REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

ISSN 2359-2842 Volume 14, número 34 – 2021 DOI: 10.46312/pem.v14i34.6362

Strategies and Conjectures Used by a Group of Elementary School Teachers in Exploratory-Investigative Algebra Activities

Estratégias e Conjecturas Usadas por um Grupo de Professores dos Anos Iniciais em Atividades Exploratório-Investigativas de Álgebra

Márcia Jussara Hepp Rehfeldt[1]

Ieda Maria Giongo[2]

Marli Teresinha Quartieri[3]

Sabrina Crisostomo da Silva[4]

Daniele Nervis[5]

Camila Bassegio Graff[6]

ABSTRACT

The present paper comes from the partial results of the research "Teaching-learning-evaluation in Mathematics in the Early Years of Elementary School: exploratory-investigative activities and teacher training", developed in a University located in the South of Brazil. Specifically, the analysis of the activity listed in this report comes from a continuing education of teachers, held in the city with which

[1] University of Vale do Taquari - Univates. PhD in Informatics in Education. E-mail: mrehfeld@univates.br. ORCID: https://orcid.org/0000-0002-0007-8639.
[2] University of Taquari Valley - Univates. PhD in Education. E-mail: igiongo@univates.br. ORCID: https://orcid.org/0000-0002-1696-0642.
[3] University of Vale do Taquari - Univates. PhD in Education E-mail: mtquartieri@univates.br. ORCID: https://orcid.org/0000-0002-9621-3830.
[4] University of Vale do Taquari - Univates. Bachelor in Languages. E-mail: sabrina.silva@univates.br. ORCID: https://orcid.org/0000-0003-4882-5776.
[5] University of Vale do Taquari. Undergraduate student in Physiotherapy. E-mail: daniel.nervis@universo.univates.br. ORCID: https://orcid.org/0000-0003-0316-5276.
[6] University of Vale do Taquari - Univates. Graduating in Pedagogy. E-mail: milagraff@gmail.com. ORCID: https://orcid.org/0000-0001-8256-0201.

the group of researchers established a partnership. Thus, the purpose of this study is to report the strategies used by teachers of the Early Years, when preparing the proposed tasks in the light of the Mathematical Investigation trend, covering the content of Algebra. The data collection is based on the transcripts of the recorders placed in the groups during the performance of the activity, as well as photos, images and notes from the researchers involved. The results indicate that the teachers expressed different conjectures for the proposed question, but the strategies had similarities among them.

KEYWORDS: Mathematical Investigation. Algebra. Continuing Education.

RESUMO

O presente trabalho é oriundo dos resultados parciais da pesquisa “Ensino-aprendizagem-avaliação em Matemática nos Anos Iniciais do Ensino Fundamental: atividades exploratório–investigativas e formação docente”, desenvolvida em uma Universidade localizada no Sul do Brasil. Especificamente, a análise da atividade elencada neste relato é procedente de uma formação continuada de professores, realizada na cidade com a qual o grupo de pesquisadores estabeleceu parceria. Assim, o objetivo deste estudo é relatar as estratégias utilizadas pelos docentes dos Anos Iniciais, ao elaborarem as tarefas propostas à luz da tendência Investigação Matemática, contemplando o conteúdo de Álgebra. A coleta de dados está embasada nas transcrições dos gravadores dispostos nos grupos durante a realização da atividade, além de fotos, imagens e anotações dos pesquisadores vinculados. Os resultados apontam que os docentes expressaram distintas conjecturas para a questão proposta, mas as estratégias tinham semelhanças entre si.

PALAVRAS-CHAVE: Investigação Matemática. Álgebra. Formação Continuada.

Introduction

Currently, some researches (CURI, 2004; SOUSA; SOBRINHO, 2010) prove that the knowledge obtained in the initial training of mathematics teachers is insufficient, because, during their academic career, the contents are focused on the curriculum grid established for that time of course, which usually takes four years (CAVALCANTE, 2011). Thus, after this training, it is necessary to reflect, together with other teachers, the new trends, practices and teachings that favor simultaneous learning, resulting from investigative activities.

With this in mind, a group of elementary school teachers sought, in partnership with a university, to offer continuing education to teachers of mathematics in the early years. Thus, the members of this group, linked to a research of the University of Vale do Taquari, with the financial support of the National Council for Scientific and Technological Development (CNPq), developed exploratory-investigative activities, focused on the trend of Mathematical Investigation, in the perspective of Ponte, Brocardo and Oliveira (2003), Trindade (2008), Fonseca, Brunheira and Ponte (1999).

For these authors, Research is associated with the idea of searching, questioning and seeking knowledge. In this context, it is up to the teacher to act as a mediator of content, seeking to provide students with autonomy and lead them to formulate their own conceptions. Based on this concept, the importance of the present article is justified, which seeks to explore the strategies and conjectures presented by teachers of the basic education network, in the second semester of 2017 and, from that, problematize the relevance of continuing education and the use of exploring new perspectives of work with Mathematics.

That said, the next section explores the vision of some theoretical contributions on Mathematical Investigation in conjunction with the approach of teaching Algebra in the early years.

Theoretical Referential

According to Trindade (2008), to investigate is to seek to know unexplored facts (explore unknown facts). Ponte, Brocardo and Oliveira (2003) complement by stating that Mathematical Investigation is a methodology that aims to work on the student's autonomy to solve questions, as well as the elaboration of hypotheses and conjectures for these problems.

In this context, problematizing implies investigating, helping and promoting the mediation of new concepts. Thus, for a moment, the tasks that usually have only one answer are ignored, opting for those that, according to Palhares (2004), have a more open character, i.e., make other answers possible.

Through these practices, we try to awaken the student's creativity so that he becomes the constructor of the knowledge acquired during the development of the tasks, and, in this way, they can be propellers in unleashing creativity. Allied to this, it is inferred that the main purpose of exploratory activities, according to Trindade (2008), is to seek to know facts, hitherto hidden. Fonseca, Brunheira and Ponte (1999, p. 4) point out that

> In a mathematical investigation, the goal is to explore all paths that emerge as interesting from a given situation. It is a divergent process. You know what the starting point is, but you don't know what the ending point will be.

Another factor that deserves to be highlighted is the importance that the path has before the result, because it is in the development of the task that the student elaborates resolutions and seeks to justify them. In this way, he develops critical thinking and becomes an investigator. As Trindade (2008, p. 154) states, "[...] the road is the goal and not the arrival". Thus, in several questions, one does not get a single answer, and sometimes it is incorrect or not original, but it is emphasized that the main goal is not the result, but the path taken to get to it. However, it is up to the teacher to mediate discussions so that the group of students finds the inconsistency in the answer provided (if any).

Moreover, when using open exploration tasks, the teacher is faced with possibilities to instigate and arouse the curiosity of students, since they are challenged to reflect and problematize issues that they deal with in everyday life and are seen as trivial. In fact, activities that contemplate this trend contribute to the development of autonomy and critical reflection in search of their own knowledge and answers to their questions.

> In investigative activities the student is encouraged to develop their autonomy, defining objectives and conducting the investigation formulating strategies, testing their conjectures, critically analyzing the results obtained. Hence comes the unpredictability character of this type of activity requires the teacher flexibility to deal with new situations that, with great probability, will arise (BANDEIRA; NEHRING, 2011, p. 3).

Finally, it makes it possible to innovate the subject of mathematics, because, as already emphasized, it values the knowledge of the students. Thus, the teacher builds mathematical knowledge together with the students through research and conjectures, which also unites them in the classroom environment. Ponte, Brocardo and Oliveira (2003, p. 47) state that

> The teacher has a determining role in research classes. [...] In monitoring the students' work, the teacher must try to strike a balance between two poles. On the one hand, to give them the autonomy that is necessary in order not to compromise their authorship of the investigation, and on the other hand, to ensure that the students' work flows and is meaningful from the point of view of the mathematics discipline.

With these theoretical foundations and imbued with the trend of Mathematical Investigation, the group of researchers sought a theme to perform the practices. To do so, the BNCC was analyzed, selecting Algebra. According to the document, the content of this part of Mathematics

> aims to develop a special type of thinking - algebraic thinking - which is essential to use mathematical models in understanding, representing and analyzing quantitative relationships of quantities and also mathematical situations and structures, making use of letters and other symbols (BRASIL, 2017, p. 268).

Also according to the document, "[...] it is essential that some dimensions of the work with algebra are present in the teaching and learning processes [...] such as the ideas of regularity, generalization of patterns and equality properties" (BRASIL, 2017, p. 268). Thus, it is thought that, for the development of Algebra in the final years of elementary school, students can already begin algebraic thinking in the early years. For Oliveira and Laudares (2015, p. 5),

> Algebraic thinking is favored when, from the early grades of elementary school, different forms of representation of mathematical ideas and relationships are valued, through various resources such as symbols, drawings, manipulative material, and grouping, classifying, and ordering activities that facilitate working with patterns.

Therefore, when working on the development of algebraic thinking, the student is stimulated to recognize patterns, generalize them, interpret them and solve several situations that may arise in their daily lives. Kieran (2004) states that algebraic thinking in the early years does not need the symbol-letter, because in these first moments of contact with this content, the student will be able to describe his thoughts orally. Like these theorists, it is thought that, in order for there to be changes in the education of a subject, reflections about the attitudinal practices of educators are necessary, aiming to make students critical and objective. To this end, it is believed that continuing education meets the aforementioned requirements.

Therefore, in these meetings, it is pertinent to exchange ideas and knowledge. For this to happen, the teacher must be open to new approaches and concepts that may have been mentioned in the initial training. Continuing education, as the name implies, is a continuation, an improvement of knowledge. Thus, the teacher will be able to understand the problems of his practice and of everyday life, aiming at a professional and practical improvement in his work context. But, as Demo (2007) states in his studies, it is necessary to invest in teacher quality in order to obtain an improvement in student learning. Thus, the school must provide moments of exchange of experiences between educators, courses, lectures, and continuing education. In this scenario, the practice occurred in a continuing education meeting, in which the content of Algebra was approached, with theoretical support from Mathematical Investigation, aiming at this part of Mathematics in the early years.

In the next section, the methodological form is presented, explaining how the study was developed.

Methodology

The activity listed in this research is based on the content of Algebra, using, for this purpose, graph paper - which can be considered an easily acquired material - for students and/or teachers. To accomplish this, the University researchers elaborated, together with some elementary school teachers, also involved in the research, proposals for exploratory tasks that contemplated fragments of Algebra, developing algebraic thinking in children. These activities were planned, written, tested and reformulated, through the use of several materials. Once the tasks were ready, the material was separated; and the meeting for the continuing education course was scheduled.

The teachers for whom the activities were prepared taught in Basic Education in a city next to the location of the Higher Education Institution. Thus, their participation in the continuing education course was voluntary. However, an incentive from the Secretary of Education of that town contributed to the workshop's effectiveness and the presence of practically all the teachers of the Mathematics area in the town.

The course took place over three nights, and other activities with different approaches were worked on. Some care was taken in all the meetings, mainly prioritizing group work. For Brunheira and Fonseca (1995, p. 4)

> Research activities provide a good opportunity for students to work together in groups. This makes it easier to combine ideas and overcome difficulties. The group also increases confidence in tackling new problems and promotes discussion among students.

It is understood, therefore, that group work becomes important for learning, because the subject (teacher in training / student in the classroom) socializes his conjectures in order to refine and justify them. Investigating with others provides an exchange of knowledge, new knowledge, since much is learned in human relationships.

For the exploration of the activity, the researchers oriented the teachers present on how the activity would be developed. Initially, the participants discussed, in the small groups they formed, their strategies and respective conjectures. Then they chose a thought to present to the larger group.

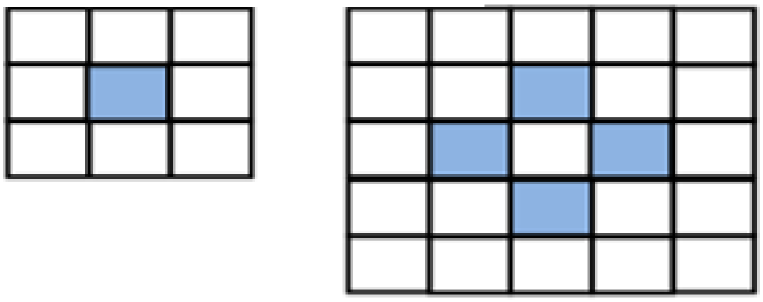
It should be noted that seventy elementary school teachers participated in this practice. To analyze the data collected during the discussions, the meeting was recorded and later transcribed. Allied to this, a photographic recording was made so that, after reading the enunciations, the research group could better understand the conjectures elaborated. It is worth mentioning that, at the end of the meeting, the material delivered, contemplating the developed activities, was collected for checking. Thus, data collection consisted of written activities, photos, recordings, and transcriptions of the recordings.

In this context, the activity called "Sequence on graph paper" aimed to evaluate the teachers' conjectures, thoughts, and doubts that arose as they performed the proposed task. The activity is fully described in Figure 1.

Figure 1 - Activity "sequence on graph paper”

Observe the sequence of the figures below:

Figure 1 Figure 2



Use the squares paper to represent this figures

Number of the figure	Total of squares	Number of colored squares	Number of uncolored squares
Figure 1	9	1	8
Figure 2	23	4	21
Figure 3			
Figure 4			
...			

Source: The authors (2018)

The activity followed the following steps: a) through the first two figures presented, teachers should observe them and follow a sequential logic in order to build a third one with the same pattern; b) according to the Figure drawn in the previous step, teachers would count how many squares were drawn, painted and not painted; c) finally, they would complete the table that appears in the activity (Figure 1), relating the total of drawn, painted and not painted squares.

To finish the task, the researchers proposed to solve, together, a mathematical generalization, because, as Ponte, Brocardo and Oliveira (2003) say, in the discussion and argumentation about their conjectures and refutations, the student is called to act as a mathematician. Given this context, it makes his role active in learning knowledge.

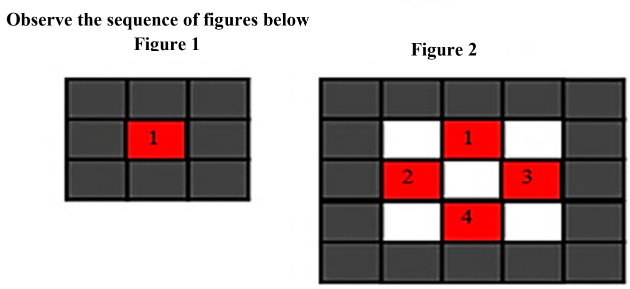
Results of the Pedagogical Intervention

In this section, we report the strategies used by teachers of the early years, during the development of the activity, with graph paper. For Ponte, Brocardo and Oliveira (2003), one of the purposes of Mathematical Investigation is to enable the development of hypotheses and conjectures for the problems presented. Thus, when the activity was given to each group, one of the researchers accompanied the discussions, aiming at the emergence of different conjectures for the problem in question. All discussions were recorded, and some strategies used by the teachers are expressed below. It is worth mentioning that, according to Palhares (2004), the questions had a more open character, allowing more answers to be given. One of the dialogues that took place is presented below, followed by an interpretation of this dialogue:

> PG:[7] In Figure one, you had a little square painted in the center of the Figure. In Figure two, what happened?
>
> D5: There was always a little square left over at the end of the figure, so we had to maintain the pattern.
>
> [...]
>
> PG: How was this thought? In the first picture, we painted this one [referring to the little square painted like in the initial picture], this one, this one, and this one [these are the squares painted on the lower left and right diagonal of the initial square, and also below the initial square, interspersed with a unit of graph paper].
>
> D5: To keep the visual pattern, there had to be a little square left on each side.
>
> PG: Back there, did you get that drawing? [researcher asking another group of teachers].
>
> D5: Yes!
>
> PG: But the strategy was different!
>
> D5: We came up with this: total squares drawn is the number of the picture times two, plus one squared; number of squares painted is the number of the picture squared, and number of squares not painted is the first formula minus the second.
>
> PG: But your drawing looked like this?
>
> D5: Yes, we thought it looked like this [the same drawing as the other group].

The first strategy analyzed emphasizes a visual pattern sequence in which the number of squares around the figure is not painted. For a better understanding, here follows Figure 2 with a schematization elaborated by the authors of the report.

Figure 2 - Authors' explanation of Group 5's thinking



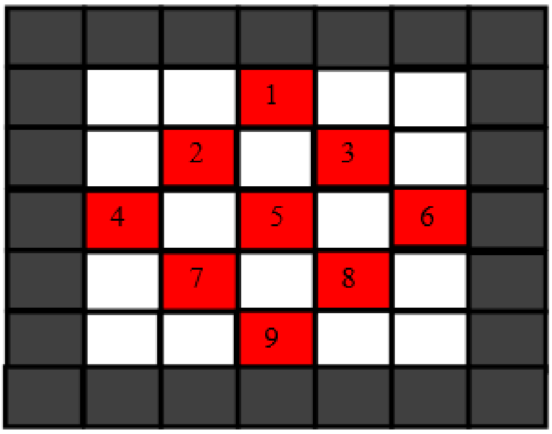
[7] In order not to expose any teachers, the researchers who proposed the practice are referred to as PG; and the teachers participating in the training are described as D.

Source: The authors (2018)

The groups thought that in the first Figure, there was only one painted square [called 1 in Figure 1]; in the second, they increased three painted squares [squares numbered 2, 3 and 4]. Thus, little square 2 and little square 3 are on the diagonals of number 1, on the left and on the right, respectively. For the number 4, there is a blank square between the 1 and this number [4]. It is noteworthy that, for them, there was always a blank line at the ends. For this explanation, the squares were painted black.

The third figure illustrates the schematization prepared by the authors.

Figure 3 - Illustration of Group 5's strategy for the third Figure

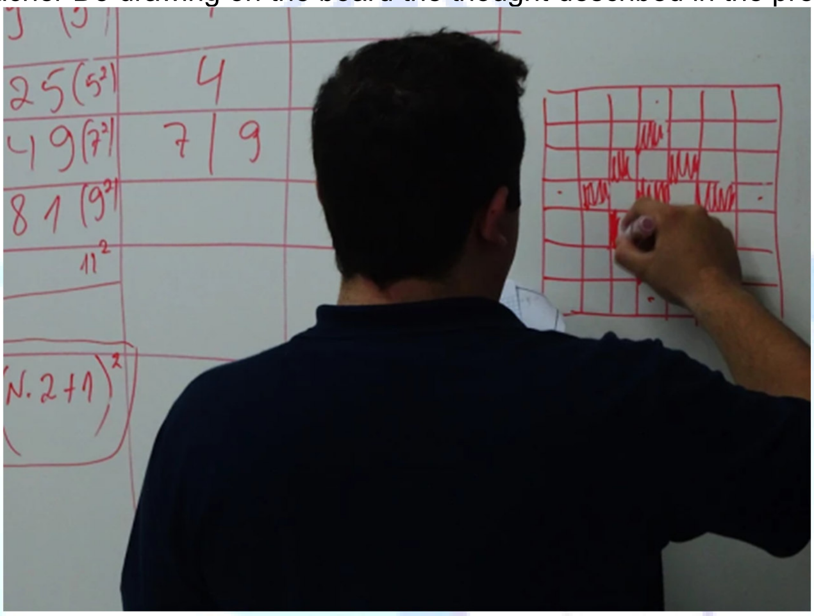


Source: The authors (2018)

Following the group's reasoning logic, squares 4 and 5 are diagonal to square number 2. Squares 5 and 6 are diagonal to number 3. It should be noted that 5 is common to both [squares 2 and 3]. To find 7, the group left one blank and painted the one below it. In the same way, they found number 8. In square 9, they left a blank between it and 5, and then painted it. According to Trindade (2008), investigative tasks help in the search for facts (in this case patterns of drawings) previously hidden or unknown to the trainee teachers.

This thought, like the others, was explained by an individual from each group on the board, exposing his thought to the training participants. According to Ponte, Brocardo and Oliveira (2003), discussion is indispensable to Mathematical Investigation because, on the one hand, students gain a "[...] richer understanding of what it means to investigate and, on the other hand, they develop the ability to communicate mathematically and to reflect on their work and their power of argumentation" (PONTE; BROCARDO; OLIVEIRA, 2003, p. 41). Thus, "[...] it is fundamental to allow students to interact with each other, learning to discuss and argue in defense of their opinions" (TUDELLA et al., 1999, p. 95). Therefore, it is confirmed that without discussion and argumentation, it is as if an investigation is lost; and no content, abstracted. Thus, it is necessary that the teacher be a mediator of this situation, leading to constructive reflection. Figure 4 represents D5, exposing the Group's thinking, including to the other teachers. According to Fonseca, Brunheira, and Ponte (1999), in mathematical investigation all the paths that arise from a situation should be explored, and this is how the participating teacher highlighted his conjecture.

Figure 4 - Teacher D5 drawing on the board the thought described in the previous excerpt



Source: Photographic collection of the researchers (2018)

According to D5, "to keep the visual pattern, there had to be one little square left on each side." For the conjecture of the total number of squares per figure, the explanation follows:

For Figure 1: $3^2 = 9$ squares;

For figure 2: $5^2 = 25$ squares;

For figure 3: $7^2 = 49$ squares.

In this way, Figure 5 illustrates the completed table, requested in the activity.

Figure 5 - Completed chart according to Group 5

Number of the figure	Total of squares	Number of colored squares	Number of uncolored squares
Figure 1	9	1	8
Figure 2	23	4	21
Figure 3	49	9	40
Figure 4	81	16	65
...			

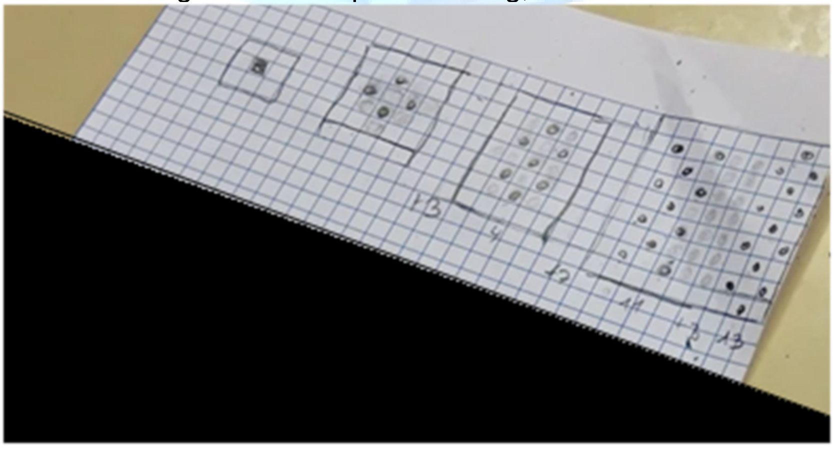
Source: Photographic collection of the researchers (2018)

To find the total number of squares, the group listed the images that had already been printed on the sheet. Its members discovered that in Figure 1 there was a 3x3 square; in Figure 2, a 5x5 square, always using odd numbers. So, to identify the first figures described on the board, they always added two, 5+2 = 7; so, the next figure would be a 7x7 square. But to find any infinite Figure, they needed a formula. In this sense, they found the following expression $(2.n + 1)^2$, where n is the number of the Figure; two is the first even number; one is the first odd number squared because it is a square - equal sides and angles. It should be noted that the expression found is already an algebraic expression, which makes use of letters, which means a process following algebraic thinking, in which the student uses resources such as symbols, drawings and manipulative material (OLIVEIRA and LAUDARES, 2015).

To find out the numbers of unpainted squares, the teachers initially found the number of painted squares and subtracted that value from the total number of squares. Thus, their formula became NT (total number of squares) - NP (number of painted squares). It should be noted that these two formulas were used by all participants, regardless of the strategy used to identify the painted squares.

Other groups also thought of always leaving "blank squares on each side", but on the inside of the figure. Thus, they did not follow a pattern like the previous group and illustrated in Figure 2. In agreement with the "style" of Investigation, Ponte, Brocardo and Oliveira (2003, p. 23), allude that "[...] since the starting points [of an investigation] may not be exactly the same, the ending points may also be different. This is because several answers can emerge from a single task. In the case of this question, two distinct conjectures emerged, expressed in Figure 6, illustrating Group 4's thinking.

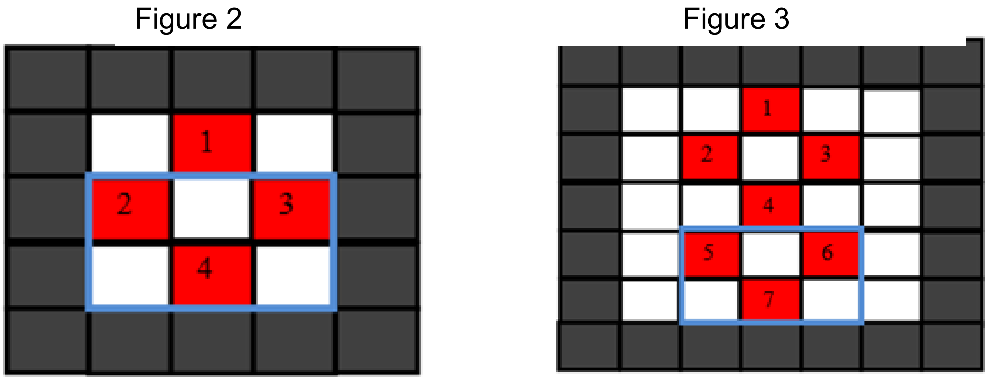
Figure 6 - Group 4's thinking, similar to 5



Source: Photographic collection of the researchers (2018)

As the group thought differently, a new Figure [Figure 7] follows with the explanation and schematization.

Figure 7 - Montage made by the authors for explanation



Source: The authors (2018)

For this group, the little square number 1 is the initial one and remains in all the following pictures. In the second, they add three painted squares that respect the contour line that is always left free [black squares]. Thus, squares 2, 3 and 4 are added below the initial one. The squares 2 and 3 are diagonal to the number 3, and the 4 is below the 1, interspersed by a white square. Then, for the third figure, they added three more below the original figure [rectangle with blue outline is always the new part in relation to the previous one]. This group completed the picture [Figure 8] as follows:

Figure 8 - Chart completed by group 4

Number of the figure	Total of squares	Number of colored squares	Number of uncolored squares
Figure 1	9	1	8
Figure 2	23	4	21
Figure 3	49	7	42
Figure 4	81	10	71
...			

Source: Photographic collection of the researchers (2018)

Instigated and questioned, the teachers arrived at the generalization, described by them as 3n - 2 or (n-1).3 + 1, finding once again an algebraic expression. Thus, they considered n to be the number of the Figure; the three, the number of squares that increases with respect to the previous one; the one, the number of squares painted in the first Figure, and the two results from subtracting the number of squares painted that increases (3), minus the number of squares painted in the first Figure (1). Thus, two different conjectures emerged, but they solved the same question. To understand the differences between the two cited thoughts, a table [Table 1] was prepared, which follows below:

Chart 1 - Differences between the Groups

	Group 5	Group 4
Concept for painting little squares	The integrant thought that from any given square two diagonals had to start.	In the Figure, only three increased, following the same parameter of the second Figure, adding two little squares painted diagonally from one that was in the center of the Figure, and another little square below the central one, interposed by a little blank square.
Formula for colored squares	N²	3n -2 or (n-1).3 + 1

Source: The authors (2018)

Thus, it is considered that the thoughts were thought-provoking and distinct to the extent that the teachers had to argue, expose and explain to other colleagues what they thought. This led them to reflect and, also, to understand the enunciations of others when describing their thoughts, making them understand the practice of Mathematical Investigation.

In addition, we should point out and corroborate with Fonseca, Brunheira and Ponte (1999) that in this trend, as illustrated by the results reported above, the paths chosen by the trainee teachers were different when making generalizations. Moreover, it was privileged the road they took when thinking about their solutions, as Trindade (2008) says, and not only the arrival. In this sense, the dialogues were inserted in order to illustrate the discussions that took place.

Still, when entering the classroom, these teachers need to know how to deal with unpredictability and flexibility (BANDEIRA; NEHRING, 2011) and having experienced this in a continuing education course can be productive.

On the other hand, difficulties were present. The first was due to the little time the group had to explore the strategies, which prevented them from reformulating them and finding other generalizations. In this sense, it was discussed the importance of more time for the research of the strategies and the conversation with the group of teachers in training, which can contribute to the search for new meanings and concepts.

As Ponte, Brocardo and Oliveira (2003) comment, the teacher needs to encourage the student's autonomy and this demands time, but it is necessary that the result to be obtained is satisfactory from the mathematical point of view. Finding this balance between the two poles is another challenge (PONTE; BROCARDO; OLIVEIRA, 2003).

Finally, it is worth mentioning that doubts and procedures were solved throughout the meetings, leaving teachers safe to insert this practice in their classroom, because according to Oliveira and Laudares (2015), inserting algebraic thinking in the early years is relevant, because it helps in the teaching of algebra in future years.

Conclusions

The research explored in this article sought to emphasize the importance of new methodologies for teaching mathematics. Through investigative and exploratory practices, teachers and students were able to reinvent themselves jointly through the discoveries obtained to justify an answer to the task. The students were able to realize that, in this type of task, there is not always one single answer, contributing to the meaning that the students superimpose on this subject. During the workshops, the moments of interaction between the participating teachers were perceptible, which makes it possible to bet that the same procedures will be carried out in the classroom with their students.

Regarding the strategies used to solve the activities, many of them showed similarities with each other. The paths that each one followed to solve the problem went beyond the ones that the researchers had found in their tests and refinements during the meetings.

It should also be noted that two conjectures were presented in the meeting. One group arrived at the generalization "3n -2 or (n-1).3 + 1"; another found "(2.n + 1) ²" as its final formula. For both, n is the number of the Figure; one refers to the squares painted in the first Figure; two results from subtracting the number three minus the number of squares painted in the first Figure; squared, because it was a geometric shape with equal sides (square). Also, for the first exposed formula, the three is the number of squares that increases from the previous Figure.

In effect, it is necessary to adapt the objectives to each class, visualizing the specifics, that is, in some, it is not necessary to ask for the final generalizations, which use advanced algebra. In others, going beyond this can challenge the students and make them critical and reflective.

The teachers who participated in the continuing education program made an effort to finish the activity and showed interest in reaching the proposed goals: investigating, testing the hypotheses, and reaching the mathematical generalization. In view of this, it is concluded that continuing education meetings are of utmost importance to favor the enrichment of knowledge about ways to teach and learn mathematics.

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Received: August 2018.

Approved: October 2020.