REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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The Additive Structures and the Learning of Mathematics: an activity analyzed in the light of Conceptual Field Theory

Translated from Portuguese: As Estruturas Aditivas e a Aprendizagem da Matemática: uma atividade analisada à luz da Teoria dos Campos Conceituais

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ABSTRACT

The conceptualization process of additive structures, carried out through the Theory of Conceptual Fields, helps us to understand how students construct mathematical knowledge. This work aimed to analyze the statement of problems and the students' production in the face of an activity applied to 46 students of 2nd and 4th years of Elementary School. The results show that students have difficulties in interpreting problems when the statements are semantically incongruous. The resolution strategy most used by 2nd grade students was drawing and, by 4th grade students, the numerical record. Some errors were frequent, such as: counting, related to the positional value and related to the inverse operation. It is hoped that this research will lead elementary school mathematics teachers to realize the importance of contemplating the different classes of problem situations, which enable the interpretation of semantically incongruous statements and differentiated resolution strategies.
KEYWORDS: Conceptual Field Theory. Mathematics Education. Additive Structures.

RESUMO

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O processo de conceitualização das estruturas aditivas, realizado por meio da Teoria dos Campos Conceituais, nos auxilia a compreender como os estudantes constroem os conhecimentos matemáticos. Este trabalho objetivou analisar o enunciado de problemas e as produções dos alunos frente a uma atividade aplicada a 46 alunos de 2º e 4º anos do Ensino Fundamental. Os resultados evidenciam que os alunos apresentam dificuldades de interpretação dos problemas quando os enunciados são semanticamente incongruentes. A estratégia de resolução mais utilizada pelos alunos do 2º ano foi o desenho e, pelos alunos do 4º ano, o registro numérico. Alguns erros foram frequentes, tais como: de contagem, relacionados ao valor posicional e relacionados à operação inversa. Espera-se que esta pesquisa conduza os professores de matemática do Ensino Fundamental a perceberem a importância de contemplar as diferentes classes de situações-problema, que possibilitam a interpretação de enunciados semanticamente incongruentes e estratégias diferenciadas de resolução.

PALAVRAS-CHAVE: Teoria dos Campos Conceituais. Educação Matemática. Estruturas Aditivas.

Introduction

Even considering Mathematics Education as a consolidated field of knowledge, with its own elements and distinct from the other areas, it is feasible that, periodically, we return to the form in which it presents itself in the didactic scenery.

In order for Mathematics Education to become as it is today, it received contributions from other areas of knowledge and not only from Education and Mathematics, as it is explicit in its nomenclature. Philosophy, Psychology, Anthropology and History are examples of areas that contributed, and continue to do so for the development of Mathematic Education.

We perceive these contributions, clearly, when we look at the different theories of Mathematical Education: by Brousseau, by Vergnaud, by Duval, by Skovsmose, as well as on their trends: Mathematical Modeling, Problem Solving, Ethnomathematics, New Technologies, History of Mathematics, Mathematical Investigation, Mathematical Games, among others.

What makes them have a common point, so that they are considered to belong to this area? What makes mathematics education a field of knowledge? These are not simple questions to solve, but it is visible that, although they variably seek different or similar paths, these surveys aim for answers to the same phenomena: the teaching and learning of Mathematics.

This work seeks to follow one of these paths, clarifying to the reader one of these theories, that of the Conceptual Fields of Gérard Vergnaud, and aims to analyze the statement of problems and the productions of students facing an activity applied to 46 students of 2nd and 4th years of Fundamental Education.

Thus, in the next sub-chapter, we will carry out a synthesis of Vergnaud's theory, which, despite the limited space for the format of this work, allows us to proceed with the desired analysis.

The Theory of Conceptual Fields and the problems of additive structures

The Theory of Conceptual Fields was developed at the 1980s in France, with the psychologist, philosopher and mathematician Gérard Vergnaud, belonging to the Piagetian tradition, with the purpose of explaining the process of conceptualizing additive, multiplicative structures, of relationships space-numbers and algebra, and thus seek to understand how students construct mathematical knowledge.

In this sense, for the author, knowledge is organized in conceptual fields, whose domain, at subjects part, occurs along a certain period, through experience, maturity and learning (VERGNAUD, 1982, p. 40).

For Vergnaud (1982) *apud* Magina; Santana; Carzola; Campo (2010), a conceptual field should be seen as a "[...] informal and heterogeneous set of problems, situations, concepts, relationships, contents and operations of thought, connected each to another and probably interconnected during the acquisition process" (VERGNAUD, 1982, *apud* MAGINA *et al.*, 2010, p. 18).

Vergnaud (2009) informs us that, from a conceptual field, one understands the development of competences of the subject. In this way, the author defines the concept as a set of three subsets: set of situations that give meaning to the concept (the reference); set of operative invariants, the concepts-in-act and theorems-in-act that intervene in the treatment schemes of the situations (the meaning); set of linguistic and symbolic representations (algebraic, graphic, natural language) that allow the representation of concepts and their relations and, consequently, the situations and schemes they evoke (the signifier).

A concept does not develop itself in a single category of situations, but in a certain variety; in the same way, we cannot analyze a situation considering only one concept, but only from several ones. Vergnaud (1993) emphasizes that a situation, however simple it may be, involves several concepts.

In order for the reader to perceive the importance of the variation of thematic problems by teacher's part, we exemplify the situations of the multiplicative structures, throw four problems proposed and explained by Magina *et al.* (2014), which culminate in the same 2x4 operation, but bring concepts with different complexities.

The first problem deals with concepts of simple proportionality; it refers to the quotient between two quantities: "Dona Maria's recipe of chocolate candies takes 1 can of condensed milk for 4 spoons of chocolate. She will make candies with 2 cans of condensed milk. How many spoons of chocolate will she use to make her sweets' recipe correctly?" (MAGINA *et al.*, 2014, p. 38).

The second problem, in turn, proposes a comparison among magnitudes of same nature, in our case, the monetary value: "A small store in the Shopping sells everything twice more expensive as the store on the corner. A sandal costs R$ 4.00 at the corner store. How much does the same sandal cost at the Shopping store? " (MAGINA *et al.*, 2014, p. 38).

The third problem involves bilinearity: the amount to be paid is proportional to the number of sons and directly proportional to the number of hours. One observes that one of these quantities is made from a continuous nature and the others of a discreet nature: "An amusement park charges R$ 1.00 for each child to play in any game for one hour. Mrs. Lulu took her 2 children to play in the park during 4 hours. How much did she pay?" (MAGINA *et al.*, 2014, p. 38).

The fourth problem, finally, deals with the idea of combination and is associated with quantities of discreet nature: "In an ice cream shop, one-ball ice cream can be served in a small cone or cup. It has 4 different flavors: mint, vanilla, cuttlefish, strawberry. Maria wants ice cream from one ball, how many different ways does she have to choose?" (MAGINA *et al.*, 2014, p. 38).

Thus, we see the importance of considering the characteristics of each situation: concepts and values. These different situations also show that the learning of a concept may need some time to materialize and, during this period, the subject goes through numerous situations in the school environment and outside, which can enable the development of schemes, prepared to deal on these situations.

Piaget *apud* Nogueira and Rezende (2014), calls the scheme "the organized activity that the subject develops under a certain class of situations", such as when working with problem-situations in additive field (addition and subtraction), are presents the schemes of joining, separating and matching one by one.

The scheme is necessarily formed by four components: an objective, subjective and anticipations; rules in action, information gathering and control; operative invariants: concepts in action and theorems in action; possibilities of interference in situations (VERGNAUD, 2009, p. 21).

Theorems in action are invariants of the proposition type and are likely to be true or false.

> Between 5 and 7 years old, children discover that it is not necessary to recount everything to find the cardinal of A∀B after counting A and B, we can express this knowledge through a theorem in action: Card

> (AUB) = Card (A) + Card (B) since A ∩ B = Ø. (VERGNAUD, 1996a, p. 163).

The different forms of representation of a concept, using different operative invariants, can bring a greater meaning to its understanding. For example, it is possible to represent the same theorem referring to additive structures in several ways, as cited by Vergnaud (1996): i) in natural language: the initial state is the final state to which is added what has been spent or lost, and what is received or gained is subtracted; ii) in algebraic writing: F = T (I) → I = T - 1 (F); iii) through the sargital scheme, which we will use later. However, the invariance of the signifier contributes to a better identification of the meaning and to its transformation into objects of thought (VERGNAUD, 1996, p. 186).

Some didactic activities can facilitate the verification of the schemes carried out by the students, through the analysis of the strategies used to find the solution and recognize the previous knowledges of the subjects, which were not evident. In this theory, a problem is characterized as being every situation in which it is necessary to discover relationships, to develop activities of exploration, hypothesis and verification, to produce a solution (VERGNAUD, 1990, p. 52).

Vergnaud (1993) establishes as a Conceptual Field of Additive Structures the set of situations that involve one or more additions and subtractions, as well as the set of concepts and theorems connected to these situations. As components of this conceptual field, Rezende and Borges (2017), supported by Vergnaud (1993), mention some concepts:

> of cardinal and measure, of temporal transformation by increase or decrease (win or lose), of quantified comparison relation (having more than), of binary composition of measures (how much in total), of composition of transformations and relations , of unitary operation, inversion, natural number and relative number. (REZENDE; BORGES, 2017, p. 333).

Magina; Campos; Nunes; Gitirana (2001) make a re-reading of the conceptual field of additive structures, classifying additive problems in three basic relations, from which the problems of addition and subtraction originate. They are: by composition, transformation and comparison.

For these three groups of problems, their extensions are broken down, which relate to different levels of complexity, and the problems whose resolution requires simpler situations are called prototypes. Next, we present the three classes of problems, with their structural forms, as proposed by Magina *et al.* (2001), together with the schemes proposed by Vergnaud:

Composition: this class comprises situations that refer to problems that involve the relations between part and the whole. Students may be presented with the values of two or more parts and questioned about the value of the whole. This type of problem is classified as a prototype of additive problems (Table 01 - Composition 01).

Table 01 – Composition 01 e Composition 02

Statement	Scheme	Statement	Scheme
João has 10 black toy cars and 7 red to cars. How many car does João have in all?	10 Part 7 Part ? All	In a fruit tray there are 13 green and yellow oranges. 8 oranges are green. How many are yellow oranges?	8 Part ? Part 13 All

Source: Elaborated by the authors

You can inform the value of the whole and one or more parts and ask about the value of the remaining part. This category is classified as problems of 1st extension of the additive structures (Table 01 - Composition 02).

Transformation: in this class of problems the temporal idea is always involved. It establishes a relation between an initial quantity and a final quantity. There are six possible situations, three related to positive transformations and three related to negative transformations.

The problems – that inform about the initial quantity and how the transformation is done (positive or negative) – are considered as prototype problems (Table 02 - Transformation 01).

Table 02 – Transformation 01 e Transformation 02

Statement	Scheme	Statement	Scheme
João had 9 candies and won 4 from Dad. Altogether, how candies did João have?	Transformation +4 Inicial State 9 Final State ?	João had 10 toffees. Ate some and remained with 7. How many toffees did he eat?	Transformation -? Inicial State 10 Final State 7

Source: Elaborated by the authors

The problems that inform about the initial and final quantities and question about the value of the transformation are considered problems of 1st extension (Table 02 - Transformation 02). The problems that offer the transformation values and the final quantity, asking for the initial quantity are considered to be more complex problems, framed as 4th extension (Table 03).

Table 03 – Transformation 03

Statement	Scheme
João had some stickers and won 5 from friend, getting 14. How many stickers did João have before?	Transformation +5 ? 14 Inicial State Final State

Source: Elaborated by the authors

Comparison: this class encompasses problems in which it is possible to compare two qualities, called referent and referred, and whose relation is always present. If the problem offers one of the quantities (referent) and the relation between them and asks about the other quantity (referred), there is a second extension problem (Table 04 - comparison 01).

Table 04 – Comparison 01 e Comparison 02

Statement	Scheme	Statement	Scheme
John is eight years old and Carlos is four years older than he. How old is Carlos?	? Referred ↑ +4 Relation 8 Referent	João is 6 years and Carlos is 11 years old. Who is older? How many years more?	6 Referred ↑ ? Relation 11 Referent

Source: Elaborated by the authors

If the problem provides both quantities (referent and referred) and asks about the relationship between them, the problems are classified as 3rd extent (Table 04 - Comparison 02). If the quantities reported are that of the referred and the relation, asking for the amount of the referent, then it is a problem of 4th extension (Table 05).

Table 05 – Comparison 03

Statement	Scheme
João has 4 shoes less than Carlos, who has ten. How many shoes does João have?	10 Referred ↑ -4 Relation ? Referent

Source: Elaborated by the authors

We realized, therefore, that the alternation between different types of problems can lead students to understand the concepts related to additive structures. Thus, below, we describe how an activity for children of 7 and 9 years was developed and applied, in order to, through a practical example, lead the reader to a better understanding of the aforementioned theory.

Empirical Field

Based on the studies by Vergnaud (1998) and the contributions of Magina *et al.* (2001), who also based their work on Vergnaud's theory, there was interest in conducting a case study between classes, one from the 2nd and another from 4th year of a municipal school in the city of Campo Mourão-Pr. This choice was due to the fact because they were groups of one of the authors. The research counted on the participation of 46 students, being 23 from the 2nd year and 23 from the 4th year, aged between 8 and 10 years.

The investigated municipal school serves the Elementary School Initial Years, special class and multifunctional resource room. The institution has 14 classes in each period (morning and afternoon), with approximately 26 students per class and a total of 650 students and 40 teachers.

The five problem situations were developed by the authors of this text based on the characteristics presented by Magina *et al.* (2001) in relation to the types of problems: composition, transformation and comparison. No more advanced problems were proposed for the 4th grade class, because the purpose of the activity was to analyze the results of different classes and seek to understand the students' development over the school years.

At the time of application, the problems were delivered to the students and read by the researcher. If any student had doubts regarding the statements of the questions, he could talk to the researcher in order to lead them to understand the statement. The data collection instrument was the written record developed by the students while participating in the observed classes.

Thus, we carry out the analysis of each problem and at the end an understanding of the performance of the classes is presented. In order to preserve the students' identities, the 2nd year students were nominated by A1 to A23 and B1 to B23 by the 4th year students.

Table 06 – Problem 1

Problem 1:	Gabriel has ten stickers and won 7 from his friend. How many stickers does Gabriel have?
Correct operation:	10 + 7 = 17
Characteristics of the problem:	Positive transformation, search for the final state – prototype.

Source: Elaborated by the authors

Table 07 – Analysis of problem 1

Student	**Did he/she record an algorithm for resolution? Which?**	**Mistakes committed**
A1, A2, A3, A7, A8, A9, A10, A12, A13, A14, A15, A16, A17, A18, A19, A20, A21 B1 B2 B3 B4 B6 B7 B9 B11 B12 B14 B15 B17 B20 B21 B22 B23.	Yes, 10 + 7 = 17	–
A4, A5, A6, A11, A22 e A23.	No, they presented drawings	–
B5	Yes, 10 + 7 = 10	Miscalculation

B8 B10 B13 B16 B19	No, they presented correct result	–
B18	Yes, 10 – 7 = 17	Miscalculation

Source: Elaborated by the authors

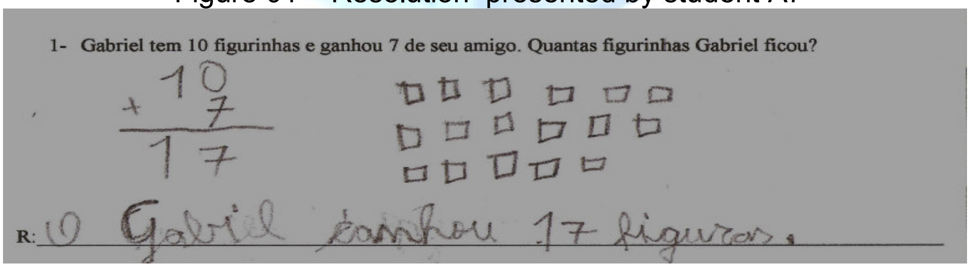
Among the 23 students of second year, who solved problem 1, 17 of them could solve the proposed situation using the addition algorithm they reached the solution of the problem and presented coherent answers. Of these 23 students, pupils A7 and A21 represented the problem using the addition algorithm and through drawing as shown in Figure 01. Students A4, A5, A6, A11, A22 and A23, correctly solved the situation suggested, only with the aid of drawings.

Among the 23 students in class of 4th year who solved the proposed problems, one can consider that 16 of them were successful in solving problem 1, presenting the calculations, correct results and coherent answers. For example, it is considered as a coherent answer that presented by student B1: “Gabriel got 17 figs”. It is noteworthy that these students put in their answers not only the final result, but phrases that explain the solution to the problem.

Students B8, B10, B13, B16 and B19 did not present calculations or drawings in the proposed problem, only the correct result was shown. It was considered that the answers presented by them differed from the previous ones, as it was not possible to identify how the problem was solved.

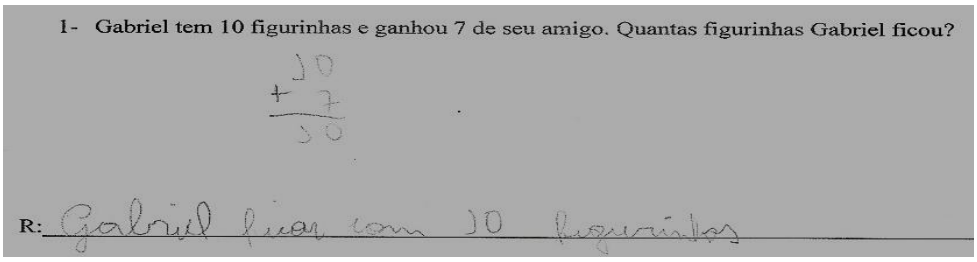
Two cases stood out: in the resolution developed by student B5, the algorithm (10 + 7 = 10) presented and the order of the data used are correct, but the solution is incorrect. It is possible that this is due to a lack of attention and, despite the error, the answer was clearly presented. Figure 02 shows student B5's resolution. Student B18 approached incorrectly the algorithm to solve problem 1: (10 + 7 = 10). Despite organizing correctly, the operation is incorrect. We can conjecture that there is a conceptual confusion regarding the use of the algorithm.

Figure 01 – Resolution presented by student A7



Source: Elaborated by the authors

Figure 02 – Resolution presented by student B18



Source: Elaborated by the authors

Following, the analysis of the resolutions for problem 02 is presented.

Table 08 – Problem 2

Problem 2:	There are 15 toys in a box. Four are small cars and the rest are dolls. How many dolls are in the box?
Correct operation:	15 – 4 = 11
Characteristics of the problem:	Composition, search for one of the parts - 1st extension.

Source: Elaborated by the authors

Table 09 – Analysis of problem 2

Student	**Did he/she record an algorithm for resolution? Which?**	**Mistakes committed**
A2, A3, A10, A12, A8, A13, A14, A15, A16, A17, A18, A19, A20, A21, A22, A23, B2, B3, B6, B7, B9, B11, B12, B13, B14, B15, B17, B18, B19, B20, B21, B22, B23.	Yes, 15 – 4 = 11	–
B1	Yes, 15 – 4 = 12	Miscalculation
A5, A11, B4	No, they presented drawings	–
A1	Yes, 15 - 4 = 10.	Miscalculation
B5	Yes, 15 -* 4 = 11	Not indicated
A7 e A9	Yes, 15 + 4 = 19	Addition algorithm
A6	Yes, 4 + 11 = 15	Addition algorithm
A4, B8, B10, B16	No, they presented correct result	–

Source: Elaborated by the authors

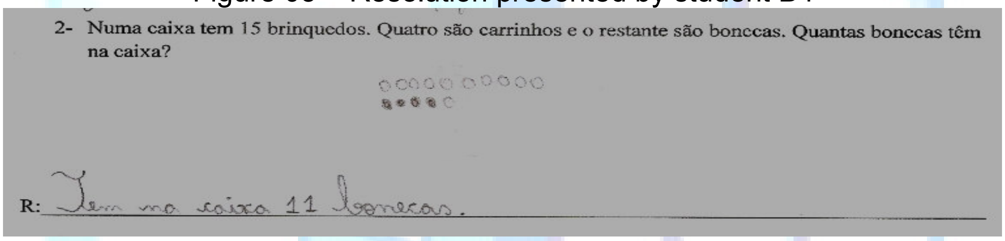
Regarding the resolutions presented by students for problem 2, it was found that 16 students of 2nd year and 19 of 4th were successful in the resolution process, since they presented in their resolutions the correct calculations, coherent and clear answers solutions to the problem. Students A4, B8, B10 and B16 did not present a solution to the problem, but only the final answer.

Students A5, A11 and B4, despite not using the subtraction algorithm (15 – 4 = 11) for the resolution, represented the problem situation through drawings. Student B4, for example, drew 15 balls that represent the total of toys and colored 4 balls that represent the total of strollers, and he concluded that the balls that were left without coloring represent the amount of dolls that are in the box, that is, 11 dolls, as can be seen in Figure 03. Student A18, on the other hand, solved the situation with the aid of drawings and through the addition algorithm. Student B5, it is believed that due to inattention, did not indicate the minus sign in the algorithm, but presented the correct answer. Students A1 and B1 built the algorithm correctly, but they erred in the calculation process, and student B1 did not present a coherent answer.

By other side, students A7, A9 and A6 presented the addition algorithm in their resolutions. Students A7 and A9 added the total of toys to the total of strollers and concluded that there were 19 dolls in the box. It is understood that these students were not able to correctly interpret problem 2. Furthermore, it is possible to notice that student A9 needs to indicate in the algorithm the letters D and U that correspond to the dozen and unit houses, respectively, for position the numbers, as shown in Figure 04.

Student A6 also presented difficulties in interpretation, being assisted by the teacher-researcher. It is interesting to note that this student did the correct mental calculation (15 - 4 = 11), but recorded the calculation 4 + 11 = 15 and, finally, presented the answer “11 dolls”. This may show that this student, despite his difficulty, was able to represent his mental calculation, of subtraction, in form of an addition operation, as a real proof of the reasoning he had performed. Thus, it is highlighted that the inverse relationship between operations may have been understood.

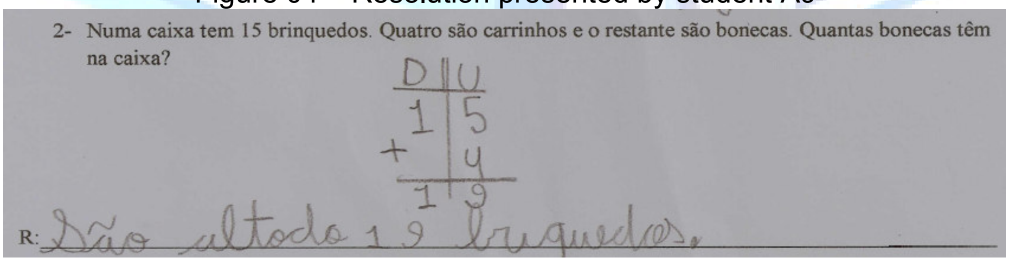
Figure 03 – Resolution presented by student B4


2- Numa caixa tem 15 brinquedos. Quatro são carrinhos e o restante são bonecas. Quantas bonecas têm na caixa?

R: Tem na caixa 11 bonecas.

Source: Elaborated by the authors

Figure 04 – Resolution presented by student A9


2- Numa caixa tem 15 brinquedos. Quatro são carrinhos e o restante são bonecas. Quantas bonecas têm na caixa?

D U
1 5
+ 4
1 9

R: São altodo 19 briquedos.

Source: Elaborated by the authors

Next, we present the analysis of resolutions for problem 3.

Table 10 – Problem 3

Problem 3:	Maria had some toffees and gave three to her cousin, keeping 19. How many toffees did Maria have?
Correct operation:	19 + 8 = 27
Characteristics of the problem:	Negative transformation, the search for the initial state - 4th extension.

Source: Elaborated by the authors

Table 11 – Analysis of problem 3

Student	**Did he/she record an algorithm for resolution? Which?**	**Mistakes committed**
A1, A2, A6, A8, A9, A10, A13, A15, A16, A17, A18, A19, A20, A21, A23, B1, B2, B4, B6, B7, B9, B13, B14, B15, B17, B19, B20, B21 B23	Yes, 19 + 8 = 27	–
A3	Yes, 28 – 09 = 19	Incorrect data
A5	No, they presented drawings	Mistake on quantity in representation by drawing
A7	Yes, 28 – 8 = 20	Algorithm of subtraction; incorrect data
A12	Yes, 8 – 19 = 31	Algorithm of subtraction: order of data
A14	Yes, 18 – 8 = 10	Incorrect data of algorithm
B11, B12	Yes, 19 + 8 = 27 e 27 – 8 = 19;	–
B3	Yes, 8 – 19 = 11	Algorithm of subtraction; mistake on calculation: order of data
B5, B18	Yes, 19 – 8 = 11	Algorithm of subtraction
A4, A11, A22, B8, B10, B16	No	–
B22	Yes, 19 + 8 = 107	Miscalculation

Source: Elaborated by the authors

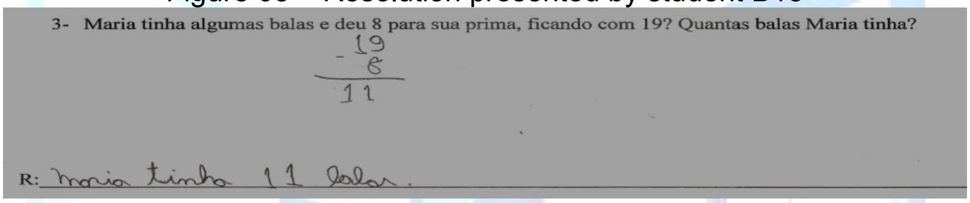
In the activities related to problem 3, it was found that 15 students from 2nd year and 16 from 4th year were successful. It was possible to verify that students the A1, A2, A8, A9, A18, A19, A20, A21, B1 and B6 did not indicate in the resolution process the number 1, at the top, which corresponds to the addition of a dozen, as can be seen in Figure 05.

It was also found that students A4, A11, A22, B8, B10 and B16 did not present a solution to problem 3. It should also be noted that students B5 and B18, possibly under the influence of the term "gave" in the statement of the problem, they solved the problem using the subtraction algorithm as shown in Figure 05. Despite this mistake, these students got the algorithm resolution right and presented a clear answer.

This semantic incongruity in statement of problem 3 may also have been the cause for the incorrect results of students A3, A7, A12 and A14. Student A3 tried to solve the problem with the aid of drawings and also through calculations, but did not interpret correctly the proposed situation: he drew 28 risks and marked 9 of them with a transverse line. It is possible that this student considered that Maria gave 9 bullets to her cousin, instead of 8 and, through attempts, intended to reach the number 19 (number of remaining toffees). This same mistake was made by student A5, who used only drawings, as shown in Figure 06.

Students A12, A14 and B3 used the subtraction algorithm and placed the numbers in the order presented by the problem: first 8 and then 9, as shown in Figure 06. Student A14 not only used the subtraction algorithm, but also supported incorrect data. Student A7, in addition to making mistakes in the calculation process, used incorrect data. One believes that this student performed mentally the calculation (19 + 8 = 28) and subsequently subtracted 8 units from the result found, thus concluding that Maria had 20 bullets. Regarding student B3, he indicated in his resolution that the operation of (8 - 9) results in 1 unit. It is believed that the student has assumed that the smallest number must be subtracted from the largest. In turn, student B28 established correctly the algorithm, however, in the resolution process, he was mistaken for not adding the ten originated from the sum of (8+9) with the ten corresponding to the number 19, and presented the value 107 as a result.

Figure 05 – Resolution presented by student B18

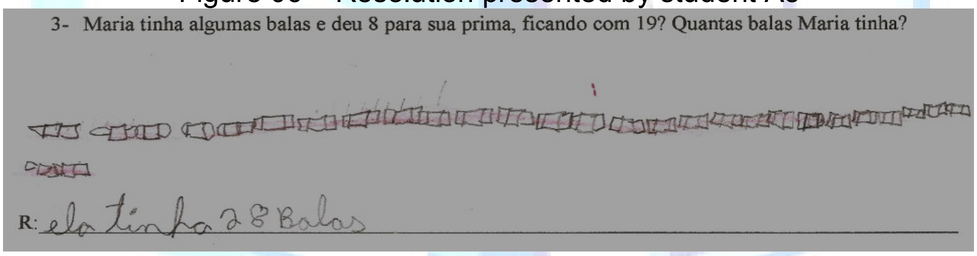

3- Maria tinha algumas balas e deu 8 para sua prima, ficando com 19? Quantas balas Maria tinha?

19
\- 8
11

R: maria tinha 11 balas.

Source: Elaborated by the authors

Figure 06 – Resolution presented by student A5


3- Maria tinha algumas balas e deu 8 para sua prima, ficando com 19? Quantas balas Maria tinha?

R: ela tinha 28 Balas

Source: Elaborated by the authors

Following, one presents an analysis on resolutions of problem 4:

Table 12 – Problem 4

Problem 4:	In a cupboard there were three green glasses and five pink glasses. How many glasses were in the cupboard?
Correct operation:	3 + 5 = 8
Characteristics of the problem:	Composition, search for the whole – prototype.

Source: Elaborated by the authors

Table 13 – Analysis of problem 4

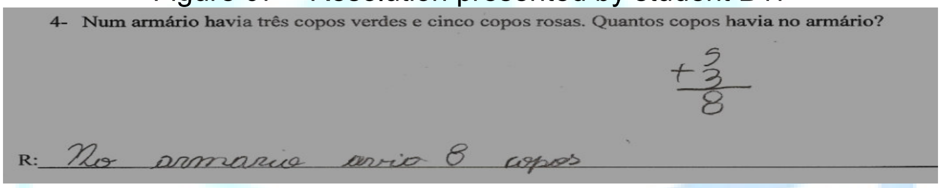
Student	Did he/she record an algorithm for resolution? Which?	Mistakes committed
A1, A2, A3, A7, A8, A9, A10, A12, A13,	Yes, 5 + 3 = 8	–

A14, A15, A16, A17, A18, A19, A20, A21, A23, B1, B2, B4, B5, B6, B7, B9, B11, B12, B13, B14, B15, B17, B18, B19, B21, B22, B23		
A6	Yes, 3 + 5 = 9	Miscalculation
A5, A11, A22.	No, they presented drawings	–
A4, B8, B10, B16, B19	No, they presented correct result	–

Source: Elaborated by the authors

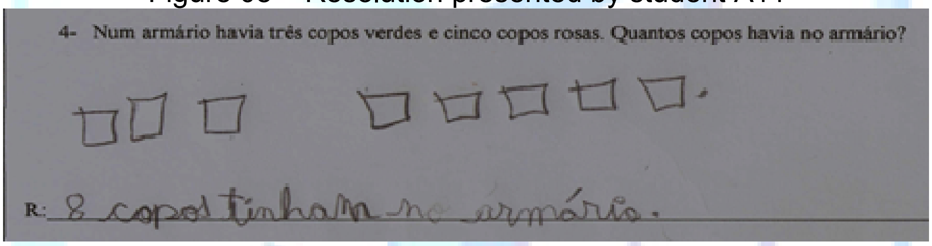
In this problem, we can see that 18 students from 2nd year and 19 from 4th year had success in the resolution process; they solved the problem correctly and presented coherent answers, according to the resolution of student B17, shown in Figure 07. Students A4, B8, B10, B16, B19 did not register for resolution, they only presented the final answer. Only students A5, A11, A22 resolved using drawings, as shown in Figure 08.

Figure 07 – Resolution presented by student B17



Source: Elaborated by the authors

Figure 08 – Resolution presented by student A11



Source: Elaborated by the authors

In sequence, an analysis of resolutions of problem 5 is shown.

Table 14 – Problem 5

Problem 5:	Karina has 9 pencils and Ana has 6 pencils more than her. How many pencils has Ana?
Correct operation:	9 + 6 = 15
Characteristics of the problem:	Positive comparison - search for the referred - 2nd extension.

Source: Elaborated by the authors

Table 15 – Analysis of problem 5

Student	Did he/she record an algorithm for resolution? Which?	Mistakes committed
A1, A2, A7, A8, A9, A10, A12, A13, A16, A17, A19, A20, A21, A23, B1, B2, B3, B4, B6, B7, B8, B11, B12, B13, B14, B15, B17, B18, B20, B21, B22, B23.	Yes, 9 + 6 = 15	–
A11, A22.	No, they presented drawings	–
A 6, B5	Yes, 9 + 6 = 3	Miscalculation
B9	Yes, 9 – 6 = 3	Sign of algorithm

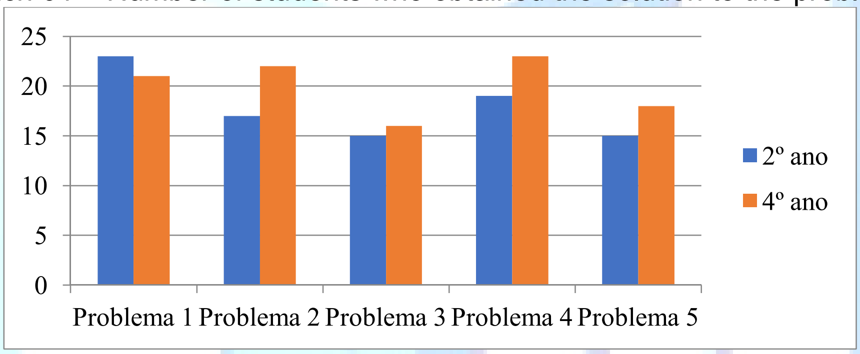
A14	Yes, 9 + 6 = 16	Miscalculation
A15	Yes, 9 + 6 = 25	Miscalculation
A18	No, they presented correct result	–
A4, B8, B10, B16, B19	Não	

Source: Elaborated by the authors

It was observed that 14 students from 2nd year and 18 from 4th year were successful in solving this activity, presenting the calculation and correct solution. Among these, only two students from 4th grade, B8 and B17, indicated the number 1 at the top in the resolution process, which corresponds to the addition of a dozen. Students A11 and A22 also achieved the correct solution, using drawings to represent the situation.

Students A6 and B5 constructed correctly the algorithm, but even with the indication of the plus sign, a subtraction operation was performed. Students A14 and A15 also performed an addition operation with the data arranged in the problem, but incorrectly: student A14 added two tens instead of one (9 + 6 = 25) and student B9 used the subtraction algorithm and presented the answer “Ana has 3 more pencils than she”. Students A4, B8, B10, B16 and B19 did not solve the problem.

Grafich 01 – Number of students who obtained the solution to the problems



Source: Elaborated by the authors

Graph 01 shows the number of students who were able to solve the problems, with the aid of algorithms, through drawings or just the correct answer. In problem 1 of the type: positive transformation, search for the final state, 2nd year students showed slightly higher performance, in the resolution process, than 4th year students. Still, the 2nd year class performed better on this problem when compared to problems 2, 3, 4 and 5.

The good performance of students in this type of problem may occur due to the semantic congruence between the keyword "won" and the addition operation, as well as the problem dealing with the same object, ‘small figures’. The lower result of the fourth year may be due to the fact that the teacher can encourage the resolution of more complex problems and the student's expectations regarding the problem may be different.

With regard to problem 2, which is configured as a composition: search for one of the parties, - we can see that the students of the 2nd year had lower performance in solving this problem, compared with the performance of the 4th grade class. On the other hand, the 4th year class increased the performance in solving this problem compared to the performance of the class in problem 1. In problem 2, the data do not refer to the same object and there is no enunciation of keywords such as “Won” or “lost”, for example, that may influence the student in choosing the signal for the algorithm. This problem is considered to be semantically congruent, since the order of numbers in the statement prevails in the assembly of the algorithm.

The performance of both classes decreased in solving problems 3 and 4, such as: negative transformation, search for the initial state and composition, search for the final state respectively. Among these problems, only in problem 3 there is a presence of semantic incongruence, in the keyword "gave" that may influence the student to perform the subtraction operation instead of an addition, the correct solution to the problem. These results show coherence with other researches in the area. We quote the result obtained in one of them:

> Problem 3 marks a sharp drop in the percentage of correct answers, especially with regard to the first three series, when the indexes, in relation to the first two, fall by at least 28%. This result confirms our hypothesis that the correctness here would be inferior to the two previous problems, since it requires more sophisticated reasoning. In fact, the child is asked "how many more balloon-toys can he buy", and yet, to resolve it, it is necessary to calculate the difference between the money that the two girls have in the bags [...] The problem’s difficulty depends on the child's ability to establish a relation between subtraction and addition [...]. (MAGINA, S; CAMPOS, T., 2004, p. 65).

Finally, with regard to problem 5, type Positive comparison, with the referent and the relation known, it was found that the 4th year students also obtained greater performance. Thus, it is concluded that, in general, the 4th year class presented greater success in resolution process, in solution found and answer presented. One believes that this fact occurred due to the level of knowledge provided by a bigger number of experiences. However, it is relevant to highlight that the 2nd year students did not obtain a considerably lower performance on problems 2, 3, 4 and 5 and even achieved a better result on problem 1.

Final Considerations

The research was carried out based on Vergnaud's Conceptual Field Theory with contributions from Magina *et al.* (2010), specifically with regard to the classes of problems of Additive Structures. From this study, five additive problems were developed, belonging to the different classes and applied to students of 2nd and 4th year of a municipal school in city of Campo Mourão-Pr. This research aimed to analyze the statement of problems and the students' strategies in face of an activity applied to 46 from 2nd and 4th year elementary school students.

The 2nd year students had more hits on problem 1, called positive transformation, search for the final state - prototype and less hits on problems 3 and 5, called, respectively, negative transformation, search for the initial state - 4th extension and positive comparison, search for the referred - 2nd extension. The 4th year students presented more correct answers in problem 4 - composition, search for the whole - prototype and less correct answers in problem 3.

The students, mainly in the 2nd year, represented their resolutions to the problems through drawings. It is conjectured that these students feel more confident in carrying out the two-way correspondence counting process: quantity of represented objects associated with the set of natural numbers. This fact that does not happen very often in activities analyzed of 4th year students, who carry out the resolutions through the algorithms, mental calculation and representing the quantities through the numerical record.

It is observed that the students had calculation errors when the statement of the problem was semantically incongruous with the elaboration of the algorithm, such as the statement of problem 3, which justifies the lower number of correct answers in both classes to this problem. Thus, we consider that students have difficulties in the interpretation of statements, especially when the results are semantically incongruous. Thus, it is expected that mathematics teachers develop with their students different classes of problem situations, which enable the interpretation of semantically incongruous statements.

When analyzing the students' strategies for solving the problem, several errors were found, such as: counting errors, errors related to the positional value and errors related to the inverse operation. This leads us to realize the importance of the teacher when he presents different problem situations, for these present different concepts and schemes, which can enable the student to develop cognitive and overcome difficulties in mathematics and, thus, contribute to his training as citizen.

These results converge with those achieved by Mendonça; Pinto; Cazorla; Ribeiro (2007) with 1803 students from the state Bahia and São Paulo enrolled from the 1st to the 4th grade (2nd to 5th grade), in which 12 problems of additive structures classified in composition, comparison and transformation were applied. As well, these results converge to results achieved by Magina *et al.* (2010) which involved 1021 students enrolled to initial years in public schools in state Bahia.

In problem 1, prototype type, two important situations occur: the 2nd year students obtained a better result than the 4th year students. This result may be due to an intense work with this type of problems with the students of the 2nd, who also obtained a good result in problem 4, another prototype.

We also highlight the fact that the result of this problem, for 4th grade students, was less positive than for the other problems, approaching the level of correctness of problem 3, of 4th extension. This fact draws attention and raises some questions: is it possible that this type of problem has not been sufficiently explored? Is it just a time gap between school age and exercise the task on this type of problem? Or, could there have been an external factor, for example, an environment that did not favor the concentration of students at the beginning of the activity?

The results obtained with the application of the problems indicate that the Theory of Conceptual Fields helps us to understand how the process of conceptualizing mathematical knowledge takes place and which schemes were used by students to solve the activity.

The teaching and learning processes therefore depend, to a large extent, on the awareness and professionalism of the mediators, their training and their professional perfecting, in order to better know and understand the theories that Mathematical Education seeks to appreciate.

We believe that the social, economic and pedagogical contexts of the students who participated in the research may have influenced the results obtained. This raises the need for researches that identify the similarities and differences among the results of studies involving the addictive conceptual field, with the aim to indicate how these factors are decisive in teaching and learning processes of mathematics.

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