

Early Years Teacher Training: Understanding Fundamental Operations from New Perspectives

Formação de Professores dos Anos Iniciais: Compreendendo as Operações Fundamentais Sob Novas Perspectivas

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ABSTRACT

This text presents part of a doctoral research project conducted at the Federal University of Uberlândia (UFU), focusing on the training of mathematics teachers in the early years. The objective was to investigate, analyze, and understand the formative process of mathematics teachers in the early years, as they establish new meanings for the four fundamental operations. The study analyzed the reflections of teachers participating in a remote extension course. From this formative space, two central themes emerged: training needs and the adoption of new approaches to teaching operations. The exchange of experiences among teachers from different regions of the country highlighted the formative potential of collective work. Redefining the teaching of operations involves rethinking pedagogical intentions and practices, making mathematics more accessible, comprehensible, and connected to students' daily lives.

KEYWORDS: Continuing Education; Remote Learning; Fundamental Operations; Historical-Cultural Theory; Mathematics.

RESUMO

Este texto apresenta parte de uma pesquisa de doutorado realizada na Universidade Federal de Uberlândia (UFU), com foco na formação de professores que ensinam matemática nos anos iniciais. O objetivo foi investigar, analisar e compreender o movimento formativo de professores que ensinam

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matemática nos anos iniciais, ao estabelecerem novas significações das quatro operações fundamentais. O estudo analisou reflexões de professoras participantes de um curso de extensão ofertado remotamente. A partir desse espaço formativo, emergiram dois temas centrais: as necessidades de formação e a apropriação de novas abordagens para o ensino das operações. O compartilhar de experiências entre professores de diferentes regiões do país evidenciou o potencial formativo do trabalho coletivo. Ressignificar o ensino das operações implica repensar intencionalidades e práticas pedagógicas, tornando a Matemática mais próxima, compreensível e conectada ao cotidiano dos estudantes.

PALAVRAS-CHAVE: Formação Contínua; Ensino Remoto; Operações Fundamentais; Teoria Histórico-Cultural; Matemática.

Introduction

This article addresses the training of elementary school mathematics teachers, a topic of relevance given the challenges of improving educational quality at this stage. We acknowledge the necessity of rethinking teacher continuing education and, to this end, begin with a theoretical framework that provides access to mathematical knowledge related to fundamental operations. We consider the concept of activity proposed by Leontiev (1978, 1983), based on Historical-Cultural Theory principles and the fundamentals of Training Guiding Activity (TGA), consistent with studies by Marco and Moura (2016).

This study was conducted through a remote training course, enabling participation by teachers from different regions of our country and broadening the debate and collective understanding of the concepts involved in fundamental operations on natural numbers. The remote modality presents new challenges and opportunities for continuing education, particularly in the current educational landscape.

Our reflections are based on Historical-Cultural Theory, which has its origins in Marx's historical-dialectical materialism (2013, p. 101). This theory understands the teacher as an individual who becomes human by appropriating the culture produced throughout history. It also considers work to be the "productive activity appropriate to an end," an activity that enables the development of the world and human individuality. Teachers become more human by performing their work and participating in learning spaces that provide opportunities for continuous professional development.

Regarding the term "continuing education," we adopt Moretti's (2007, pp. 23–24) approach, which defines continuing education for teachers as "a process that occurs in the continuity of initial training and aims to transform school reality by articulating theory and teaching practice." Based on these considerations and the theoretical foundations adopted for our research, we posit that continuing education for teachers occurs after initial training. It enables teachers to actively engage with

general models of teaching organization, thereby fostering students' development of theoretical thinking and scientific concepts. Thus, continuing education aims to contribute to the psychological development of both teachers and students.

It is important to emphasize that the appropriation of new meanings of fundamental operations does not occur in isolation. Rather, it is directly linked to teaching practice and the collective learning context. This study emphasizes the importance of collaborative spaces in prompting teachers to revisit their conceptions and transform their pedagogical practices, thereby promoting more effective articulation between theory and practice in mathematics teaching, even in remote environments.

This article is an excerpt from a doctoral research project defended in August 2022. The project's motivation was the question, "What evidence of new understandings of fundamental operations is revealed by mathematics teachers when they participate in a distance learning training activity?" To answer this question, our main objective was to investigate, analyze, and understand the training process of early-years mathematics teachers as they establish new meanings for the four fundamental operations (Moura et al., 2010; Moretti & Souza, 2015).

We achieved this goal with the help of the following specific objectives: 1) Understanding the training needs of teachers regarding the theoretical and methodological aspects of the four fundamental operations. 2) Identify the initial meanings teachers attribute to the fundamental operations in the early years of elementary school. 3) Analyze the transformation of meanings attributed by teachers, highlighting the process of acquiring new knowledge about the four fundamental operations in the context of remote training.

Theoretical framework

The historical-cultural theory, which originated from Marx's historical-dialectical materialism (2013), has two precursors: Vygotsky (2017) and Leontiev (1978, 1983, 2017). Based on Marx's work, Vygotsky (2017, p. 32) emphasizes that "man is a social being who, outside of social interaction, will never develop those qualities and properties that have been developed as a result of the historical evolution of all humanity," and Leontiev (1978, p. 267) adds that "the individual learns to be human. What nature gives him at birth is insufficient for living in society. He still needs to acquire what has been achieved in the course of the historical development of human society."

Rigon, Asbahr, and Moretti (2010) explain that, according to Marx, human beings are the result of the interaction between the individual's biological aspects and social and cultural factors. Through the appropriation of culture and everything that humanity has historically developed, the individual becomes human. This concept assumes that human development occurs throughout history, in both the collective dimension of the species and the personal trajectory of each individual. This perspective allows for a theoretical analysis of the social nature of humans and their socio-historical development.

From this perspective, humans cannot be understood only biologically but also as social beings whose formation occurs through cultural and collective interaction. Work is mediated by physical and social instruments that express shared societal modes of action. Leontiev (1978) points out that human activity is driven by a motive that differentiates it from simple action and that the personal meaning attributed to the activity is fundamental to its conscious realization.

According to Leontiev (1978, p. 296), activities are "[...] processes psychologically characterized by what the process as a whole is directed toward (its object), always coinciding with the goal that stimulates the subject to perform this activity, that is, the motive." According to the author, a subject must initially feel the need to act for the action to be considered active. The motive is what drives them to continue until the objective is fulfilled. The motive for the activity is "that which, when reflected in the human brain, excites one to act and directs the action to satisfy a specific need." The motive stimulates an isolated action corresponding to the object of the action, which is achieved when the action is performed" (Leontiev, 2017, pp. 45–46). The actions by which the activity is performed are one of its fundamental components. These actions represent the process by which the activity is performed. The means by which these actions are performed are called operations. Thus, according to Leontiev (1983), the elements that constitute the unity of an activity are the elements of orientation (need, motive, object) and the elements of execution (goal, action, and operation).

According to Leontiev (1978), motives can only be understood when they are external to the subject's activity, or "effective motives," which are no longer external. Motives are linked to the personal meaning that subjects attribute to their activities because "[...] to find personal meaning, we must discover the corresponding motive" (Leontiev, 1978, p. 97). Personal meaning is one of the main components of human consciousness, alongside significance and sensory content.

Leontiev (1978, p. 95) defines meaning as

[...] the generalization of reality that is crystallized and fixed in a sensitive vector, usually a word or phrase. It is the ideal, spiritual form of humanity's crystallized experience and social practices. Its sphere of representations of a society, its science, and its language exist as corresponding systems of meanings. Meaning, therefore, belongs, first and foremost, to the world of objectively historical phenomena.

Throughout their lives, humans adopt the culture of previous generations. In other words, by the time a person is born, this system of meanings already exists, and they must adopt it. This process occurs through activity. Leontiev (1978, p. 95) adds that "meaning mediates man's reflection of the world to the extent that he is aware of it, that is, to the extent that his reflection of the world is based on and integrates the experience of social practice." The author further adds that "meaning is, therefore, the form in which a person assimilates generalized and reflected human experience" (p. 95).

For example, when viewing an algorithm for subtraction, my consciousness may or may not reflect the synthesis of historical knowledge from previous generations about removing a quantity from a given group. In other words, it may or may not reflect this meaning. Historically, algorithms are instruments produced by humanity that aim to synthesize knowledge, but the subjects who use these algorithms do not necessarily appropriate these syntheses. The relationship between algorithms and historical meaning is not direct; otherwise, there would be no need for teacher-student training. These instruments are often used in crystallized ways that do not allow for awareness of these processes of meaning.

In our research, continuing education processes are understood to intervene in this process. They highlight existing meanings and enable the appropriation of new ones, allowing this algorithm to be used consciously and fostering awareness of actions. According to Leontiev (1978), when humans satisfy their need for knowledge, they appropriate the meaning of a concept. Furthermore, Piotto, Asbahr, and Furlanetto (2017, p. 105) emphasize that "the concept is nothing more than generalization, the meaning fixed in the word."

The appropriation of systems of meaning is an important component of social consciousness, as it depends on the personal meanings each individual attributes to these systems. According to Leontiev (1978, p. 98), personal meaning "precisely translates the subject's relationship with conscious objective phenomena," meaning "it is a relationship created in life, in the subject's activity" (p. 98). Meaning that which is not yet conscious has not been objectified or concretized; it does not yet fully exist for

humans. Moura, Sforzi, and Araújo (2011) add that human-produced knowledge only establishes meaning for other humans when it is inserted into human activity mediated by this knowledge. In other words, it establishes meaning through the process of attributing social and personal meaning.

"When we think about the professor's activity, we realize that it is permeated by the relationship between meaning and significance. In order to perform an activity, individuals need to understand that it will satisfy their needs. The activity must have personal meaning because it was triggered by something that motivated them" (Moura, 2004, p. 259). We understand that a dialectical relationship exists between meaning and sense, which permeates the individual-collective relationship and becomes possible when the subject is active. Sense cannot be taught; it is produced in the subject's life (Leontiev, 1978) through activity. Meaning is part of this process of producing and appropriating meaning. They are constituted in these units of meaning and, at the same time, permeate groups with more stable, generic meanings. Through personal meaning, subjects adapt to change and alter their relationships with these meanings. According to Piotto, Asbahr, and Furlanetto (2017, p. 107), "meaning is not constituted in an individualized way; on the contrary, it is constituted in a dialectical, social, and historical relationship."

But how do the reflections outlined so far relate to school education, and more specifically, to teacher training? We consider school education to be a way for humans to generalize their learning. According to Araújo and Moraes (2017), school education finds the modes of action that contribute to humanization in Pedagogical Activity. The authors identify two fundamental internal links of Pedagogical Activity that must be understood: the logical-historical movement of concepts and the modes of action through which new generations appropriate these concepts. These links can be strengthened by considering the teaching-learning-development unit in school education.

Thus, considering the teacher in the context of teaching activity means recognizing them as a human being who, through conscious organization of teaching, experiences a process of humanization alongside the student's development. Thus, pedagogical activity requires the teacher's intentionality, enabling the student to engage in learning activities that contribute to the psychological development of both. In other words, the teacher's teaching activity "[...] is characterized by a direct relationship with knowledge. By acting intentionally and developing actions that

promote their students' learning, teachers focus on the motive that drives them in their activity" (Rigon, Asbahr, & Moretti, 2010, p. 35).

Teachers essentially work with concepts, so they need access to tools that help them understand their subject matter precisely. They must attribute meaning to what they teach, so their students can see the significance of what they say is important to learn (Moura, 2004, p. 258). The same author describes the teacher as "a creator of meaning for what is taught, and their main tool is the word," and the school as "a place of production and exchange of meanings that constitute the meaning of the actions of the subjects who carry out the educational activity" (p. 260).

We emphasize that a teacher's work can be compared to alienated labor (Marx, 2013) when their objective (teaching and learning) does not align with their motivation. Instead of appropriating knowledge accumulated historically by students and teachers, they are driven by career advancement and a monthly salary, among other things. Leontiev (1978) uses capitalist production as an example in which the worker's activity becomes something other than what it really is. Unfortunately, in the current capitalist model, teachers can adapt to this system by organizing teaching activities in which students' personal meanings do not align with the social meanings that define the essence of scientific concepts. This occurs, for example, in the mathematics curriculum, thus emptying teaching of its social meaning. Therefore, teacher training is necessary, as students depend on teachers and teaching organizations to develop (Brito & Araújo, 2018).

In light of the above, we advocate for teacher training that considers the characteristics of non-alienated work, as defined by Marx (2013). We conceive of teacher training as a productive activity that is appropriate for achieving a goal. Personal meaning depends on the motive, which is expressed in social meanings. This enables the teacher's own humanization and the humanization of students.

Methodology

The empirical material analyzed was based on the reflections of twelve professors who participated in a remote extension course using the mediation tools available in the Moodle virtual learning environment and the web conference platform (MConf). The professors came from various states in Brazil: 41 from Minas Gerais (mostly from Uberlândia), eight from São Paulo, five from Goiás, two from Rio de Janeiro, and two from Mato Grosso do Sul. The course, entitled "Continuing Education for Teachers Who Teach Mathematics in the Early Years — Focopemai," was organized by a team of 14 professors. This team included six professors from the

Mathematics Department at the School of Basic Education at the Federal University of Uberlândia (Eseba); one professor from the Institute of Mathematics and Statistics at the Federal University of Uberlândia; three mathematics professors from the Uberlândia Municipal School System; two mathematics professors from the State School System; one teacher who worked as a technician at the Federal University of Uberlândia; and one doctoral student on scholarship.

This team was formed due to the close relationship between the Study and Research Group on Mathematics Teaching and Pedagogical Activity (Gepemape) at IME/UFU and the Study and Research Group for Teaching and Learning in Mathematics Education (Gepeaem) at Eseba/UFU. The extension course was part of the "Rede de Extensão #UFUEMCASA" (Extension Network #UFUEMCASA) program in partnership with the Distance Education Center (CEaD) and the Office of Extension and Culture (Proexc) at the Federal University of Uberlândia (UFU) according to Proexc Ordinance No. 17 dated March 25, 2020.

The trainers took several actions, including organizing the pedagogical space in the Moodle virtual learning environment, developing media materials such as videos, PowerPoint presentations, and Word documents, participating in online reflection sessions as planned in the course modules, and acting as individual tutors to accompany and guide the participating professors in the technical, pedagogical, and theoretical parts of the course.

The eight-week course took place in the second half of 2020. The 40-hour workload was divided into four modules. In each module, participating professors had access to online classes taught by guest lecturers from other universities and reflective sessions organized by course trainers. Each module contained various materials with a theoretical basis, available on Moodle. Tasks included reflections and sharing of pedagogical practices by the participating professors using the available Moodle tools.

The modules were divided as follows: Module 1 (5 hours): Theme: The Concept of Number; Module 2 (5 hours): Theme: Decimal Number System; Module 3 (10 hours): Theme: Meaning of Addition and Subtraction Operations; Module 4 (20 hours): Theme: Meaning of Multiplication and Division Operations.

The course proposal was justified by the historical and human condition of objective reality: the need for continuous training in the four fundamental operations for mathematics professors teaching early-level courses remotely during the social isolation experienced in Brazil in 2020 due to the Covid-19 pandemic.

A total of 34 participants completed the course, and 12 were selected for analysis. These representatives were selected based on their activity on the Moodle and MConf platforms and their level of involvement in the 40-hour extension course. Two professors with the following ranges of teaching experience were also considered: 0 to 2 years, more than 2 to 5 years, more than 5 to 10 years, more than 10 to 15 years, more than 15 to 20 years, and more than 20 to 25 years.

This criterion was established to determine if professors' training needs in the early years of mathematics end after initial training or after years of teaching. Training is continuous and occurs whenever the subject is active (Leontiev, 1978, 1983), aiming to appropriate general modes of teaching organization that contribute to the psychological development of teachers and students (Marco; Moura, 2016). We preserved the identities of the research participants by using fictitious names when they were mentioned in the transcripts of the available empirical materials on the Moodle platform.

Since the extension course was entirely remote with synchronous and asynchronous components, we used Moodle (AVA) for asynchronous activities and MConf for synchronous activities, such as online classes and chats, which we call reflective sessions. All synchronous moments were recorded and made available on Moodle later on, meaning all the empirical material for our research was present in the virtual learning environment (VLE) for analysis. Both synchronous and asynchronous interactions allowed us to understand the phenomenon, as both included interaction between course participants. Asynchronous interaction was facilitated and monitored through Moodle via individual participation reports for each task and an access report for each resource provided in the VLE.

To analyze the phenomenon in motion in the extension course, we used Caraça's (2000, p. 112) concept of isolation, assuming that "[...] in the impossibility of embracing, in a single stroke, the totality of the universe, the observer cuts out and highlights a set of beings and facts from this totality, abstracting from all others related to them." Thus, the isolated elements of our research that emerged from our analyses are: Training Needs and Appropriation of New Meanings.

We believe that the isolated element contains the characteristics of a totality — a reality in which, as Moura (2004, p. 167) asserts, "all the factors that are interdependent and have a significant impact on the phenomenon being studied" persist. To reveal the formative actions of the research participants within each element, we used Moura's (2004) formative episodes because we believe they contain

essential relationships within the elements that are indispensable for explaining the phenomenon. According to Moura (2004, p. 276), episodes are "[...] written or spoken phrases, gestures, and actions that constitute scenes that can reveal interdependence between the elements of a formative action." These scenes "seek to reveal the multiple determinations and essential relationships that make it possible to understand the phenomenon beyond appearance and the immediate" (Araújo & Moraes, 2017, p. 68).

Structurally, the isolated training needs contemplate two episodes: "Need and Reasons for Participating in the Training Course" (Episode 1) and "Initial Personal Meaning in Relation to the Concept of Number and Fundamental Operations" (Episode 2). Episode 1 has two scenes: 1.1: The need for theoretical and methodological appropriation to organize teaching, and Scene 1.2: Training needs regarding the teaching of curricular content in mathematics. Episode 2 consists of two scenes: Scene 2.1: The Concept of Numbers and the Decimal Number System (DNS). Scene 2.2: The Concept of the Four Fundamental Operations. These episodes reveal and analyze the training needs of the research participants and their initial understanding of numbers and the four fundamental operations.

The isolated appropriation of new meanings consists of one episode. Appropriation of Theoretical-Methodological Instruments (Episode 3). Its scenes are: Scene 3.1: The Concept of Number and the SND; Scene 3.2: Addition and Subtraction; Scene 3.3: Multiplication and Division; and Scene 3.4: Sensory Instruments in the Organization of Teaching Operations. This episode aims to reveal the new meanings that professors attribute to the four fundamental operations in the signification process.

Discussion of Results

We know that the emergence of a need is the first condition for the teacher to be active, and the motive drives it. According to Leontiev (1978), to ensure the structure of the activity, the motive must coincide with the object. In episode 1, we explore the journey of understanding teachers' training needs regarding the theoretical and methodological aspects of the four fundamental operations. The results of the analysis showed that not all professors who participated in the study enrolled in the extension course for an effective reason that coincides with the objective of our study: the acquisition of new meanings related to fundamental operations. According to our theoretical framework, merely understandable motives are insufficient to give meaning to the action (Leontiev, 1978; Longarezi & Franco, 2013).

Analysis of the empirical material produced revealed that the professors' understandable motives evolved into effective motives over the course of the action,

thereby ensuring the conditions for the activity and the Guiding Training Activity. We infer that, to transform the motives that drive teachers' activity, they must have access to historically produced knowledge and become aware that their actions often lead to alienated work rather than to teaching activities that enhance the development of theoretical thinking and higher psychic capacities.

The analyses also indicated that professors have training needs throughout their teaching careers, as their actions dissociate personal meaning from the social meaning of fundamental operations. Some professors revealed that their undergraduate education did not cover what they needed to learn to carry out their teaching activities. They said that the learning experiences they had in the extension course, through dialogue and experience-sharing, provided them with opportunities for formative reflection.

For example, Vera, a professor for over 20 years, identified a theoretical void in her initial training. "A few years ago, teaching degrees had a curriculum focused on systematization. As a result, we did not acquire the theoretical arguments to deal with our students," adding that this training gap affects students because "those of us who have been teaching mathematics for several years did not have the basic requirements in our training to promote our own logical mathematical thinking, let alone that of the children we teach." Vera reports that she initially thought difficulties in organizing teaching were only a concern for professors with education degrees. However, through her experience with mathematics professors, she realized this challenge also affects them.

Dealing with the four operations for mathematics graduates always seemed obvious to me. But it isn't! Much less so for educators. Even though we are now (2020) in the final years of elementary school, dealing with this issue in the classroom and in research with professors in the early years has enabled me to identify weaknesses in teachers, students, and researchers (Vera).

These records show that Vera recognizes the existence of training needs related to teaching fundamental mathematical operations for both early-year and late-year teachers. We recognize that this need is the starting point for the possibility of teachers being active, and

[...] the possibility of sharing meanings in joint activities is a premise of historical-cultural theory, which considers knowledge to be the result of the construction and appropriation of knowledge and know-how in the process of giving meaning to the objectives of shared activities. (Moura, 2021, p. 18).

The analysis of the empirical material also enabled us to identify that training based on know-how rather than thinking skills in solving higher-level math exercises does not make sense for those who graduate and intend to pursue a career in teaching. As an example of this, we identified that Rose, a professor for over 15 years, seems to have realized that the extension course provided new meanings for the use of sensory instruments in teaching mathematical content, when she states:

I was teaching the way I had learned. We didn't have any practical training in teaching mathematics during our undergraduate studies, only theory. Despite my knowledge of golden material and the abacus, I was introduced to an innovative way of teaching, which also improved my knowledge and understanding of the whole process (Rose).

In episode 2, the analyses revealed that the initial meanings that most professors attributed to the concept of number and the Decimal Number System (DNS) were dissociated from their essences: sometimes these meanings were limited to their experiences as students in basic education, and sometimes they were restricted to the didactic textbook.

When I started this course, I was overwhelmed by questions, including those about DNS. It's incredible how much we can know and still be so limited and superficial. My knowledge of DNS was limited to what I learned in elementary school. Over time, I forgot about it (Luciene).

Regarding the meaning of the concept of numbers and operations, I would highlight that it was more related to my experience as a student in the early years, my experiences at school, my own studies, and the needs I felt when teaching certain concepts to children in the classroom.

I had only heard a little about the subject in the didactic textbook I used for teaching, and it was not clear enough for me to feel confident mediating lessons related to numbers (Laís).

We infer that when professors recognized their limitations in forming these concepts, they were motivated to remain in the learning space and reframe their understanding. Regarding fundamental operations, the initial meanings were largely confined to an understanding of operational techniques that were completely divorced from the process of appropriating "[...] the essential features of the embodied activity, accumulated in the object" (Leontiev, 1978, p. 268). Vera's record shows evidence of her appropriation of ways of operating with the algorithm. However, she did not appropriate the meanings of fundamental operations because she did not participate in environments designed for this purpose. She states that:

[...] perhaps what they did not allow us to do was to think about the meaning of these operations. So I consider that, when starting module 3, in addition to anxiety, I assess myself at that moment as insecure

and instrumentalist, only knowing how to operate the algorithms, without paying attention to the importance of their meanings (Vera).

That said, we believe that when teaching fundamental operations, professors need to give meaning and significance to historically objectified knowledge. This knowledge is the result of human activity seeking solutions to problem situations involving the control of variation in groups of quantities. Since professors organize teaching activities, they must have this understanding to intentionally promote student development.

Episode 3's analysis of empirical materials revealed professors appropriating new meanings. The records showed changes in professors' initial meanings, approximating the social meanings of theoretical-methodological concepts set in motion during the extension course. We understand that this process involves an individual-collective relationship because sharing meanings during training can enable professors to create and adopt new meanings.

For example, Professor Elena emphasized in her records the importance of these interactions during the extension course. She also pointed out their potential influence on the teaching organization if a professor does not "like" mathematics. She mentioned the richness of the discussions that took place in the chat, where everyone could express their doubts, desires, certainties, achievements, and their "mathematical practice."

I believe that interaction between professors in the early and later years is fundamental to building a solid foundation... I believe that the greatest difficulty for children is often the professor... who shows that they don't like math. What makes me happy is seeing this movement of professors in the early years mobilizing to learn more every day and contribute to building a solid foundation in this subject. (Elena)

We believe that Elena's reported "liking" or "disliking" of mathematics may be linked to the educational limitations of early mathematics teachers. It is not enough for professors to be aware of their limitations. They must take action to reduce and overcome them. This requires intentional, continuous training to acquire the theoretical and methodological skills needed to teach mathematics in the early years. Bruna, a professor for over 20 years, states that

[...] there is no point in making our students skilled at performing calculations randomly if, when challenged to think about the actions of adding and subtracting, for example, instead of analyzing the transformations that are occurring, these students only think about reproducing a set of finite procedures. (Bruna)

Thus, we understand that when professors' understanding of addition and subtraction is limited to algorithms, their ability to teach is compromised because they

cannot teach what they have not mastered. Sforni and Galuch (2016, p. 483) clarify this point, stating, "When there is a mismatch between the social and personal meanings of an action, the possibility of developing personally and professionally through work is reduced."

We understand that fundamental operations are a way to quickly control the variation of groups of quantities, freeing oneself from any sensory trace. Algorithms are a synthesized way to perform this movement. Mastering the steps of an algorithm does not mean that one has understood the social meaning of the operations. In view of this, Alice notes: "Before, calculation was fundamental for me, but today I see that, in both cases, calculation is just a way of representing the algorithm, and it doesn't necessarily help students understand what it means to subtract or add." Luísa admits,

From the beginning of my student life, I realized that I did not fully understand the ideas behind the two operations covered in the course so far, because at school I was taught the operations themselves, and I learned to perform them through repetition. I changed my understanding after taking the course, where I learned about algorithms, the meanings of operations, and the ideas behind addition and subtraction. (Luísa).

Rose, a professor for over 15 years, reports: "I taught the process mechanically, as I was taught, starting directly from the calculation," and Vera, a professor for over 20 years, also reveals:

After the module, I understood that operating requires meanings, thoughts of additions in the sense of joining and adding, that adding requires these thoughts, these relationships. And subtracting too. It requires thinking about the relationship between removing, completing, and comparing the initial values. (Vera)

On the other hand, Jaqueline, with less than 5 years of teaching experience, recognizes that her understanding of addition and subtraction is limited. This is the result of her training prior to participating in the continuing education course. In fact, this teacher has negative feelings about her math learning from her childhood: "I studied in the early grades in a rural area in a multi-grade classroom, which was horrible. I learned through psychological pressure, by force." However, through participation in the learning space, Jaqueline was able to change her negative view of mathematics and set new goals for her teaching practice by adopting new perspectives.

From now on, I believe that I will not be the same professor of mathematics. My background is very different today than it was before the training course. Every class is a new learning experience. Sometimes I think I lacked knowledge and skills in the strategies I used in the classroom. From now on, I will implement all the knowledge and

suggestions presented in this course. It is a pleasure to learn math in a way that is so different from how I learned it as a child. Not even in college. I was never given such a good perspective on teaching math before. (Jaqueline)

We understand that the methods used to organize mathematics teaching are influenced by the way professors understand concepts and can contribute to either an alienating or a humanizing education for the individual (Leontiev, 1978; Martins, 2010; Rigon; Asbahr; Moretti, 2010).

Considerations

Our analysis revealed that mathematical operations involve developing mental strategies that ensure the fundamental properties of the Decimal Numbering System (DNS). The DNS's purpose is to control the variation of groups of quantities. This variation occurs when performing operations such as addition, subtraction, multiplication, and division. This entire process is essentially linked to the DNS's structure. We also infer that there is a gap between teaching the synthesis of this knowledge, as reflected in algorithms, and the formation process resulting from human activity when controlling variation in large groups.

It is important to note that, although algorithms are a fundamental human invention for performing calculations more quickly, they should not be the starting point for teaching fundamental operations. Beginning teaching with algorithms risks promoting mechanized learning, which hinders the psychological development of professors and students.

Another important issue that arose from the analysis was the validation of teaching operations that incorporate mental calculation strategies, thereby ensuring and elucidating the characteristics of the SND. We believe that organizing teaching in this way, which involves solving problem situations that emphasize mental calculation, allows students to develop mental actions by creating strategies, testing hypotheses, and establishing relationships between inverse operations. Accordingly, we believe that working with the social meanings of operations should be the starting point for teaching them. Students can then identify which operation to use to solve a given problem. The social meanings of the operations are as follows: addition involves joining and adding; subtraction involves removing, completing, and comparing; multiplication involves adding equal parts, rectangular arrangement, combining elements, and proportionality; and division involves successive subtractions and measurement.

Regarding teacher training as a means of transforming the motives that drive their activity, analyses revealed that professors need access to historical knowledge to

recognize that their actions often lead to alienated work rather than to teaching activities that enhance theoretical thinking and higher mental abilities.

The analyses also indicated that professors have training needs throughout their teaching careers, as they dissociate the personal meaning from the social meaning of fundamental operations. Some professors revealed in their records that their undergraduate studies did not cover what they needed to learn to carry out their teaching activities. They also said that the learning experiences in the extension course, which included dialogue and experience-sharing, provided them with opportunities for formative reflection.

Based on our understanding of the need for continuous teacher training, we assumed at the outset of this research that continuous training occurs after initial training. It enables professors to remain active and to adapt the general teaching organization to foster theoretical thinking and scientific concepts in students. This contributes to the psychological development of professors and students.

Additionally, we highlight that this study provides evidence that continuous remote training can promote the transformation of motives and the awareness of the meaning of operational algorithms by producing personal meanings that align with social meanings. This broadens access and promotes interaction between professors, strengthening the training process even when it is conducted remotely.

To ensure the movement of meaning in this remote learning space, the following were necessary: a theoretical basis based on THC, intentional organization of training activities, constitution of a training collective consisting mainly of Mathematics professors, collective study and planning meetings among teacher trainers, participation of professors from other universities who are scholars of the discussed topic, and technical support from the #UFUEmCasa program in partnership with the Distance Education Center (Centro de Educação a Distância- CEaD) and the Dean of Extension and Culture (Pró-Reitoria de Extensão e Cultura - Proexc) at the Universidade Federal de Uberlândia (UFU).

It is essential to consider these aspects to ensure effective training practices that raise awareness among professors. This is important because we will increasingly deal with technology and remote teaching methods and denying them is no longer an option.

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