# Análise de Erros na Produção dos Alunos na Prova da 12a edição da OBMEP: o Caso das Escolas do Oeste do Estado do Pará - Brasil 

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#### Abstract

The objective of this investigation was to analyze errors made in the twelfth test of the Brazilian Olympiad of Mathematics in Public Schools-OBMEP by students of the public schools from Pará's western region, who were in the 8th and 9th grades of elementary school, in the perspective of Error analysis methodology. For this purpose, a sample of 620 tests from a universe of 1477 was taken. The subject evaluated was Arithmetic (numbers and operations), discussed in question 4 of the test. From the analysis of the solutions, the errors were classified according to their type (errors due to misinterpretation, deficiency in basic concepts, unfamiliarity with multiples and numerical sequences and application of mistaken knowledge) and some examples were examined and discussed. The main results point to fragilities in the learning of multiples, divisors and basic operations, as well as strategies of resolution misused, besides difficulties presented by the students in the interpretation of the command of the question.


KEYWORDS: Error Analysis. OBMEP. Arithmetic.

## RESUMO

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#### Abstract

O objetivo desta investigação foi analisar erros cometidos na prova da $12^{a}$ edição da Olimpíada Brasileira de Matemática das Escolas Públicas-OBMEP por alunos das escolas públicas da região Oeste do Pará que estavam cursando o $8^{\circ}$ e $9^{\circ}$ anos do Ensino Fundamental, na perspectiva da metodologia de análise de erros. Para tanto, foi tomada uma amostra de 620 provas de um universo de 1477. O assunto avaliado foi Aritmética (números e operações), abordado na questão 4 da referida prova. A partir da análise das soluções, os erros foram classificados de acordo com o seu tipo (erros devido à má interpretação, deficiência nos conceitos básicos, desconhecimento de múltiplos e sequências numéricas e aplicação de conhecimento equivocado) e alguns exemplos foram examinados e discutidos. Os principais resultados apontam fragilidades no aprendizado dos conteúdos de múltiplos, divisores e operações básicas, assim como estratégias de resolução mal utilizadas, além de dificuldades apresentadas pelos alunos na interpretação do comando da questão. PALAVRAS-CHAVE: Análise de Erros. OBMEP. Aritmética.


## Introduction

The analysis of the students' productions occupies - or should occupy - a position of great highlight in the practice of teachers and teaching researchers. This analysis works not only as a means of diagnosing performance (evaluation), but also as a guiding parameter for teaching practice, which, in turn, includes, in addition to the evaluative dimension, the dimensions of planning and teaching itself.

In fact, the analysis of students' responses can be focused both as a teaching methodology, when used in the classroom in order to promote teaching based on detected errors and leading students to question their answers (CURY, 2007), as well as research methodology, from the investigative perspective of the student's subjective processes and elaborations, their most recurring errors, resolution strategies employed, etc. In this article, our focus is more on the second approach, notably from the perspective of error analysis.

Popularized in Brazil by Helena Noronha Cury's ${ }^{4}$ researches, error analysis, in general terms, consists of giving special attention to students' written production, in order to understand their reasoning, especially in the context of wrong answers. Diagnostics and elements that can help compose models of reasoning employed in the solutions are sought, as well as the analysis of methods of resolution and application of previous knowledge (relationships, hypothesis construction, inferences, simulations, etc.).

Our analysis was developed based on the productions presented in the 12th edition of OBMEP's test. The Olympiad is a social inclusion project that aims to discover, encourage and recognize talents in the process of training in the various areas of knowledge throughout the national territory. It has been held since 2005 by

[^1]the Ministry of Education (MEC), the Ministry of Science and Technology (MCT), in partnership with the National Institute of Pure and Applied Mathematics (IMPA) and the Brazilian Mathematical Society (SBM). The target audience of OBMEP is composed of students from the 6th grade of elementary school until the last year of high school, with wide participation throughout the national territory (BRASIL, 2017).

This study investigates and examines the solutions presented by students in the 8th and 9th grades of elementary school in question 04 of the 12th edition of OBMEP's test (Level $2^{5}$ ), that involves the content of Arithmetic ${ }^{6}$. From the investigation of the solutions presented, we seek to analyze the mistakes made from the perspective of the error analysis methodology.

## Error Analysis

When analyzing the students' written productions, we are mainly evaluating the content of that production. In other words, we are employing a data analysis methodology known as content analysis. In general terms, content analysis can be defined as
(...) a set of analysis techniques that aims to obtain, through systematic procedures, quantitative or qualitative indicators that allow the inference of knowledge related to the production / reception of messages. It is, in last case, of an effort of interpretation that oscillates between the rigor of objectivity and the fruitfulness of subjectivity. (CASTRO, ABS e SARRIERA, 2011, p. 816).
Due to its broad definition, it is subject to several forms of operationalization. Bardin's ideas (1979), which suggested that this operationalization take place in three main moments, served as a guiding parameter for this research. The first one corresponds to the pre-analysis, when the hypotheses are defined, the objectives outlined and the analysis criteria indicators that will be used. In this phase, a "floating" reading is made in which "the researcher allows himself to be impregnated by the material" (CURY, 2007, p. 65).

The second moment corresponds to the exploration of the material and refers to the transformation of the raw data (original evidence) into a structure for the manifestation of the data - in our case, semantic cuts of the answers, categorization of the observed errors and, finally, the proper enumeration of these categories, which

[^2]considers, among other things, the previous theoretical understanding about the emergence or not of meanings in the analyzed answers.

Finally, the third moment, which consists of the treatment of the results, the phase in which the description of the categories occurs, with the presentation of the charts and tables produced in the form of frequency distributions of the classes, or even the application of standardized statistical tests. Then, the productions are interpreted, seeking to reach a deeper understanding of the resolutions presented through inferences in the student's subjective field, in the search for new understandings and possible diagnoses.

We agree with Santos (2015), Castro, Abs and Sarriera (2011) and Fiorentini and Lorenzato (2009) on the idea that operationalize such an analysis, mainly with the elaboration of categories that describe errors, it is an exercise, at the same time, deductive, as it is operationalized based on previous knowledge (notably the results obtained in other investigations); and inductive, with concepts, meanings and categories emerging from the data. It is also an essentially interpretative effort, which consists not only in the collection and analysis of data, but above all in the production of new data from the generated analyzes.

Thus, we can say that error analysis is an investigative proposal that, methodologically, is based on content analysis, just as the methodological principles established by Bardin (1979). In listing the types of documents that could be part of a research of this nature, the author points out, for example, responses to questionnaires, experience reports and, above all, tests or discursive tests.

Although the analysis of students' written production can commonly be confused with diagnostic assessment procedures - and it is a fact that the two activities have their intersections - there are some important differences. The first, most obvious, consists in the fact that the analysis of production does not aim at attributing a concept or grade; the second difference lies in the intrinsic goals. While, in the diagnostic evaluation, errors are discarded and the successes are rewarded with a standardized score, necessary for progression between the series, in the analysis of errors, what counts is not the success or the error itself, but the forms of to appropriate a certain knowledge, which, in turn, can show both learning difficulties and the diagnosis of possible didactic obstacles.

## Methodological Procedures

This research is classified as descriptive, according to its objectives, since it seeks to "describe or characterize in detail a situation, a phenomenon or a problem"
(FIORENTINI, LORENZATO, 2009, p. 70). According to the data collection process, it is characterized as documentary, since it relies on documents that have not received a previous analytical evaluation ${ }^{7}$.

The research subjects are students from public schools in the western region of Pará who were attending the 8th and 9th years of Elementary Education in 2016 and participated in the second phase of the 12th edition of OBMEP, held on September 10, 2016.

The research analysis corpus consisted of a sample taken from a total of 1477 tests. The sample was obtained based on the calculation of sample size for the interval estimation of the proportion, using the stratified proportional sampling technique, in which the cities in the researched region were taken as strata. Samples of size directly proportional to the size of the population strata were taken. Thus, we proceeded to calculate the sample size, as given by expression (1).

$$
\begin{equation*}
n_{0}=\frac{a_{a / 2}^{2} p^{*}\left(1-p^{*}\right)}{E^{2}} \tag{1}
\end{equation*}
$$

where ${ }^{n_{\text {ais }}}$ the sample size for an unknown or infinite population; $z_{\alpha / 2}^{2}$ corresponds to the degree of confidence, given by the confidence level ( $\alpha$, the success rate of the procedure); $p^{*}$ is the population proportion of individuals belonging to the categories we are interested in studying ${ }^{8}$; and $E$ is the margin of error, or maximum desired error. Thus, for $\alpha=95 \%, z_{\alpha / 2}^{2}=1,96, p^{*}=0,5 \mathrm{e}^{E=3 \%}$, the expression (1) is:

$$
\begin{equation*}
n_{0}=\frac{1,96^{\mathrm{D}}, 0,5(1-0,5)}{(0,08)^{2}}=1067,111 . \tag{2}
\end{equation*}
$$

For known population size, we take the correction factor given by:

$$
\begin{equation*}
n=\frac{n_{0} N}{n_{0}+N-1} \tag{3}
\end{equation*}
$$

where n represents the true sample size and N is the population size. Thus, when we take (2) and (3) together, we obtain:

$$
\begin{equation*}
n=\frac{1067,111,1477}{1067,111+1476}=619,7618 \cong 620 \tag{4}
\end{equation*}
$$

With the sample size n calculated in expression (X.4), we proceeded to collect the sample according to the stratified proportional sampling procedures. The sizes of

[^3]each population stratum $(N)$, their proportion $(p)$ and their respective sample sizes ( $n$ ) are shown in Table 1.

Table 1: Strata (cities), population sizes ( N ), proportions ( p ) and sample sizes ( n ) for a sample of size 620 OBMEP tests.

| Cidade | $\boldsymbol{N}$ | $\boldsymbol{P}$ | $\boldsymbol{N}$ | Cidade | $\boldsymbol{N}$ | $\boldsymbol{P}$ | $\boldsymbol{N}$ |
| :--- | :---: | :---: | :---: | :--- | :---: | :---: | :---: |
| Alenquer | 72 | 0.049 | 30 | Mojuí dos Campos | 18 | 0.012 | 8 |
| Almerin | 28 | 0.019 | 12 | Monte Alegre | 101 | 0.068 | 42 |
| Altamira | 95 | 0.064 | 40 | Novo Progresso | 23 | 0.016 | 10 |
| Anapú | 25 | 0.017 | 10 | Óbidos | 86 | 0.058 | 36 |
| Aveiro | 21 | 0.014 | 9 | Oriximiná | 73 | 0.049 | 31 |
| Belterra | 21 | 0.014 | 9 | Pacajá | 50 | 0.034 | 21 |
| Brasil Novo | 21 | 0.014 | 9 | Placas | 23 | 0.016 | 10 |
| Cachoera da Serra | 4 | 0.003 | 2 | Porto de Moz | 12 | 0.008 | 5 |
| Castelo dos Sonhos | 7 | 0.005 | 3 | Prainha | 69 | 0.047 | 29 |
| Curuá | 29 | 0.020 | 12 | Rurópolis | 22 | 0.015 | 9 |
| Curuai | 9 | 0.006 | 4 | Santarém | 359 | 0.243 | 151 |
| Faro | 7 | 0.005 | 3 | Senador José P. | 18 | 0.012 | 8 |
| Itaituba | 73 | 0.049 | 31 | Terra Santa | 21 | 0.014 | 9 |
| Jacareacanga | 16 | 0.011 | 7 | Trairão | 11 | 0.007 | 5 |
| Juruti | 94 | 0.064 | 40 | Uruará | 8 | 0.005 | 3 |
| Medicilândia | 31 | 0.021 | 13 | Vitória do Xingú | 30 | 0.020 | 13 |
|  |  |  | Total | $\mathbf{1 4 7 7}$ | $\mathbf{1}$ | $\mathbf{6 2 0}$ |  |

Source: The authors (2018).
From the tests analyzed in the sample, a question was selected that deals with the content of Arithmetic. The choice of this content was due to the fact that it presents important initial basic concepts, such as basic operations, factoring, minimum common multiple, positional base system, among others. We are also in line with the National Curriculum Parameters - PCN (BRASIL, 1998), when they state that it is important to emphasize that the abandonment of arithmetic cannot be configured in the final years of elementary school. The document also points out that arithmetic problems are practically not posed as challenges for students in those grades and that the situations worked on generally favor only algebraic concepts.

The resolutions were then analyzed and categorized according to the type of error made. For that, we proceeded in the elaboration of its own categorical system. Naturally, the error classification process has a high degree of complexity, as it is necessary to dive into the student's subjective universe of thought, in the analysis of his particular intentions and strategies for solving them. To objectively interpret this phenomenon and to propose an adequate categorical system that is capable of describing and explaining the variety of errors made in a given question is a difficult
task, since the manifestation of the error occurs from a multiplicity of causes that, in practice, are generally very difficult to perceive or describe accurately. So we recognize the weakness of the proposal for a categorical system of errors that, in turn, encompasses, among other things, our personal interpretation of the student's reasoning as well as the limitations of our vision and understanding about his cognitive universe.

Anyway, in order to objectify our analysis, we developed a system of classifying errors and categorized the students' resolutions according to what was manifested to us as an emphasis, within the limited perceptible spectrum in each case. The analysis and discussion of the results are presented in the following section.

## Analysis and Discussion of Results

We started the discussion by reproducing the statement of question 4 , of the 12th edition of OBMEP's test -2016 - Level 2.
(OBMEP - 2016) Question $4^{9}$ In the figure, the letters $A$ and $B$ represent the possible digits that make the product of numbers 2A5 and 13B a multiple of 36 .

Figure 1 - Illustration of question 4


Source: 2nd phase of the 12 th edition of OBMEP.
a) In all possible results for the product of these numbers, the number of the units is the same. What is that number?
b) What are the possible values of $B$ ?
c) What is the highest possible value for this product?

As question 4 of the test has three items ( $a, b$ and $c$ ), from the sample of 620 tests analyzed, a total of $3 \times 620=1860$ resolutions results, between right, wrong and blank answers. Table 2 shows the frequency distribution for the 1860 correct,

[^4]wrong and blank answers to question 4 of the 2 nd phase of the 12th edition of OBMEP-Level 2, held in 2016.

Table 2: Frequencies of correct, wrong and blank answers, per item, of question 4 of the 2nd phase of the 12th edition of OBMEP-level 2, 2016.

| Answers | Item $\mathbf{a})$ |  | Item $\mathbf{b})$ |  | Item $\mathbf{c})$ |  | Total |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | fi | $\mathrm{fr}(\%)$ | fi | $\mathrm{fr}(\%)$ | fi | $\mathrm{fr}(\%)$ | fi | $\mathrm{fr}(\%)$ |
| Correct | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 | 0 | 0.00 |
|  | 123 | 19.84 | 99 | 15.97 | 115 | 18.55 | 337 | 18.12 |
| Wrong | 497 | 80.16 | 521 | 84.03 | 505 | 81.45 | 1523 | 81.88 |
| Total | $\mathbf{6 2 0}$ | $\mathbf{1 0 0 . 0 0}$ | $\mathbf{6 2 0}$ | $\mathbf{1 0 0 . 0 0}$ | $\mathbf{6 2 0}$ | $\mathbf{1 0 0 . 0 0}$ | $\mathbf{1 8 6 0}$ | $\mathbf{1 0 0 . 0 0}$ |

Source: The authors (2018).
From Table 2, we establish the interval estimates for a 95\% confidence level and a $3 \%$ margin of error. Figure 2 shows the constructed confidence intervals.

Figure 2 - Confidence intervals for the estimates of correct, wrong and blank answers for question 4 of the 2nd phase of the 12th edition of OBMEP-Level 2, 2016.


Source: The authors (2018).
From the analysis of Table 2 and Figure 2, we found that there was no correct answer to question 4 of the test in any item. 18.12\% of the total answers were left blank (error margin of $\pm 3 \%$ ). And, finally, a population estimate of $80.84 \%$ for the wrong answers, with a margin of error of three percentage points more and less. Each item ( $a, b$ and c), in turn, also maintained estimates close to $80 \%$ for the population proportion. The discussions are, therefore, generated on the basis of the 1523 wrong answers $(81.88 \% \pm 3 \%)$, which are the object of our analysis in this study.

For that, we conducted the discussions following a pre-established analysis script according to the following order: first, we presented the error classes identified in the attempts to resolve the question and the respective categories of answers. The error classes, more general and comprehensive, explain the nature of the error made, while the response categories highlight more specifically the actions taken by the student when trying to solve the question. Next, we listed the main skills needed to solve the issue. And, finally, we selected some examples of resolution attempts observed in the research analysis corpus and discussed the solution strategies used
in the light of the theory of error analysis. We sought to examine and understand the student's thinking when proposing a particular solution, discuss the emerging perspectives and produce knowledge from this process.

The sample of 620 tests was analyzed and the errors categorized according to their type, giving rise to what we call here error classes. Altogether, we identified four main classes, not necessarily disjoint from each other. Naturally, errors can manifest themselves on a comprehensive scale, enabling the same attempt at a solution to fall into two or more different classes of errors. In the drive to produce objective data that would help us to explain and describe the phenomenon studied, we sought to categorize each resolution into a single class according to the emphases perceived in the errors.

Table 3 explains the error classes and response categories for the 1523 resolutions analyzed.

Table 3: Error classes and response categories for the 1523 responses analyzed in the tests of the 12th edition of OBMEP- Level 2, 2016.

| Error classes | Answer categories | fi | fr(\%) |
| :---: | :---: | :---: | :---: |
| Error related to misinterpretation | Disagrees with the statement Answer without justification | $\begin{array}{\|c} 4 \\ 1029 \end{array}$ | $\begin{gathered} 0.26 \\ 67.56 \end{gathered}$ |
| Subtotal |  | 1033 | 67.83 |
| Error related to deficiency in basic concepts | Digit with more than one number <br> Try to operate with the unknowns The student assigns a value to $A$ and $B$ and repeats it <br> Answer depending on $A$ and $B$ Assign values, do the wrong multiplication | $\begin{gathered} 104 \\ 10 \\ 21 \\ 220 \\ 8 \end{gathered}$ | $\begin{gathered} 6.83 \\ 0.66 \\ 1.38 \\ 14.45 \\ 0.53 \end{gathered}$ |
| Subtotal |  | 363 | 23.83 |
| Error related to ignorance of multiples and number sequences | Do not understand the concept of multiple <br> Assign values to $A$ and $B$, lose operations <br> Reached at least one possible value of $B$ List all digits | 34 <br> 47 <br> 12 <br> 14 | $\begin{aligned} & 2.23 \\ & 3.09 \\ & 0.79 \\ & 0.92 \end{aligned}$ |
| Subtotal |  | 107 | 7.03 |
| Error related to the application of wrong knowledge | Uses Roman numerals Take the highest number and make the product <br> 36 because it is his multiple | $\begin{gathered} 11 \\ 3 \\ 6 \end{gathered}$ | $\begin{aligned} & 0.72 \\ & 0.20 \\ & 0.39 \\ & \hline \end{aligned}$ |
| Subtotal |  | 20 | 1.31 |
|  | Total | 1523 | 100.00 |

Source: The authors (2018).

From the analysis of Table 3, we found that the most recurrent type of error is related to the incorrect interpretation of the question command, which corresponds to more than half of the analyzed solutions ( $67.83 \%$ ). This fact is noteworthy, because it implies that most students were not even able to achieve what requested the command of the question, which led them invariably to take two measures, disagree with the statement or simply submit an answer without justification. In the section of concepts and procedures involving numbers and operations, the PCN encourage
(...) interpretation (...) of problem situations, comprising different meanings of operations involving natural, integer and rational numbers, recognizing that different problem situations can be solved by a single operation and that eventually different operations can solve the same problem (BRASIL, 1998, p. 71).
The PCN also promote the "establishment of relationships between natural numbers, such as 'being a multiple of 'and 'being a divisor of'" (BRASIL, 1998, p. 71). We verified that $7.03 \%$ of the mistakes made were directly related to the lack of knowledge about the concepts of multiples and numerical sequences. The deficiency in the domain of basic concepts was present in $23.83 \%$ of the wrong answers, while $1.31 \%$ of the analyzed cases contained errors related to the application of mistaken knowledge.

In the sequence, we listed the skills necessary to solve question 4 of the test. These skills were assigned according to the solution disclosed by the OBMEP organization and are explained below:
a) identify the number of natural number units;
b) know when the number is a multiple of 5 ;
c) know when a number is a multiple of 36 ;
d) know the criterion of divisibility by 4 ;
e) know the criterion of divisibility by 3 ;
f) know the criterion of divisibility by 9 ;
g) know how to factor a natural number;
h) have mastery of elementary operations involving natural numbers.

With the skills listed, we took some examples of solutions from the corpus of analysis of the research in order to evaluate and discuss them. We sought to examine in detail the steps presented in each solution in order to understand the mechanisms and strategies of resolution employed by the students.

Figure 3 - First example.
a) Em todos os possiveis resultados para o produto desses números, o algarismo das unidades \& o mesmo. Qual e esse
algarismo?
Oalgoviramo ie nuimero durzais númorés io mukns. por usso

- aigarivano desse nuimmo ч 5 .

Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.
The example shown in Figure 3 is classified as an error related to misinterpretation. In the answer presented, the student does not seem to understand the statement. The question command indicates that the number of units obtained from the product between the presented values is the same, however the student seems to understand that the number of units of the two values to be multiplied (2A5 and 13B) are the same, stating that the answer is 5 , because 5 is the number of the 2A5 units.

In Figure 4 below, we present an example of an error related to deficiency in basic concepts.

Figure 4 - Second example.


Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.
In this resolution, it is clear that the strategy employed was to use the multiplication algorithm. It is possible that this choice was encouraged by the image that accompanies the statement. The student chooses to do the multiplication with the letters (A and B), which created difficulties, since he used properties of addition and multiplication (distributive and associative) to justify the steps, however these are not valid in the way that were taken in the algorithm (treating the figures as terms of an equation). Thus, as a result of the product, an expression given in terms of $A$ and $B$ is obtained, which is presented as an (incorrect) answer. When suggesting a solution based on the multiplication algorithm, instead of conjecturing possible values
for $A$ and $B$ that could satisfy the conditions of the problem from the use of divisibility criteria, it is evident the search for a more algebraic solution, which can satisfy all imposed conditions. Naturally, the arguments adopted are fragile insofar as the used multiplication algorithm is not adequate in the presence of unknowns, replacing the digits.

Next, we present in Figure 5 an example of an error related to unfamiliarity of multiples and numeric sequences.

Figure 5 - Third example.


Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.
In this example, we can notice that, by answering " 2 because it is a multiple of 36 ", the student demonstrates both the lack of understanding about the concept of multiple of a number, and also seems to make a confusion between the concepts of multiple and divisor.

Figure 6 - Fourth example.

```
b) Quais săo os possiveis valores de B?
    lls porsiveis volor be B povile sen oquadquet
    numere suer cidar em sus calsetespoder
    ser 2U ou 1 ou stc.
```

Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.
In the exposed solution, the student states that the number $B$ can be any number. This demonstrates a lack of understanding of the problem, once the statement literally establishes that figures $A$ and $B$ make the product of the two numbers to be a multiple of 36 . So we conclude that $B$ cannot be any number, that is, it must be taken into account that for the product of $2 A 5$ and 13B to be a multiple of 36 , the first condition is that this product is even. As the number of units for one of the factors is 5 , the number of units for the other factor $(B)$ must be even. Therefore, an error is established by misinterpretation.

Figure 7 - Fifth example.


Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.

In this other solution, it is seen that the student assigns any value to $A$, and simply discards B. Starting with an attempt to use the multiplication algorithm; he misses the product between 235 and 13. In view of the above, the first error established was to assign any arbitrary value to $A$, then discard $B$ as it had no function or importance in the problem, and, finally, perform the multiplication incorrectly. Often, the student, when he does not understand the statement correctly and does not reach the complexity of the problem, then promotes arbitrary simplifications, eliminating elements of the problem, or deliberately assuming hypotheses that do not directly match the statement.

In the case explored in the previous example, the student chooses to discard the unknown $B$, because working with two unknowns seemed too complicated. Having to deal with the second unknown, A, it seemed plausible to simply replace it with an arbitrary value (three) and try to carry out the multiplication.

Figure 8 - Sixth example.


Source: Tests solved, 12th edition of OBMEP - Municipalities of Western Pará.
In the answer presented here, the student states that the highest possible value for the product is the letter y , justifying that y is the maximum value, since it can be the result of "a high product". The argument indicates that the student may have mistakenly related concepts of quadratic function to the fact that the value is maximum, referring to the vertex ordinate $\left(\left(Y_{v}\right)\right.$. Therefore, this error is attributed to the application of mistaken knowledge.

In problems involving the maximum - or minimum, depending on the case - of quadratic functions, it is common to obtain the vertex ordinate directly as an answer. The idea that any problem that asks for the maximum amount, whatever the context, is associated with obtaining this order, consists of a gross simplification of the diversity and complexity of cases, the product of automatisms commonly encouraged in traditional education. So, in this perspective, when asked to give the maximum
value, the student immediately returns the idea of $Y_{w,}$, to the detriment of the critical and systematic analysis of each case in the search for the appropriate solution.

All of these examples keep among them a similarity, the premise that the teacher can and should learn from the students' mistakes. The strategies adopted reveal sometimes the absence of concepts necessary for resolutions, sometimes the mobilization of existing concepts, but poorly constructed in the student's intellectual universe. The analysis of these productions allows us to enter, although with narrow and well-defined limits, on the student's cognitive level and seeking to understand their resolution strategies, the hypotheses built, the way they correlate and intercalate concepts, the mobilization of basic concepts, in short, their modus operandi.

## Final considerations

This study aimed to analyze errors made in the 12th edition of OBMEP's test by students from public schools in the western region of Pará who were in the 8th and 9th grades of elementary school, from the perspective of the error analysis methodology. The analyzes took place over question four of the test, which involves Arithmetic content.

The main results point to the fact that, throughout the researched region, there were no hits recorded on this issue. The blank answers corresponded to $18.12 \%$ (with a margin of error of $\pm 3 \%$ ) of the total, while the wrong answers, here the object of our attention and analysis, corresponded to $81.88 \%$ (with a margin of error of $3 \%$ ).

The main mistakes made are related to misinterpretation (67.83\%), deficiency in basic concepts $(23.83 \%)$, ignorance of the concepts of multiples and sequences (7.03\%) and application of mistaken knowledge (1.31\%). Most students gave answers without justification (67.56\%).

From the analysis of the students' productions, some strategies emerged that demonstrated, among other things, the difficulty manifested in interpreting the commands of the items, the misapplication of concepts and procedures (such as the confusion between the concepts of multiple and divisor, the misapplication multiplication algorithm), besides the mobilization of malformed concepts.

To learn from the possible recurring errors in the student culture of Mathematics, the teacher does not necessarily need to be involved in research guided by the methodology of error analysis. Practice and experience in the classroom usually provide several of these elements, and the teacher attentive to
observing them will not be unaware of the difficulties commonly faced in the construction of the concepts taught.

Common examples observed daily in the routine of any classroom are errors such as confusion regarding the concepts of multiple and divisor, as we were able to ascertain in this study. There is also a very common doubt regarding the participation of zero as a multiple of all-natural numbers as a routine in the life of the mathematics teacher. The same idea can be extended to number one as a divisor of all-natural numbers. Other recurring examples include the dilemma of division by zero, or the confusion between the concepts of Minimum Common Multiple (M. M. C.) and Maximum Common Divisor (M. D. C.).

All of these are frequent manifestations, which, under the watchful eye of the teacher who perceives them, become care and emphasis when teaching. In general, the analysis of errors made by students can be taken as a tool of formative function, aiming at the positive restructuring of previous schemes, taking the idea away from the general imaginary that error is something execrable and punishable. Often, in fact, it can teach us as much or even more than the correct answers.

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[^1]:    ${ }^{4}$ Especially from Cury (1988), in her dissertation, in which the author approaches Error Analysis in demonstrations of plane geometry in the university context.

[^2]:    ${ }^{5}$ The OBMEP organization categorizes participants into three levels, according to their level of education, namely: Level 1-6th or 7th grade of elementary school; Level 2-8th or 9th grade of elementary school; and level 3 - High School.
    ${ }^{6}$ As Cury (2007) points out, in a survey of 40 Brazilian surveys on error analysis, the content of Arithmetic has been extensively explored in investigations involving the analysis of the production of elementary school students.

[^3]:    ${ }^{7}$ In contrast to bibliographic research, which is the one that uses already prepared materials, such as books, theses, articles, etc.
    ${ }^{8}$ When the true value of $p^{*}$ is unknown in the population, it is prudent to use the conservative guess $p^{*}=0,5$. The sample size ( $n_{0}$ ) does not change much when you change $p^{*}$, as long as $p^{*}$ is not too far from 0,5.

[^4]:    ${ }^{9}$ The solution to question 4 was released by the OBMEP organization on its website and is available at://www.obmep.org.br/provas.htm.

