us to intervene better.*

The reflections happen in the dialogue and the group through writing pedagogical narratives and their sharing, creating a space in which different forms of language support professional development. We believe that the power of collaboration is related to decreasing relationships of oppression and power. The more meaningful the trust and respect relationships among the members are, the better the processes of knowledge production.

Some members pointed out the role of reading colleagues' narratives in learning. Gilda, for example, highlighted the discussion in the analysis of Suzi's narrative:

> *Then, in the first dialogues, as happened many times with me, we induce the answers, because we are teachers. So, it seems that I'm watching myself.[...] But it's something I saw myself too, because we are teachers, so, we have to police ourselves, I saw myself here, "wow, I also do that". Because I'm transcribing my narratives now, and I notice this too, in which I induced the child in some situations.*

When having the possibility of reflecting on their colleagues' practices, through their pedagogical narratives, Gilda sees herself faced with her own challenges in the classroom. The analysis of the dialogue established between Suzi and her students made Gilda reflect on the teachers' mediating role in the classroom and how this plays out in her classes.

Carlos, on the other hand, when discussing Beatriz's narrative, highlights:

> *For me, it is always an interesting experience to read the narratives of elementary and early childhood education because I always compare it with the 50-minute mathematics class, with the things to do…and, to me, it is basically another world, and I find it very interesting how each thing is a mega event.*

Carlos teaches middle school and confronts the practices of elementary teachers with those of specialist teachers, who often cannot do interdisciplinary work due to time constraints and the amount of content to be studied.

In the discussion about Beatriz's narrative, Serena also shows the role of writing in the narrative:

> *Here in the group, we can see that very strongly, the level of reflection about the pedagogical practice when we write, when we read, when*

> *we bring it to the group. This movement is very rich. These writing strategies are also important for teacher education.*

Being part of a collaborative group that privileges shared work allows different types of learning to be built throughout the group's trajectory. Carlos provides evidence of this when discussing an excerpt written by Suzi in her narrative:

> *This part I think was very interesting really is when you wrote, Suzi, that "the material combined with the educator's guidance", I think this part is very important because, sometimes, there is an idea, like: ow, put something concrete there to mix up and it seems that only this will make everything develop.[...] I also think that it matches the group dynamic we have here, the interventions and mediations that take place.*

At different points, we raise the discussion of the role of manipulable materials in mathematics classes – the question appears in the narrative writing and is reflected by Carlos during the collective discussion.

At the end of the discussion about Suzi's narrative, Ana interpellated the group: *"But the process is quite nice, right? I'm loving it!"*; Suzi answers*: "I'm loving it! Learning this way... I'm loving it!"*; and Ana completes: *"learning mathematics, learning how to write a narrative, learning how we analyze a narrative"*.

In the Bakhtinian perspective, discourses affect and are affected by otherness. Ana's and Suzi's final words show the role of the group in different types of learning and in the appropriation of colleagues' practices and discourses, leading to a practice and a discourse typical of the group. Each one, on their own way, affects and lets themselves be affected in the discursive web, creating a "common place" in which practices and discourses are shared (Smolka, 2000).

Final remarks

In the current context, the formative processes held on a large scale by the educational system have not been enough to promote teachers' professional development. In this sense, spaces in collaborative groups have shown how much teachers learn and how they build a knowledge repertoire regarding mathematics and their teaching. When this group gathers pedagogue-teachers and specialist-teachers, knowledge sharing becomes extremely powerful in promoting different types of learning. Pedagogue teachers contribute to questions related to classroom organization, the careful observation of students, the concern to include neurodivergent or disabled students, among other types of knowledge; on their turn, the specialist teachers contribute to broadening the mathematical knowledge of all groups. However, the group needs a work methodology, and, in this regard, the

flexibility offered by the design research methodology has been favorable for creating tasks that enhance mathematics learning in the classroom.

From this perspective, the production of pedagogical narratives has been establishing itself not only as a possibility for producing knowledge, but also as a formative process, mainly when the narratives are shared, discussed, and analyzed in the group.

In this text, we present part of the work developed by the group when sharing narratives. The colleagues' exotopic gazes (Bakhtin, 2011) over the narrative enable reflections and (re)significations of practices and mathematical concepts. We learn with others and from them (Vigotski, 2000, 2009) to produce a pedagogical narrative and analyze students' mathematical thinking.

The group has contributed to participants' education, valuing and strengthening principles such as having pedagogical intentionality; creating environments conducive to classroom learning; listening to students and valuing their thinking; providing mediations that foster development; and establishing different forms of records for teachers and students.

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