REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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Didactic Video: some perceptions of supervised internship between academic and school mathematics

O Vídeo Didático: algumas percepções da prática docente inseridas entre a matemática acadêmica e a matemática escolar

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ABSTRACT

We present in this article the analysis of one of the videos produced during a research that had as goal to investigate how Mathematics is presented in the videos formulated by Mathematics students taking the subjects in Supervised Internship of a distance online Mathematics degree with emphasis in education. As methodological procedures we adopted participant observation in the virtual learning environment Moodle, natural environment of the subjects; we interviewed the teachers, responsible for the subjects investigated, and the coordinator of the course; we applied two questionnaires, one for the students of the subjects investigated and the other for the teachers working in the distance online pre service teacher education program; we used information from the pedagogical project of the course as data and analyzed the videos produced by the students. For the organization and analysis of the data produced we use the Grounded Theory. From this analysis we infer that there is a relation between Academic and School Mathematics and that students use videos as didactic and pedagogical resources.

KEYWORDS: Distance Education, Use and Production of Videos, Mathematics Education, Didactic Videos.

RESUMO

Apresentamos neste artigo a análise de um dos vídeos produzidos durante uma pesquisa que tinha como objetivo investigar como a Matemática é apresentada nos vídeos elaborados por licenciandos, discentes das disciplinas de Estágio Supervisionado, de um curso de Graduação em Matemática na

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modalidade a distância. Como procedimentos metodológicos adotamos a observação participante no ambiente virtual de aprendizagem Moodle, ambiente natural das disciplinas; entrevistamos as professoras responsáveis pelas disciplinas investigadas e o coordenador do curso; aplicamos dois questionários, um para os cursantes das disciplinas investigadas e outro para os professores que trabalham na modalidade a distância; utilizamos as informações do projeto pedagógico do curso como dados e analisamos os vídeos produzidos pelos licenciandos. Para organização e análise dos dados produzidos utilizamos a Teoria Fundamentada nos Dados. Dessa análise inferimos que existe uma relação entre a Matemática Acadêmica e a Matemática Escolar e que os estudantes utilizam vídeos como recursos didáticos e pedagógicos.
PALAVRAS-CHAVE: Educação a Distância. Uso e Produção de Vídeos. Educação Matemática. Vídeos Didáticos.

Introduction

In Brazil, undergraduate courses were, theoretically, remodeled from the version known as “3 + 1” - in which, in the first three years, the specific subjects of the course were worked on and the last year was destined to work on a set of didactic-pedagogical techniques for the teaching of Mathematics in Basic Education - for a course in which the pedagogical subjects are worked on since the undergraduate students joined the course. In this sense, an attempt was made to deepen the formation of the teacher as an educator, since “from the 1970s onwards, in the midst of an intense discussion on the social and political role of education, structural changes began to take shape in education courses with emphasis in education” (MOREIRA; DAVID, 2007, p. 13).

According to these authors, gradual changes are beginning to be noticed in undergraduate courses, in which the teaching of techniques does not represent the locus of pedagogical training, including subjects such as Sociology of Education, Educational Policy and others, from which the graduate starts to be recognized as a teacher (of Mathematics, for example). They point out that the current model came into force in the 1980s with the inclusion of integrating subjects. However, they highlight questions about the extent to which the “3 + 1” model was broken and how the relations between theory and practice are distributed in the specific subjects of undergraduate courses, particularly in undergraduate courses in Mathematics. Moreira and David (2007, p. 14) emphasize that, in the specific case of undergraduate courses in Mathematics,

> [...] since the 1990s, several works have been developed on these courses, including dissertations and theses [which] are rarely focused in a specific way on the relationships between the mathematical knowledge conveyed in the training process and the mathematical knowledge associated with the school teaching practice.

Pimenta and Lima (2012, p. 34) point out that specific knowledge represents the “disciplinary knowledge in training courses, which in general are completely disconnected from the professional field of future trainees", being necessary, as a transforming element, the theory to be developed from investigations with the purpose of presenting reflections in relation to school activities, on teaching practices and in relation to the procedures adopted by students. Such knowledge is established by Tardif (2010, p. 38) as corresponding "to the speeches, objectives, contents and methods from which the school institution categorizes and presents the social knowledge defined and selected as models of erudite and educational culture for erudite culture".

The supervised internship[3] can represent the moment in teaching education when it becomes possible to carry out research procedures that are aimed at observing the teaching practice (Mathematics) of the future teacher, as well as reflecting on theory and practice that enhance the knowledge of the undergraduate. In order to bring the university closer to Basic Education, and to observe, in the sense of investigating, nuances of this practice as well as the knowledge of the future Mathematics teacher, we opted to choose the scenario of investigating subjects in the Supervised Internship of a distance online Mathematics degree with emphasis in education.

Thus, we seek in this article to present an excerpt of the analysis of the data produced, that is, we present and discuss the analysis of one of the twenty-three videos that were produced in the research that had the Supervised Internship I, II and III of the Mathematics Degree with emphasis in teaching in the distance mode at the Federal University of Alagoas (UFAL), in which, as a methodological procedure, participant observation was carried out in the virtual learning environment (Ambiente Virtual de Aprendizagem - AVA), in which the subjects are taught, as well as we interviewed the teachers responsible for these subjects and the course coordinator. In addition to applying a questionnaire to teachers responsible for the subjects operating in the distance modality and another questionnaire to their students, subjects of the research, we also use the information present in the Pedagogical Course Project (PPC) and the analysis of the videos produced by the students as the final activity of the subjects.

[3] In some moments, we present Supervised Internship with Initial Capitals, when referring to the discipline offered in the Mathematics Course with emphasis in teaching, and in lower case when it represents the set of activities developed by the students as real situations of practices and theories as professional contribution.

We investigate the mathematical content that was presented in the videos produced by the undergraduates, which represent one of the objectives of the research. To carry out the investigation, interpretation procedures were used that value the subjects' speech and behavior and were in line with the research objectives, generating more descriptive characteristics (BORBA; MALHEIROS; AMARAL, 2013), in which it was possible to assign meanings (BOGDAN; BICKLEN, 1994) to the researcher's concerns (BICUDO, 1993), pointing out the phenomena that emerged from the data. Such subjective characteristics led us to use the qualitative research procedures for the investigation.

As supporting elements for the research, we use some systematic models for the use of videos in the Basic Education classroom (MORAN, 1995; FERRÉS, 1996; TUCKER, 2013; BRAME, 2015), as well as theoretical resources aimed at analyzing the mathematical teaching knowledge, presented by the students in the videos (MOREIRA; DAVID, 2007; TARDIF, 2010). And, in this work, we opted to choose one of the videos that makes it possible to analyze the subject knowledge and teaching training of students in mathematics. This article, as already highlighted, emanates from a doctoral research in which the organization of the data produced used the Grounded Theory, especially the one presented by Strauss and Corbin (2008), as well as analytical models of videos that contributed in organizing and viewing videos (POWELL; FRANCISCO; MAHER, 2004; GINO; MILL; NAGEN, 2013).

Theoretical resources and models for analyzing the videos produced

The way in which undergraduate students teach and learn Mathematics can be conditioned to the knowledge accumulated in previous, curricular and disciplinary experiences linked to the teaching knowledge (TARDIF, 2010), and also to the technologies used during the production of knowledge (BORBA; VILLARREAL, 2005). In this section we present some theoretical developments used in the research that contributed to the analysis of one of the videos, which will be presented later in this article, such as those broadcasted by Moreira and David (2007), when they present significant differences between the Mathematics that is developed in school of Basic Education and Mathematics that the student sees in their undergraduate courses.

The scholar and the academic: different views of mathematical knowledge

In this work we will refer to Mathematics Teachers, to both the ones who work in teaching in Basic Education and the ones who develop their teaching in Higher

Education in undergraduate courses in Mathematics. With regard to the Basic Education teacher, we understand, like Moreira and David (2007), that he needs to work with School Mathematics, which represents a set of significant knowledge that are associated with the development of basic school education. For teachers of undergraduate courses, as these authors claim, it is more common than the

> [...] types of objects to work with, the levels of abstraction in which the questions are posed and the permanent search for maximum generality in the results make the emphasis on abstract structures, the rigorously logical co-deductive process and the extreme precision of language are, among others, essential values associated with the view that the professional mathematician builds on mathematical knowledge (MOREIRA; DAVID, 2007, p. 21).

Put in a primarily different way, the Mathematics Teacher of Basic School presents a more educational view, with a more descriptive interpretation, in which he seeks to present alternative and accessible forms to students in each of the school stages “for demonstrations, arguments or presentation of concepts and results, deep reflection on the origins of students' mistakes, etc.” (MOREIRA; DAVID, 2007, p. 21). These authors also emphasize that the definitions and statements represent important elements in the process of differentiating School Mathematics and Academic Mathematics. As Academic Mathematics is axiomatically structured in a set of postulates and primitive concepts, which support established definitions and theorems, it requires a profoundly precise formulation, since a mistake in relation to any mathematical object would imply an incompatibility in mathematical theory. Moreira and David (2007, p. 23) also emphasize that

> formal definitions and rigorous statements are important elements both during the process of shaping the theory - at times when the community assesses and eventually accepts a new result, thus ensuring its incorporation into the group of those already accepted as valid - and in the process of systematized presentation of the theory already elaborated.

According to these authors, there is a counterpoint between Academic Mathematics and School Mathematics. School Mathematics is not positioned in the direction of presenting validation of results that are already configured as characteristics of Academic Mathematics, based on the use of definitions and tests based on an axiomatic structure. In contrast, School Mathematics is guaranteed to be valid by Academic Mathematics itself. As an example, Moreira and David (2007, p. 23) highlight that in Academic Mathematics the uncertainty as to whether the “number **χ (P)** - characteristic of Euler-Poincaré - was equal to two for every convex polyhedron remained until it appeared a demonstration considered correct of Euler's theorem". In the case of School Mathematics, this type of concern is not discussed, as well as whether the product of two natural numbers is commutative.

> The problem that arises in school education is not to demonstrate a fact like this rigorously, from precise definitions and results already established, as in the scientific axiomatic process. The fundamental question for School Mathematics [...] refers to learning, therefore, to the development of a pedagogical practice aimed at understanding the fact, the construction of justifications that allow the student to use it in a coherent and convenient way in their school and extra-school life (MOREIRA; DAVID, 2007, p. 23).

That is, in Mathematics Taught at School the mathematical rigor of Academic Mathematics is not the only way to verify and validate mathematical results. Less formal explanations, without the deductive and axiomatic procedures of Academic Mathematics, can be accepted by presenting an understanding of Mathematics with convincing arguments that would not be accepted in Academic Mathematics. Among the production of mathematical knowledge, associated with School Mathematics and Academic Mathematics, we highlight the teaching knowledge that is associated with the possibilities of producing knowledge of this mathematical knowledge, for example.

Teaching mathematical knowledge

For Tardif (2010), teaching knowledge can be associated with the various aspects of being a teacher. This knowledge can be associated with professional knowledge, understood as that transmitted by institutions that have the prerogative to train teachers, such as universities or colleges of education. Such knowledges are "plural, composite, heterogeneous, because they bring to light, in the very exercise of work, knowledges and manifestations of know-how and know-how being quite diverse and coming from varied sources, which we can also assume to be of a different nature" (TARDIF, 2010, p. 61).

For this author, such knowledges are also curricular, which are directed to the "speeches, objectives, contents and methods from which the school institution categorizes and presents the social knowledges defined and selected as models of erudite culture and training for the erudite culture" (TARDIF, 2010, p. 38). Such knowledges are conducted from school programs; knowledges that teachers need to develop and apply.

In the investigation carried out here, we found that some undergraduates had already been working as teachers at the time of the researh, despite not having completed their graduation. Thus, we understand that the school experience, as a teacher in Basic Education, is part of the teaching knowledge constructed by such undergraduates. Tardif (2010) points out that such knowledges are built throughout the profession from the practice/work of the undergraduates/teachers in the Basic Education. According to this author, such knowledges can be produced at work during interactions with other more experienced teachers and “they are necessary within the scope of the teaching profession and that they do not come from educational institutions or from curricula" (TARDIF, 2010, p. 49).

As we highlighted in the previous section, the Academic Mathematics produced in the training of future teachers is conditioned and expressed in their teaching practice in the classroom. Teaching knowledges can be named, in the same way, as disciplinary knowledges, which are produced from university subjects. Such knowledges “are also integrated into teaching practice through the training (initial and continuous) of teachers in the various subjects offered by the university” (TARDIF, 2010, p. 38).

One of this author's questions is that the knowledges of professional training, curricular knowledges and disciplinary knowledges are incorporated into teaching practice without being legitimized by it, without being produced by the teachers themselves - he names them as second-hand knowledges. In the same way, they argue that teachers remain in the role of transmitters and carriers of such knowledges. Another criticism pointed out by Tardif (2010) is that disciplinary knowledges are not used in the Basic Education school, in the same way that curricular knowledges are not discussed at the University in undergraduate courses, and that both are not articulated with professional knowledges. Meanwhile,

> [...] technical knowledge and know-how are gradually being systematized into bodies of abstract knowledge, separated from social groups - who become atomized executors in the universe of capitalist work - to be monopolized by groups of specialists and professionals, and integrated into public training systems. In the 20th century, science and technology, as a fundamental core of contemporary erudite culture, were considerably transformed into productive forces and integrated into the economy. The scientific community is divided into groups and subgroups dedicated to specialized tasks of restricted knowledge production (TARDIF, 2010, p. 43).

We understand that teaching knowledge, with its heterogeneous characteristics, represents plural knowledge, formed by diverse knowledge originating from training institutions, professional training, curricular knowledge, teaching practice, student experiences, as highlighted by Tardif (2010) in his consultations with Basic Education teachers:

> [teachers] talk about various knowledge, skills, competences, talents, ways of know-how, etc., related to different phenomena related to their work. They speak, for example, of knowledge of the subject and of knowledge related to the planning of classes and their organization. They also deal with the knowledge of the great educational principles and the education system, commenting on the programs and textbooks, their value and their usefulness (TARDIF, 2010, p. 60-61).

Finally, we highlight that this amount of knowledges produced by the participants of the study was evidenced in the data produced in the research, partially presented here. They were found in the actions of the graduates and undergraduates/teachers, in the discussions in the AVA, in the videos produced by them and in the interviews and questionnaires answered.

Undergraduates-with-videos: a didactic and pedagogical resource

Based on the understanding given in the studies of Levy (1993) and Tikhomirov (1981), Borba and Villarreal (2005) describe that knowledge is produced by a collective thinking formed by human and non-human participants, "a collective formed by human beings-with-medias or human-beings-with-technologies and not, as suggested by other theories, by solitary or collective human beings formed only by human beings" (BORBA, 2002, p. 139). This author stresses that he has no interest in carrying out improvement checks, or not, regarding the use of a certain media, such as information technology, writing or language, because due to the perspective of technology concatenated with the production of knowledge, it becomes very complex to make comparisons on results and evaluate them, in order to identify whether it is better or worse.

Borba (2002) and Borba and Villarreal (2005) turn to Tikhomirov (1981) to affirm that "thinking is not only having the capacity to solve a given problem, but also involves the way used to solve it, the values involved in its resolution, and also the choice of the problem as part of the thought" (BORBA, 2002, p. 137). This thinking is qualitatively organized in a different way, considering the advent of information technology as opposed to that carried out only with pencil and paper.

Levy (1993) presents reflections on the relationship between technique, knowledge and writing. Borba (2002) and Borba and Villarreal (2005) emphasize that this author characterizes the notion of technologies of intelligence in three great techniques developed by human beings and that are associated with memory and knowledge, orality, writing and information technology. They report that the history of mankind has always been intertwined with the use of medias and, in the same way, human beings are impregnated with techniques.

The research that was developed, and that is being highlighted in this article, investigated the mathematical knowledge presented by undergraduate students of a Mathematics course in the distance form in the videos produced by them. However, as pointed out in the previous paragraphs, the video media is qualitatively different from the discussion forum media or chat media, and also different from the student solving a certain problem using pen and paper and photographing it to share in the discussion groups that are part of the AVA. Chiari (2015) classifies the material made available and, many times built by the most varied participants in the distance form, as Interactive Digital Teaching Material (Material Didático Digital Interativo - MDDI), and these participants are described by Almeida (2016) as "polidocentes-com-media" (teachers who are able to work with different medias).

The procedures used for the production of the research data emerged two categories of analysis, which were developed through Grounded Theory, namely: videos as pedagogical potential for undergraduates/teachers in the classroom and videos as potential didactics for the undergraduates of a distance mathematics course. The first is aimed at the undergraduates/teachers who use the video in the Basic Education classroom, and the second one is related to the use of videos by undergraduates in order to contribute to university studies in the most varied subjects, since the course has its operation in distance learning, and students do not have face-to-face contact with teachers.

Ferrés (1996) points out that the use of video by undergraduates/teachers, as highlighted in the previous paragraph, can be planned in the same way that the teacher plans a class using other technological resources, such as orality and writing, that is, he can use excerpts from videos by making editions, trying to adapt them to the content he wants to present to his students, as well as adapting videos previously used to the students' reality. In the data produced in the research, the undergraduates named the videos they used as "didactic videos". Thus, we adopted this name for the videos, with mathematical content, produced by the undergraduates.

Presentation and analysis of the video “square root by subtraction”

For the presentation and analysis of the video shown in this section, we chose to make some adaptations to the models presented by Gino, Mill and Nagen (2013) and Powell, Francisco and Maher (2004). Gino, Mill and Nagen (2013) expose, in the work "Writings on education: challenges and possibilities to teach and learn with emerging technologies", a table showing the video, which the authors name by technical sheet. In this table they present the main characteristics of the video, such as title, format, script and production, year, production and award. However, no analytical positioning is performed, and the video description is developed after this presentation board. We understand by description of a video in written form something that, after reading, allows us to have an objective idea of the production.

In the adaptation we prioritize, due to the characteristics of the research that was developed, to highlight in the technical file the mathematical content explored in the video, explicitly or implicitly, as developed by Oechsler (2018), when this author investigated the nature of the communication developed by students of three municipal schools in the city of Blumenau, when videos were produced in mathematics classes. In addition, we seek to present some particularities of the videos, such as images of the moments of production, classification of the filming procedures - slides with or without narration, animation, screen capture, videos with other videos, staging a problem, Digital Mathematical Performance (Performance Matemática Digital - PMD), video class, video with manipulative material, photographs with stop motion and the explanation of content without the teacher (OECHSLER, 2018).

We also chose to present the description after the technical file, in the same way as described by Gino, Mill and Nagen (2013). We also try to prioritize, in the technical sheets, the human and non-human resources used by the undergraduate during the production of the video, our indication for the use of the video in the classroom, as well as the title, the time of the video and the address (link and QR Code) for complete visualization of the material.

Regarding the analysis, Powell, Francisco and Maher (2004) present a model for analyzing videos that recorded the action of students of Mathematics. Of the seven steps highlighted by these authors, we chose five, similarly to the way Scucuglia (2006), Scucuglia (2012) and Oechsler (2018) prioritized.

The first step is related to viewing and describing the video. As already highlighted, we chose to describe the video after the technical sheet and we prioritized viewing the video several times in order to make it familiar. The second step is aimed at data encoding. As we have already pointed out, we chose to use Grounded Theory (Teoria Fundamentada nos Dados - TFD) in the research, which presents its own method of categorization and coding. We emphasize that the categories that emerged are based on the data produced by all the procedures used, not only on the videos that were produced by the undergraduates. The third step is related to the identification of critical events that are part of the phenomena identified in the research and that are related to the development of the categories that emerged. The fourth step indicates the transcript. We emphasize that transcription is different from description. The first presents analytical characteristics, while the second details the video information. Finally, in the fifth step, the authors understand that it is necessary to carry out the constitution of the narrative, reporting the history of events.

Having made these considerations, in table 1 below we present the main characteristics of the video, such as format, duration, theme and content covered, indication for use in the classroom and the address on YouTube. Then we show some reflections on the Mathematics presented in the video by Mirla.

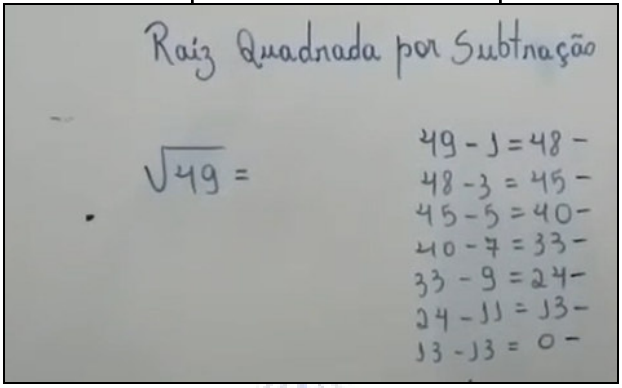
Table 1: Technical sheet of the video "Square root by subtraction"

Títle	“Square root by subtraction”
Format/Ressources Used	Videoclass. The student uses a marker and a white board to solve an activity with mathematical content.
Video time	1 min. and 40 sec.
Them/Contents covered	Square root. Arithmetic Progressions.
Indications for use in the classroom.	We understand that this video can be used in the classroom, as it presents another possibility for calculating the square root of a perfect square integer. However, reflections can be presented in order to carry out investigations for the calculation of square roots for whole numbers that are not perfect squares or even for non-integer numbers. A second point that we highlight is related to the validity check of the algorithm used by undergraduate Mirla, in the sense of exploring other themes, such as, for example, arithmetic progressions. Research can be carried out by the students, guided by the teacher, in order to find other methods of determining the exact, or approximate, square root of real numbers.
Address and *QR Code*	https://www.youtube.com/watch?v=9lWGW4KxG1c&index=23&list=PLXAu57dw5ErhkQtLVA9oOiFeJnHcNUruz&t=0s

Source: Data from the Author (2018)

In figure 1 we highlight the steps that undergraduate Mirla used to find the square root of the integer 49, by the method she calls “square root by subtraction”.

Figure 1: Scheme with the steps used in the video “Square root by subtraction”

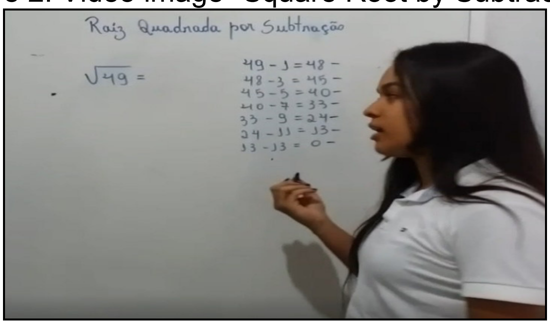


Source: Data from the Author (2017)

In the video, the undergraduate states that to determine the square root of the integer **49**, it is enough to perform successive subtractions on it of consecutive odd integers from the odd number **1**. The student highlights in her speech that, if the result of this subtraction is null, the initial number, in case **49**, has an exact square root and is given by the number of subtractions performed in this procedure. In the case of number **49**, **7** consecutive subtractions were made, that is, the square root of number **49** is equal to **7**.

The undergraduate Mirla uses a marker and a whiteboard to present an activity in an expository way, qualified as videos. With few corporate movements, we highlight the movements with the arms, a slightly shaky voice, staging the video with apparent confidence keeping the viewer aware of what she is talking about. Background music can be smoothly perceived as well as the noise made by passing cars. It seems that the environment chosen for the video production may be a pole of the Universidade Aberta do Brasil (UAB) or a school in which the student works. A slight "shake" at some points in the footage that indicates that the cell phone or digital camera is being handled by someone else without using a tripod or other type of fastener. Throughout the video, perhaps due to the topic addressed, Mirla uses the digits of the decimal numbering system, with the exception of the title, which is displayed in the mother tongue, which is the Portuguese language (Figure 2).

Figure 2: Video image "Square Root by Subtraction"



Source: Data from the Author (2017)

The undergraduate starts the video presenting a justification regarding the algorithm chosen to stage the video, as described below:

Mirla: [...] *Well, guys, let's answer the square root by subtraction, which is an easier method to answer. There is another way that is decomposing, and some people find it difficult to divide*.

Tucker (2013) highlights the importance of presenting the objectives and justifications at the beginning of the video and ending with a given conclusion. However, the video ends abruptly without presenting a conclusion about the content covered. Observe that figure 2 shows the moment in which Mirla finalizes the successive differences of the consecutive odd numbers, performed initially on the integer **49** and, continuously, on the differences obtained previously. The number of differences is pointed out by the undergraduate as the square root value:

Mirla: [...] *there is something even cooler, we can obtain the square root of forty-nine, we just need to count how many times we subtract, and we will have the result, [she counts indicating the odd numbers] 1, 2, 3, 4, 5, 6 and 7, so the square root of forty-nine is seven*.

We understand that some questions can be asked about the method chosen by Mirla when producing the video under discussion, besides having some reflections regarding activities that can be developed in the classroom, as well as future research can be carried out: a) Is it ensured that this procedure is valid for other perfect square integers? b) What procedures can be developed in the classroom in order to provide answers to the previous question? c) Once this is solved and the answer to item 'a' is positive, how to explore this method to determine the approximate square root of whole numbers that are not perfect squares? And for non-integer numbers? d) What reflections can be addressed for numbers where multiple iterations are needed to determine the square root value?

Moreira and David (2007) highlight that teaching practice produces knowledge, produces adaptations of knowledge constituted outside that practice, making them useful or usable. These authors also emphasize that

> [...] the training process in the Mathematics degree conveys certain knowledge that is considered “useless” for teaching practice. Likewise, other knowledge works "inappropriately", with reference to this practice [...] often refuses [...] to develop a systematic discussion with the undergraduates regarding the concept and processes that are fundamentals in basic school education in Mathematics (MOREIRA; DAVID, 2007, p. 42).

However, these authors point out that only teaching practice is not capable of producing all the teaching knowledge necessary for the teacher's pedagogical practice.

> [...] in the same way that the question of knowing the nature of the knowledge produced in teaching practice arises for the teacher training process, it is also necessary to understand the nature of the "non-knowledge" associated with that same practice. But, for that, it is necessary to place these "non-knowledges" within the school mathematical education process instead of conceiving it, purely and simply, as a lack in relation to scientific mathematical knowledge. In the same way that the knowledge produced in the teaching experience is not seen as a contribution to scientific mathematical knowledge, these "non-knowledge" can also be situated in relation to School Mathematics and not to Academic Mathematics (MOREIRA; DAVID, 2007, p. 43).

Taking the topic covered in the video as an example, what elements of the undergraduate's training are really accurate and priority? Why are there several contents worked in the degree that are not part of the teaching practice in Basic Education? According to Iezzi, Dolce and Murakami (1977), from a natural **n** and a real number **a ≥ 0**, it is possible to demonstrate that there is always a positive real number **b**, such that $\mathbf{b^n = a}$. Such number **b** will be called the nth root of **a** and will be indicated by:

$$b = \sqrt[n]{a}$$

However, neither the way approached by these authors, nor the way used by Mirla in the video are presented in the main textbooks that drive teaching methodologies used in the Basic Education classroom. As an example, Centurión and Jakubovic (2015a, p. 44) emphasize that the square root of forty-nine is equal to seven, because the number seven when squared, or the second power, results in forty-nine, symbolically,

$$\sqrt{49} = 7, as\ 7^2 = 49$$

That is, these authors present examples using integers in the definitions, while the previous ones generalize algebraically. Finally, we present a possibility to check the model presented by Mirla in the video. In Table 2 below, we will initially take the sum of consecutive odd number sequences.

Table 1: Square root for difference: a justification

S_n	Σ	n^2	Quantity of odd integers added
S_1	1=1	1^2	1
S_2	1 + 3 = 4	2^2	2
S_3	1 + 3 + 5 = 9	3^2	3

S_4	1 +3 + 5 + 7 = 16	4^2	4
S_5	1 +3 + 5 + 7 + 9 = 25	5^2	5

Source: Data from the Author (2018)

It is important to note that the sum of consecutive odd numbers is a perfect square, in which the number of consecutive odd numbers added represents the square root of the sum made. From the previous table, when we subtract consecutive odd integers, as the inverse property of the addition, we will always obtain zero. However, the question remains: is the sum of **n** consecutive odd integers equal to the square of **n**? That is,

$$S_n = n^2?$$

By adding n odd and consecutive integers we are performing the sum of the terms of an arithmetic progression of ratio **2**, that is:

$$S_n = 1 + 3 + 5 + \ldots + [1 + (n-1).2] = 1 + 3 + 5 + \cdots + (2n-1)$$

Which can be rewritten as

$$S_n = \frac{(a_1 + a_n).n}{2} = \frac{(1 + 2n - 1).n}{2} = \frac{2n^2}{2} = n^2$$

Therefore, the algorithm used by the undergraduate in the video is valid and, in addition, we observed that this activity can be developed as an application to the theme of arithmetic progression and represents a teaching practice that is part of the activities developed in the classroom by teachers, and that the video, seen as a pedagogical and didactic resource, was used for the presentation.

This resource, among others present in the Basic Education classroom for a longer time, modifies spaces and brings students closer. Ferrés (1996, p. 32) points out that “the school, understood as an ecosystem, became aware of the threat posed to the teacher by the incorporation of modern audiovisual technologies, opted for subjection: audiovisuals were converted into assistants”.

This author also points out that, in the use of video, the most important is the process. During the production, the construction of the script, the choice of the scenario, the contact with the text and the assembly, say, of the speech, all this planning makes the content worked on easier to assimilate. The author also points out that “reducing the didactic use of video to showing programs represents a mutilation of the expressive and didactic possibilities that are available” (FERRÉS, 1996, p. 40).

In this process, contact with technology leads producers to reorganize their thinking, thus bringing educational changes with more diverse ideas (BORBA;

VILLARREAL, 2005). These authors emphasize that “visualization has been the main change in the interface of computers through monitors that were introduced as an essential part of computers” (BORBA; VILLARREAL, 2005), besides representing the main form of providing feedback.

They also point out that, in mathematical activities, the use of diagrams and visual reasoning procedures, in addition to dynamic and interaction representations, represent characteristics of the mathematical reasoning that is used and, however, such visual representations are not accepted as part of formal test by some communities formed by mathematicians.

Final Considerations

In almost all sectors of our society, technological means are present and affect the way we communicate, how we make investments, how we take care of our health, in short, how we live. The educational sector, made up of social relations, receives external processes and needs to verify how the evolutions (or non-evolutions) can be managed in the pedagogical procedures. For this absorption to be carried out, it is important that researches are developed in order to investigate how these changes will help in Education.

In the research we carried out it was possible to observe that the videos are part of the school life of the undergraduates in a Mathematics course in the distance form. The fact that they take subjects without face-to-face contact with the teacher makes them use the videos as a didactic resource to answer questions about the content studied in the subjects. In addition, we observed that many of the undergraduates are teachers of Basic Education and use videos in their classes. The way these videos are used is not among the objectives of this research, and new works may be developed in the future in order to observe this use.

The video that we analyzed in this work is part of the pedagogical resource category, in which undergraduates/teachers use videos in the Basic Education classroom to introduce a certain subject or to broaden discussions on a certain topic under debate. However, discussions about the teaching practice of the Mathematics teacher are carried out in order to identify and relate what the initial training is proposing and to navigate between the proposals presented by Academic Mathematics and School Mathematics.

As this video was produced in the Supervised Curricular Internship discipline, we understand that the production developed by Mirla can be used so that the teacher of the internship discipline can analyze the pedagogical skills of the undergraduate, and the researcher to make inferences about the teaching action and the specific knowledge of the mathematics teacher (TARDIF, 2010), as well as the technological resources that contribute to the production of knowledge (BORBA; VILLARREAL, 2005). In addition, we understand that such videos can still be used for discussions between the undergraduates in discussion forums in the AVA, in order to disseminate such productions, based on the idea and type of the video (OECHSLER, 2018), the contents covered and possible submissions to video festivals.

We emphasize that this article presents a snippet of data from a survey that investigated how videos with mathematical content contribute to the teaching education of undergraduates when studying in a distance learning course. Thus, we understand that this work discusses possibilities experienced by the undergraduates in a Mathematics course in which videos of mathematical content enhance the production of teaching knowledge, as well as exposing their accumulated experiences in relation to mathematical knowledge aiming at the classroom practice with technological resources.

We hope that this research can contribute to the investigations of the community formed by mathematical educators in scenarios such as Distance Education, Supervised Internship and mainly in the pedagogical procedures in the classroom using video technology.

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