REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

ISSN 2359-2842 Volume 19, número 52 – 2026 DOI: https://doi.org/10.46312/a0e46c42

Forms of generalization in the teacher education process that include elements of algebraic knowledge in early school

Formas de generalização no processo formativo de professores envolvendo elementos do conhecimento algébrico nos anos iniciais

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RESUMO

Este artigo tem como objetivo analisar as formas de generalização mobilizadas por professores dos anos iniciais ao se envolverem com Situações Desencadeadoras de Aprendizagem que incorporam elementos do pensamento algébrico. Fundamentada no conceito de pensamento teórico e na Teoria da Objetivação, a investigação foi conduzida por meio de um experimento formativo com que envolveu 18 docentes que lecionam matemática nos anos iniciais do Ensino Fundamental. Os resultados evidenciaram que problemas cuja resolução pode ser feita por meio de contagem tendem a favorecer uma generalização de natureza aritmética, enquanto aqueles que exigem a superação da contagem para alcançar a solução propiciam o desenvolvimento de generalizações de caráter algébrico. Com isso, a pesquisa busca contribuir tanto para o aperfeiçoamento da formação docente quanto para a promoção do pensamento algébrico nos anos iniciais do Ensino Fundamental.

PALAVRAS-CHAVE: Pensamento Algébrico. Generalização. Formação de professores dos Anos Iniciais. Teoria da Objetivação. Pensamento Teórico.

ABSTRACT

This article aims to analyze the forms of generalization employed by early-grade teachers when engaging with Learning Trigger Situations that incorporate elements of algebraic thinking. Based on the concept of theoretical thinking and the Theory of Objectification, the research was conducted through a formative experiment involving 18 elementary school mathematics teachers. The results showed that problems solved through counting tend to favor arithmetic generalizations, while those that require overcoming counting to reach the solution foster the development of algebraic generalizations. Thus,

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research seeks to contribute both to the improvement of teacher education and to the promotion of algebraic thinking in the early grades of elementary school.
KEYWORDS: Algebraic thinking. Generalization. Early childhood teacher education. Theory of Objectification. Theoretical thinking.

Introduction

Research since the 1980s has highlighted the importance of introducing algebra in the early years of Elementary School, rather than waiting for arithmetic knowledge to mature (Filoy & Rojano, 1989; Fiorentini; Miorin & Miguel, 1993; Lins & Gimenez, 1998; among others). Based on these studies, and with the explicit inclusion of algebra as a thematic unit in the early years by the BNCC (Brazil, 2018), interest in research involving this topic is growing in Brazil.

Given the recent inclusion of this topic in official documents and understanding the importance of organizing teaching in a conscious and intentional way for the development of algebraic thinking, it is evident that there is a need for training teachers who teach mathematics in the early years in order to promote pedagogical practices that can enhance the development of this special way of thinking in students at this stage of education.

In this context, the Study and Research Group on Production and Education from a Historical-Cultural Perspective[3] conducted a collective research project with teachers who teach mathematics in the early years of elementary school, entitled "O desenvolvimento do pensamento algébrico nos anos iniciais". As an unfolding of this collective movement, this article will present an excerpt from a doctoral research project (Romeiro, 2023) that aimed to investigate the forms of generalization manifested by teachers in the early years when engaging with learning-trigger situations, from the perspective of the Teaching-Orieentering Activity (Moura, Araújo & Serrão, 2018). The choice of the research objective was due to the importance attributed to generalization when it comes to the development of algebraic thinking present in official documents and research related to the topic (Brazil, 2018; Radford, 2018; Vale & Barbosa, 2019; Proença, 2019, among others).

The research is based on the concept of theoretical thought from the works of Davídov (1988) and on the Theory of Objectification by Luis Radford (2021), regarding

[3] In portuguese: Grupo de Estudos e Pesquisas sobre Produção e Educação na perspectiva Histórico-Cultural - Geppedh-Mat. More information about the group is available in the CAPES Research Group Directory, at the following address http://dgp.cnpq.br/dgp/espelhogrupo/35714.

the theoretical and methodological contributions involving the studied theme, since we understand that generalization is embedded in the movement of thought in general, and algebraic thought in particular.

Based on the principles of the Theory of Objectification, the analysis of reality in motion, which allows the object to reveal its essence (Vigotski, 2007; Davídov, 1988), was carried out from a multimodal perspective (Radford, 2015; Moretti & Radford, 2021), since we understand that subjects manifest thought with the aid of various semiotic means, such as gestures, verbal language, symbolic representations, among others.

Given the objective outlined for this article, we will seek to present the process of generalization in the thought process of teachers, aiming to identify in their semiotic manifestations the type of generalization involving algebraic knowledge.

2 Forms of Generalization in mathematical thinking involving elements of algebraic knowledge.

Starting from the understanding that algebra, as well as various mathematical concepts, is produced from human need, and that this continues to be produced and transformed based on the historical and cultural reality of a society, we base our research on the historical-cultural perspective in general, and on the concept of theoretical thinking and the Theory of Objectification in particular.

The Theory of Objectification (TO), proposed by Luis Radford (2021), is a contemporary approach that investigates the teaching and learning processes in the classroom, and has as its fundamental elements knowledge, understanding, and learning. Unlike contemplative or cognitivist approaches to the learning process, TO assumes that this process occurs through human activity, according to Leontiev's conception (1978), which Radford (2021) calls joint labor. Joint labor in TO is not limited to simple interactions among subjects, but involves a conscious collective action between teachers and students, who share a common goal: to transform knowledge into understanding. In addition to incorporating the structuring elements of the activity: the motive, the objective, and the object (Leontiev, 1978), joint labor, from the perspective of TO, also includes a community ethical dimension, based on respect, commitment, and care for the other (Radford, 2021).

In this framework, thought is not configured as an operation in an isolated mind and in a spontaneous way, since thought articulates embodied social practices, symbolic mediations, the use of signs, and the use of cultural artifacts, which only occur when it involves otherness, that is, in the relation with oneself and with the other. From this same perspective, Davídov says that thought "is the movement of forms of activity of society historically constituted and appropriated by those [the subjects of society]" (Davýdov, 1982, p. 279). In this way, thought involves a dialectical relation between meaning and cultural context and personal meaning, with its foundational basis being the productive, practical, object-based activity: the work."

In his theory, Davídov (1988) distinguishes two modes of thought that can be developed in school pedagogical practice: empirical thought and theoretical thought. Empirical thought operates from the apparent and observable qualities of the object, that is, "[...] it recognizes as common the similar qualities in all objects of the same type and class" (Davídov, 1988, p. 100). From this classification, an abstract and general mode of constitution of the object is induced, an empirical generalization. Empirical generalization determines a general form of solution for some types of particular situations, initially presented through models. According to Davídov (1988), this type of thought is limited because it does not analyze the object in its fullness and universality, does not consider the tensions and contradictions of the historical production of this object, and does not reveal its essence, observing only the logical process, ready and finished, as if the object had always existed in the way in which it was presented.

Theoretical thought, on the other hand, seeks to understand the "idealization process of one of the aspects of object-practical activity, the reproduction in it of the universal forms of things" (Davídov, 1988, p. 125). This thought aims to reveal the genesis of objects from their essence, that is, from the totality of their external and internal determinations. This special type of thought follows two dialectical paths: the reduction of the chaotic and immediate concrete to the abstract and the ascent from the abstract to the real concrete. It is in the movement of ascent that theoretical generalization is inserted, which makes it possible to reveal the essence of the object and, from it, to solve various particular problems of the same essence. In this way, the movement of theoretical thought is mediated by elements of the concept itself in order to understand it consciously and be able to use it to understand and act in the world.

For both Davídov and Radford, human activity is the means by which the subject understands the objects present in the world in a way conscious, enabling their critical and transformative action on reality. However, TO offers an explicit contribution on the role of human activity in the teaching and learning process as a dialectical unit, a joint action of teachers and students, in order to enable the constitution of the human subject, ethical, critical, political, poetic.

In TO, the encounter with knowledge is the objective of the teaching and learning process in activity in motion (Radford, 2021). We understand that as the subject enters into motion from the activity, thought also comes into motion. In this way, we argue that in the search for the encounter with knowledge in order to develop the maximum human potential, thought comes into motion. Since objectification is "social processes of progressively becoming aware of the historical-cultural systems of thinking and doing – something that we gradually perceive and at the same time endow with meaning" (Radford, 2021, p. 109), we argue that, in seeking the encounter with knowledge, thought must follow the movement of theoretical thought.

During this process of objectification, mediated by human activity, makes it possible to materialize knowledge – understood as human potential – transforming it into knowledge, which is configured as a singular manifestation of this knowledge. Knowledge, therefore, has an internal essence, which can be revealed through collaborative work, allowing its reproduction in other situations that contain the same essence. As we have described, in the objectification process, thought also comes into motion, which, in turn, is mediated by elements of the concept. In view of this, we consider that the encounter with knowledge, although it does not reflect it in its entirety, is mediated by the elements that constitute the concept of this knowledge.

Considering that algebra is knowledge produced in the historical and cultural course of humanity, and that it can be materialized in the form of knowledge, we argue, as do Santos (2020) and Moretti, Virgens and Romeiro (2021), that algebraic thinking is theoretical thinking mediated by an algebraic concept. In this sense, the theoretical generalization of the concept becomes a constituent element of algebraic thinking.

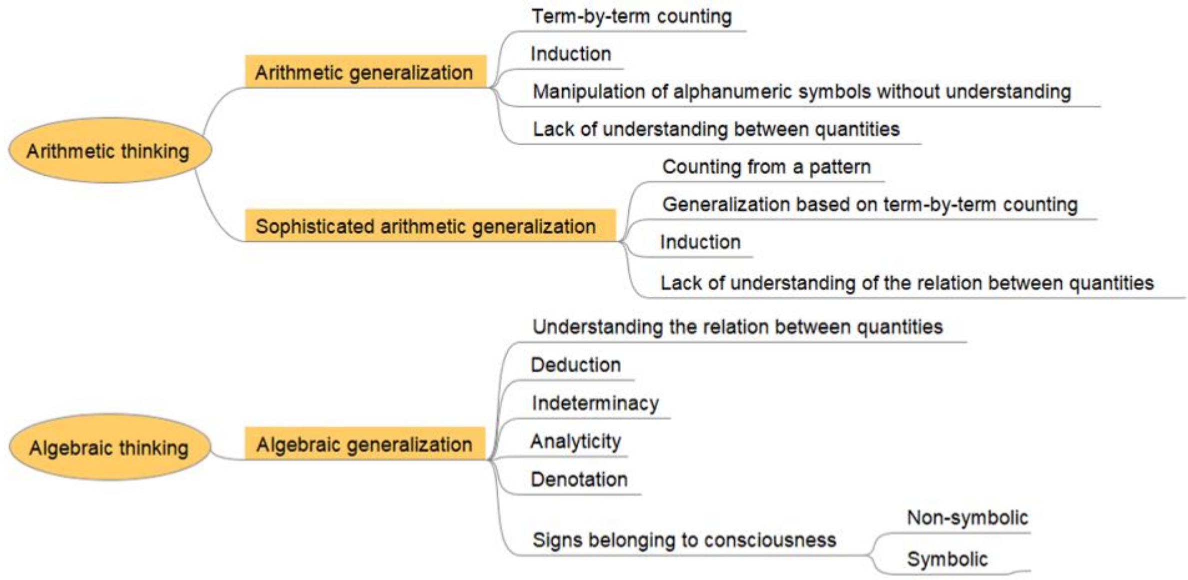
From the perspective of TO, the algebraic generalization, characteristic of the movement of algebraic thought, is characterized by indeterminacy, analyticity, and the ways of representing or symbolizing these indeterminate quantities and their operations; that is, an “analytic way in which we think when we think algebraically” (Radford, 2018, p. 9). For the author, indeterminacy is associated with the use of variables, unknown terms, parameters, among others. Analyticity refers to the manipulation of indeterminate quantities as if they were determinate; that is, one operates with unknown terms and their operations, as if the terms were known. Idiosyncratic modes of representation encompass the semiotic means of explaining and representing generalization and thought (Radford, 2018). This form of representation can be traditional, using symbolism, or non-traditional, using other semiotic means such as natural language.

Radford (2018) also notes that, when seeking the result of a situation containing elements of algebraic knowledge through trial and error or through induction, generalization is not considered algebraic, that is, it remains in the field of arithmetic, since attention and activity are focused on "finding" a general way to solve the problem or "finding a formula," and not deducing, through analyticity, the general way to solve it. We consider that when dealing with algebraic knowledge, this form of generalization can be classified as a generalization belonging to empirical thought and not to algebraic or theoretical thought.

Vergel, Radford, and Rojas (2022) also identify the existence of another way to generalize situations, mainly encompassing sequences, which they call "sophisticated arithmetic generalization." According to the authors, this form of generalization is based on proto-analyticity, that is, a primary mode of analyticity that does not involve deduction in its process, until it reaches a generalization of the object involving the elements of the algebraic field, but still within the arithmetic field, since it starts from arithmetic data through counting to establish a general way of defining any term of a sequence.

Figure 1 summarizes our understanding of the relationship between thought and the generalization process containing algebraic conceptual elements.

Figure 1 - Relation between thought and types of generalization involving algebraic conceptual elements



Source: Romeiro, 2023, p. 95

Radford (2018) complements his analysis of algebraic thinking by proposing different forms of manifestation of algebraic generalization with what he calls layers of generality. For the author, there are three levels of generalization in the development of algebraic thinking: factual generalization, contextual generalization, and symbolic generalization, which are progressively perceived by students.

Factual generalization, according to Radford (2018), refers to the first level in which the general idea of a concept, in the case of his studies, the general form of a pattern, is expressed through facts, through particular examples, with generalization being strongly linked to the concrete. In contextual generalization, which occurs in a new direction in the movement of algebraic thought, subjects begin to detach themselves from dependence on particular examples and start to express their understanding of the object using a more general semiotic representation (Moretti, Virgens & Romeiro, 2021), that is, “the formula is expressed at a more general level; the variables and their relationship become explicit and are referred to through contextual elements – spatial linguistic divisions” (Radford, 2018, pp. 22-23). Finally, symbolic generalization uses symbolic semiotic means, mainly literal ones, to represent the general way a sequence is formed. This way of thinking and generalizing is presented in a rather sophisticated field of algebraic thought, so that particular situations are not necessary to understand or explain the sequence and its pattern. This level of generalization “changes the way the subject thinks and is, within the context of algebraic thought” (Moretti, Virgens & Romeiro, 2021, p. 1468).

With each layer of generality, a new movement of thought is constituted and, in this movement, “the empirical becomes theoretical, and conversely, what at a certain stage of science was considered theoretical becomes empirically accessible at another, higher stage” (Kopnin, 1978, p. 153), encompassing elements had already validated and overcoming the limits of these elements for the formation of new conceptions.

These three layers of generality allow us to understand the path and movement of the subjects' thinking in relation to algebraic concepts, revealing the complexity involved in the movement of theoretical thought in this field of knowledge. It was in light of this understanding of generalization, involving algebraic knowledge, that we focused our analysis of the reality produced with teachers in the formative process. In the next section, we will present the methodological path of the research.

3. The Research Process: The Methodological Path

This research was developed using the historical-dialectical method, articulated with the multimodal semiotic approach of Objectification Theory (Radford, 2015; 2021), aiming to understand the forms of generalization manifested by teachers in collaborative work. To this end, we jointly developed with the researchers of Geppedh-

Mat a formative experiment (Davídov, 1988) focusing on the development of algebraic thinking in the early years of Elementary School.

The experimental part of the research was carried out in the second semester of 2018 and the second semester of 2019. In total, 20 face-to-face meetings were held during the period designated for in-service teacher education, called "activity hour" in the Guarulhos municipal system. Additionally, two readings were assigned to be done at a location of the teachers' choice and subsequently discussed in the face-to-face meetings. Eighteen teachers from the municipal public school system of the city of Guarulhos participated in this formative experiment. The majority of them had a degree in Pedagogy, with only one teacher, who did not participate in the entire experiment, also having specific degree in mathematics. All signed the Informed Consent Form and authorized the use of images and testimonies. In order to ensure confidentiality, the names used in this article are fictitious.

Based on criteria previously and intentionally defined by the researchers, the teachers were separated into four groups that remained together until the end of the experiment. To allow for an analysis consistent with the assumptions of multimodal semiotic analysis (Radford, 2015; Moretti & Radford, 2021), the meetings were recorded on video using five cameras and four recorders, one for each group. The productions made on the blackboard by the teachers or at moments that the researchers considered important were also photographed. In addition, sheets for individual and/or collective recording were made available to the teachers. All materials produced were digitized and organized into virtual folders by date and file type. For a better understanding of the data, the audio and video were synchronized.

The analysis of the reality apprehended in the formative experiment followed the three methodological moments proposed by Radford (2015). Initially, the “salient segments” were identified and transcribed, that is, passages that seemed to contain evidence of learning, of objectification (Radford, 2015, p. 561). These transcribed segments were analyzed in light of the theoretical framework, taking into account all semiotic manifestations, seeking to reveal the phenomenon studied, that is, the forms of generalization manifested by the teachers, in motion.

In order to structure the actions in a collective and collaborative context between the participating researchers and teachers in training, five Learning Trigger Situations were developed (Moura et al., 2010) composed of conceptual elements of algebraic thinking, namely: generalized arithmetic and variables (variation field, unknowns and functional relations). The second part of the experiment also required teachers to analyze the strengths and weaknesses of some teaching materials used in daily school life for the development of algebraic thinking in students. In the end, teachers were asked to develop a pedagogical action plan with students, considering the knowledge and reflections produced throughout the teacher education process.

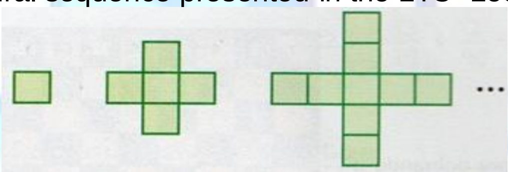
In the following section, we will present a section of the analysis of these manifestations identified in the education process of teachers who teach mathematics in the early years.

4 Forms of generalization manifested by teachers involving elements of algebraic knowledge in the early years.

The data analysis in the research was organized into episodes through salient segments, in accordance with the structure of the teacher education experiment. In order to identify the data analyzed in the salient segments, we will use the sequential numbering of the dialogues containing the transcription of the teachers' speeches and some interpretive comments from the researcher.

In this article, we have chosen to highlight excerpts relating to the reality apprehended within the moment entitled: “Forms of generalization in situations involving sequences”, involving the Learning Trigger Situation “Leo's fantastic adventure”, worked on over two meetings. The LTS contained a virtual story with three questions of progressive complexity, having as essential conceptual elements the relation between magnitudes, the variable, and the functional relation, presenting a recursive figural sequence, as shown in the following figure:

Figure 2: Recursive figural sequence presented in the LTS “Leo's Fantastic Adventure”

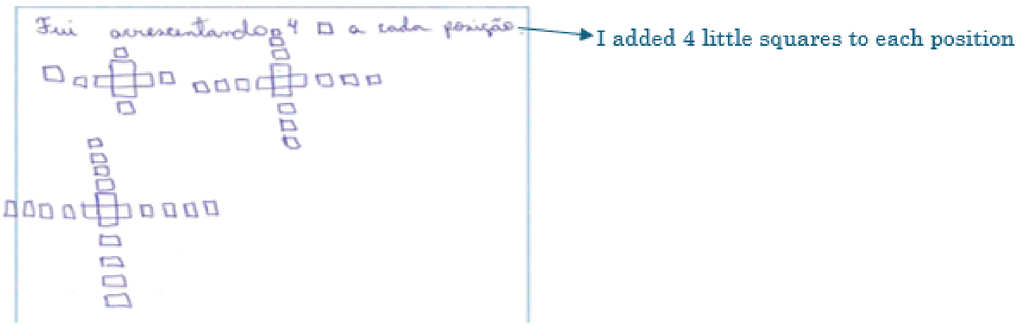


Source: The researchers’ Archive of the GEPPEDH-Mat (Romeiro, 2023)

The virtual story presented the story of five boys who got lost in a strange land called "Papelândia". To get out of this place, it was necessary to type a code into the elevator, which determined that, every 5 positions, one person would exit "Papelândia However, all the people who entered together had to exit simultaneously, since the elevator would only open once.

The first question proposed invited the teachers to determine the number of little squares[4] present in the 5th term of the sequence. In order to aid in visualization and understanding, little cubes of golden material were made available that could serve as a concrete instrument. The teachers began solving the LTS by observing the sensory and concrete data of the sequence, with some using the concrete material to support the continuation of the next terms. In general, the teachers solved the first question by counting term by term and arithmetically generalized the pattern of the sequence, that is, they recognized that with each term there was an increase of four squares, as recorded by teacher Eloísa.

Figure 3: Solution to the first question of the LTS by teacher Eloísa



]Source: Eloísa, 5, RI

The teachers generalization method indicated that the sequence pattern was found; however, the deduction of the sequence's structure was not revealed in the teachers' thinking, failing to demonstrate analyticity, one of the characteristics of algebraic generalization. This way of thinking about the sequence did not allow the teachers to quickly find remote terms, but rather, through the support of the previous term added to the pattern, that is, counting each term from the pattern. This type of thinking does not reveal the structural relations of the sequence, the relations among the magnitudes, thus remaining in the empirical arithmetic field, regarding algebraic knowledge.

The second question proposed determining the number of little squares needed for all the boys to leave Papelândia, that is, the number of little squares needed for the five boys to leave together in the elevator. This question involved two distinct relations between quantities: position in relation to the number of people and the number of

[4] In this text, we will use the term "little square" to refer to the flat figure represented on the SDA sheet, and the term "little cube" to designate the concrete material offered and manipulated by the teachers.

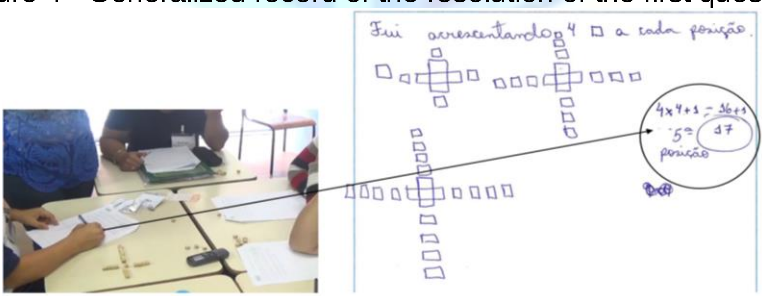
squares in relation to the position in the sequence, which involved a new way of thinking about the sequence, since the answer was not quick to find.

Initially, the teachers had to think about and identify which position they would find the number of little squares in. A long period of dialogue was necessary until the teachers were able to reach a consensus and understand this relation. After this conclusion, they returned their focus to the relation between the number of little squares as a function of position and sought to find the number of little squares in the 25th term of the sequence.

One of the teachers suggested that by continuing to add little squares term by term, they would reach the desired position, and in this way, it would be possible to obtain the answer. However, teacher Débora pondered, "But are we going to spend the rest of our lives adding little cubes? There has to be a logic to it" (Débora, Meeting 5, Group 4, Video: 15’26”-15’44”). This reflection spurred a new dialogue among the teachers, in which they began to seek a general way to understand and resolve the issue. This joint movement was quite lengthy, and concrete materials were heavily used, both to validate hypotheses raised and to deepen the understanding of the sequence's structure.

During the manipulation of the little cubes, the teachers observed that it was not necessary to add a little cube to each side of the central little cube, since the addition was repeated on each side of the cube corresponding to the first position. Based on this finding, they concluded that, to determine the number of squares in each position, it was necessary to know the number of squares on one side of the central square and multiply that value by four, thus maintaining the pattern of the sequence. This understanding of the sequence's structure allowed the teachers to understand the sequence in a new way, as recorded in the figure below:

Figure 4 - Generalized record of the resolution of the first question



Source: Eloísa, 5, VTC_19’38”/ Eloísa, 5, RI

According to the virtual story, five positions needed to be added for each person. In this way, the teachers organized the little cubes into groups to find the answer to the proposed problem.

The salient segment 1, shown in Figure 5, showed that, although the teachers managed to solve the second question, they still did not demonstrate an understanding of the functional relation of the sequence: position as a function of the number of boys. There are indications of a proto-analyticity, approaching a sophisticated arithmetic generalization, since, despite understanding the spatial structure (multiplying the blocks formed for each position by four), they still did not demonstrate an understanding of the general numerical structure, based on the arithmetic counting of blocks of cubes.

In both the first and second questions, the generalization shown by the teachers remained within the scope of arithmetic, since the teachers' attention was focused on concrete data and term-by-term arithmetic counting, even when working with blocks of cubes grouped for each child exiting the elevator.

Figure 5: Generalization method involving sophisticated arithmetic.

	Salient segment 1	*Interpretative comments*
1	*Katia:* How much does it be here? 	Referring to the number of little cubes on one side of the cube corresponding to the 1st position.
2	*Eloísa:* But why do these have 5 and this one only has 4?	Questioning the first block formed by 4 little cubes and the others formed by 5 little cubes, representing the increase of 5 positions for each boy's starting position.
3	*Paula:* Because this is the first position.	Pointing to the little cube corresponding to the 1st position, she restarts to count of each little cube that was linked to a position up to the little cube corresponding to the 25th position.
4	*Katia:* So it will be 24 [...]  [...]times 4 plus 1 [...]  [...]What does that give? 97 [...]. So, actually we answered question 2 here. For all 5 to leave together, they will leave in the 25th position, right? So it's 24 x 4 + 1.	At the end of her conclusion, Professor Eloísa agrees that the number of squares to be typed is 97 (24x4+1), but it seems she still hasn't grasped the functional relation of the sequence: position as a function of the number of boys, thus requiring a sophisticated arithmetic generalization.

Source: Meeting 5, Group 4, Video (36' - 36'48")

In order to move them away from the dependence on the blocks of little cubes used by the teachers in the second question, and to present the teachers with the need to understand the structure of the sequence, the researcher/trainer proposed that they think about the number of squares corresponding to the 23rd position in the sequence.

Figure 6: Dialogue about the solution for the 23rd position

	Salient segment 2	Interpretive comments
5	*Irineu:* I did this here, to find the 23rd: (23 – 1) x 4 + 1. 	Showing her sheet with the notes to the teachers. The teachers carefully observe the arguments she has developed to solve the problem.
6	*Katia:* How much did it be?	
7	*Irineu:* 89.	
8	*Eloísa*: Here you are on the 20th [cube], and he said over there it was the 23rd. Add 3 more [little cubes]. 	Dialogue with teacher Paula observing the concrete material cubes arranged in the 20th position, counting them one by one.
9	*Paula:* Count	Teacher Katia begins counting the cubes one by one. This demonstrates the foundation of arithmetic thinking and arithmetic generalization. Teachers Irineu and Eloisa observe the counting. 
10	*Katia:* There are 23. So you have to do 23 x 4 [...].	At this moment, Teacher Eloisa interrupts her.
11	*Eloísa:* [...]Here are 22.	Teacher Eloisa demonstrates confidence in her speech, stating the number of squares. This shows evidence of her understanding of both the spatial and numerical structures of the sequence.
12	*Katia:* So, it's 22 x 4 which equals 88, plus 1, which equals 89.	
13	*Eloísa:* It is.	The statement suggests a shift in algebraic thinking, including factual algebraic generalization.
14	*Irineu:* You always have to subtract 1 from the position number.	Oral expression shows evidence of the movement of algebraic thought, including, in this case, a contextual algebraic generalization.

Source: Meeting 7, Group 4, Video (18'20" – 19'33")

Lines 10 and 12 revealed evidences that the teachers understood the three characteristics belonging to the movement of algebraic thinking, becoming able to calculate any term of the sequence. As the manifestation of this way of thinking is linked to particular terms, in this case the 23rd term, it is possible to interpret that the movement of algebraic thinking includes a factual algebraic generalization.

Based on this collective understanding, teacher Irineu, in the 14th line, establishes the relation between the magnitudes of position and quantity of squares.

The use of the expression "always" signals a generalization valid for any term of the sequence, denoting analyticity, since he treats the unknown term as if it were known. Furthermore, this way of expressing an understanding of the sequence approaches a contextual algebraic generalization, as the particularity gives way to a more general way of representing the sequence, and the variables become explicit through verbal deictic expressions (Radford, 2018), becoming conscious for the teachers.

In order to provide a new movement of thought, the third question of the LTS asked teachers to leave a message that revealed the secret of the elevator, valid for the exit of any number of people who were in Papelândia, that is, to record the general way of representing the sequence. In order to answer this question, the teachers had to start the movement of thought without the aid of counting, since this procedure proved insufficient to solve the question.

In the salient segment 3 (figure 7), it was possible to observe the movement of algebraic thought and algebraic generalization already at another level of complexity. Contextual generalization gives way to a more general mode, in which signs overcome verbal aspects, making the use of concrete material unnecessary, deducing the symbolic register to represent the sequence (Figure 8).

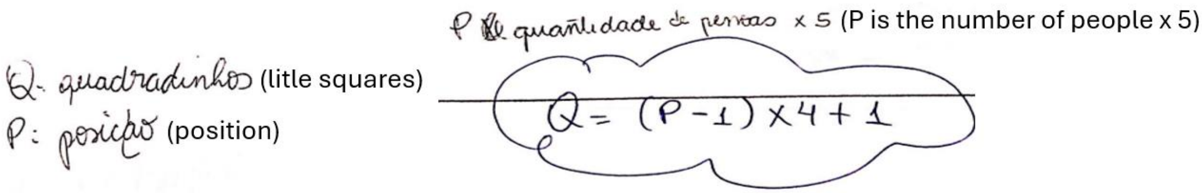
The record made by the teachers (Figure 8) represents the teachers' way of understanding the structure of the sequence, revealing a movement of thought that goes through a symbolic algebraic generalization. The movement of ascent from the abstract to the concrete, mediated by generalization, allows teachers to revisit concrete situations and solve problems involving the same conceptual structure. An example of this is the determination of the number of squares in the 23rd position, expressed in line 25 in Figure 7, even when this task is not directly linked to the previously worked functional relation, as in the case of the position as a function of the number of people. This return to the concrete, supported by generalization, demonstrates the consolidation of conceptual links inherent to algebraic knowledge.

Figure 7: Forms of generalization manifested by teachers

	Salient segment 2	Interpretive comments
5	*Irineu:* I did this here, to find the 23rd: (23 – 1) x 4 + 1. 	Showing her sheet with the notes to the teachers. The teachers carefully observe the arguments she has developed to solve the problem.
6	*Katia:* How much did it be?	
7	*Irineu:* 89.	
8	*Eloísa*: Here you are on the 20th [cube], and he said over there it was the 23rd. Add 3 more [little cubes]. 	Dialogue with teacher Paula observing the concrete material cubes arranged in the 20th position, counting them one by one.
9	*Paula:* Count	Teacher Katia begins counting the cubes one by one. This demonstrates the foundation of arithmetic thinking and arithmetic generalization. Teachers Irineu and Eloísa observe the counting. 
10	*Katia:* There are 23. So you have to do 23 x 4 [...].	At this moment, Teacher Eloisa interrupts her.
11	*Eloísa:* [...]Here are 22.	Teacher Eloisa demonstrates confidence in her speech, stating the number of squares. This shows evidence of her understanding of both the spatial and numerical structures of the sequence.
12	*Katia:* So, it's 22 x 4 which equals 88, plus 1, which equals 89.	
13	*Eloísa:* It is.	The statement suggests a shift in algebraic thinking, including factual algebraic generalization.
14	*Irineu:* You always have to subtract 1 from the position number.	Oral expression shows evidence of the movement of algebraic thought, including, in this case, a contextual algebraic generalization.

Source: Romeiro (2023, pp. 194-195)

Figure 8 - Recording using alphanumeric symbolic notation to represent the relation among quantities



Source: Irineu,7, RI

The record prepared by the teachers highlights the two functional relations involved in the LTS: position as a function of the number of people (P is the number of people x 5), and the number of little squares as a function of the position Q = (P – 1) x 4 + 1.

In addition to the movement of thought, the posture assumed by the teachers in these excerpts demonstrates the fundamentals of collaborative work, characterized by collective engagement around a common goal. It is observed that the teachers demonstrated a conscious understanding of the functional relations, through a synthesis shared by the group, indicating an advance in the movement of algebraic thinking, passing through algebraic generalization. In other words, it was in the collaborative work among the teachers that the objectification of algebraic knowledge in the form of knowledge occurred, in a mediated process of development of theoretical thinking that takes place in unity with algebraic generalization.

5. Final Considerations

This research aimed to identify the forms of generalization manifested by teachers in the early years of schooling when engaging with learning-trigger situations (LTS) that include elements of algebraic knowledge.

The research was based on the Theory of Objectification, which assumes that knowledge is produced through collaborative work in the dialectical relation between teachers and students to achieve a common goal. We also relied on the concept of theoretical thinking, understanding that in the materialization of knowledge in the form of knowledge, thought enters into movement in the direction of the movement of theoretical thought. Thus, in this research, we assume algebraic thinking to be theoretical thinking mediated by algebraic concepts.

In the context of algebraic knowledge, Radford (2018) states that algebraic generalization is characterized by indeterminacy, analyticity, and the ways of representing or symbolizing these indeterminate quantities and their operations, an “analytic way in which we think when we think algebraically” (Radford, 2018, p. 9). Radford (2018) also proposes three layers of generalization in the process of objectifying algebraic knowledge: factual, contextual, and symbolic.

As an excerpt from this article, we present the data from the reality apprehended in a formative experiment with teachers of the early years of Elementary School, who encountered the virtual story and the problems contained in the LTS “Leo's Fantastic Adventure”. Through a multimodal analysis, we observed that the first two questions proposed in this Learning Trigger Situation (LTS) initially mobilized arithmetic thinking, even allowing for a form of generalization considered more sophisticated, albeit based on counting. The third question, by asking for the formulation of a general rule for the presented sequence, prompted the teachers to examine its structure from both a spatial and numerical point of view. This analysis favored the objectification of algebraic concepts, such as the identification of patterns, the relation among quantities, and the range of variation. This third moment also propelled the teachers towards a thought process closer to algebraic theory, in line with the concrete-abstract-concrete path. The passage from the concrete sensory level, related to the perception of the pattern and spatial organization of the sequence, to the abstract level was mediated by analyticity, thus enabling the development of an algebraic generalization.

In short, when solving problems with defined positions in the sequences, such as adjacent terms, the teachers resorted to counting. This type of thinking is in the field of arithmetic thinking, encompassing an arithmetic generalization or sophisticated arithmetic. However, when faced with questions that required a more general understanding of the sequence's structure, the thought process shifted towards theoretical thinking, which incorporates algebraic generalization into its various layers, highlighting the numerical and spatial structure through analyticity.

The apprehended reality showed that the path of algebraic thought occurs during the objectification process when algebraic knowledge becomes an object of knowledge, considering that objectification encompasses the process of understanding knowledge in motion from human activity embedded in a historical-cultural context, the collaborative work.

Finally, we understand that the results of this research offer important contributions to the structuring of the education process of teachers who teach mathematics, especially with regard to the approach to algebraic knowledge in the various stages of basic education. This in-service education process should consider teachers and students as historical-cultural subjects, envisioning human formation in its maximum potential from a humanizing education perspective, which aims at teacher training beyond appropriation of concepts, but the transformation of human beings.

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Submetido em:25/07/2025

Aceito em:22/09/2025