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Contextualizing geometrical transformations for Early Childhood education

Contextualizações de transformações geométricas na Educação Infantil

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ABSTRACT

This study seeks to describe the value that future preschool teachers grant to certain contexts in the construction of the notion of transformation. It is analyzed a professional task in which a reflexive view of two narratives of school experiences is requested carried out with children from 2 - 8 years old about geometric transformation. The results show a weak knowledge of the properties and mathematical definitions of the transformations. From the contexts analyzed, motivation is valued more than mathematical constructed objects. The future teachers have difficulties in the recognition of adequate phenomena to contextualize the transformations and fail to justify the value of the experiences in order to promote the geometric reasoning.

KEYWORDS: Early Childhood Education, Geometric transformations, Prospective Teacher Training

RESUMEN

En este estudio se busca describir el valor que futuros docentes de educación infantil otorgan a ciertos contextos en la construcción de la noción de transformación. Se analizan las producciones de los participantes a una tarea profesional en la que se pide una mirada reflexiva de dos narrativas de experiencias escolares realizadas con niños de 2 – 8 años sobre transformaciones geométricas. Los resultados muestran un conocimiento débil de las propiedades y definiciones matemáticas de las transformaciones. De los contextos analizados se valora más la motivación que los objetos matemáticos que pueden ser construidos a partir de ellos. Los futuros docentes tienen dificultades en el reconocimiento de fenómenos adecuados para contextualizar las transformaciones y no consiguen justificar de forma argumentada el valor de las experiencias para fomentar el razonamiento geométrico.

PALABRAS-CLAVE: Formación de profesores, educación Infantil, transformaciones geométricas

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RESUMO

Este estudo procura descrever o valor que os futuros professores da educação infantil dão a determinados contextos na construção da noção de transformação. As produções dos participantes são analisadas para uma tarefa profissional em que se requer uma visão reflexiva de duas narrativas de experiências escolares feitas com crianças de 2 a 8 anos sobre transformações geométricas. Os resultados mostram um conhecimento fraco das propriedades e definições matemáticas das transformações. Dos contextos analisados, a motivação é mais valorizada do que os objetos matemáticos que podem ser construídos a partir deles. Os futuros professores têm dificuldades no reconhecimento de fenômenos adequados para contextualizar as transformações e não justificam o valor das experiências para promover o raciocínio geométrico.

PALAVRAS-CHAVE: Educação Infantil, Transformações geométricas, Formação de professores

Introduction

Current curricular approaches focused on the development of competencies as well as various investigations have pointed out the relevance of the study and analysis of contexts in the construction of mathematical and scientific meanings at different levels. Authors such as Davis & The Spatial Reasoning Study Group (2015) argue that this is key to the development of geometric thinking, particularly when studying geometric transformations. However, they also point out that the study of these notions at early ages is not usually taken into account.

In the field of teacher training, it is considered necessary to clarify the nature of mathematical and didactic knowledge about geometric transformations (GENKINS, 1975) as a preliminary step to the analysis of the difficulties in teaching and learning them. Regarding geometry, Espinoza, Barbé and Dinko (2007) point out that teachers tend to postpone their teaching due to the lack of mathematical and didactic training they have on this topic.

Sarama and Clements (2009, 2011), argue that geometry and spatial concepts are often ignored or minimized at the beginning of school education, and this can be explained, by the teachers' conception, that suppose that children do not they can learn certain contents because of their complexity and level of abstraction, or because teachers have difficulties in building geometric learning opportunities.

On the other hand, Canals (1997); Hoffer (1981); Clements & Battista (1992); Battista (2007); Giménez and Vanegas (2007) and Alsina, Novo and Moreno (2016) state that it is important to promote cognitive processes such as visualization, reasoning and the representation of geometric objects from the earliest ages, in order to carry out quality training. In this same sense, Harper (2003) alludes to the need for the formation of primary school teachers to develop informal geometry experiences supported by computer

visualizations. While Thaqi and Giménez (2014) point out the relevance and need to consider contextualization that imply cultural aspects.

We therefore consider it pertinent to analyze how future teachers of early childhood education recognize the mathematical knowledge associated with certain phenomena, particularly how they relate different contexts with certain geometric notions. We assume that if future teachers recognize the characteristics of good contexts and the mathematical ideas they involve, they can be aware of their potential modeling and, therefore, design rich and appropriate activities for children to develop their geometric thinking.

In this study we seek, on the one hand, to describe what mathematical ideas a group of future teachers of early childhood education associates with school experiences developed and narrated by practicing teachers, and on the other, to recognize what value is given to the context of said school experiences in the construction of the notion of transformation.

This text is organized in four blocks, this brief introduction, a second block that reflects on what phenomena are suitable to be associated with the notion of transformation, a third in which some important features of the exploratory study carried out with future teachers of infant education and, finally, in the fourth block some results and final considerations.

Natural phenomena and contexts for working with transformations

From realistic mathematical education, it is considered relevant to associate the phenomena with the mathematical ideas that underlie it (FREUDENTHAL 1983, 1991) And, this as it is proposed by Alsina, Novo and Moreno (2016) is key, to identify and use good contexts in Infant education. Contextualizing is important because in order to develop geometric thought it is necessary to start from reality initially and then arrive at abstraction. The work from contexts of everyday life can be an appropriate way to build mathematical notions with children of the first ages. These contexts can motivate students; can help them understand why mathematics is useful and necessary; and they can contribute to the fact that students understand how mathematics is used in society and in their environment (REEUWIJK, 1997).

We consider as proposed by Heuvel-Panhuizen (2002) that the value of context is to be a source for mathematization, and that this process can be understood as a search and problem-solving activity, but also as a source of organization for a theme as reinvention

process. In the case of transformations, various phenomena in the real world evoke the idea of symmetry and projection. Thus, observation of space is a fundamental process to be developed in Early Childhood Education to initiate and promote the construction of children's mathematical ideas. Itacarambi and Berton (2008) point out the importance of recognizing transformations in nearby environments. Also, Veloso, Bastos and Figueirunhas (2009) highlight the need for the use of manipulative materials to study symmetry, rotation and translation

A look at artistic contexts can be an appropriate scenario to study the characteristics of objects and begin to recognize the properties of transformations. Two concrete examples of this are: the banners of the Festivities of San Juan de Volpi, in which various symmetries can be seen, and the sculptures by Claes Oldenburg where processes of enlargement or reduction of objects that maintain their shape but change in size can be seen (See figure 1).

Figure 1 - Examples of designs of children inspired by the work of Volpi (left). Sculpture from Oldemburg in the Art Museum of Denver (right)

Source : The authors

Barbosa (2008) states that contextualization establishes a dialogue with works of art, facilitating the reading of images to recognize geometric transformations. In our case, in previous studies, we have used the artistic context to work different geometrical notions with students aged 11-12 (BADILLO, GIMÉNEZ, VANEGAS, 2011). We believe that it would be appropriate to use these contexts with children of early ages to promote processes such as visualization. The look at the transformations should begin with an exploration in which the children describe what they see and what they recognize that is repeated in different works.

Villarroel, Nuño, Anton, Zuazagoitia (2016) show that even before seven years of age children express pictorial abilities, among which is the spontaneous representation of symmetries. In the real world, relations of physical and aesthetic balance can be considered,

typical of many natural phenomena such as natural mirrors or observation of the symmetry of the bodies of some animals or plants. These phenomena should have a significant influence on the processes of teaching and learning in children where the exploration of biological phenomena is associated with the study of geometric notions that allow and improve their understanding.

As already mentioned, an adequate context that allows demonstrating the transformation between an element and its counterpart is the use of mirrors. In these cases the non-immediacy of the phenomenon suggests a visually independent initial and final idea. In children's education this idea can be combined with the introduction of stories where characters or objects must be searched, for which tools such as mirrors can be used (see Figure 2). The action of drawing what is discovered, what is seen, offers the opportunity to materialize the transformation.

Figure 2 - A tale about the use of mirrors (left) and a child designing with magic mirror (right)

Source: The authors

In everyday situations with mirrors, young children in general are attentive to repetition and change, more than in the phenomenon itself (something that I put in front of the mirror, it is seen inside the mirror, if we move an arm, the arm moves also in the mirror, etc.). But, we must go further and use the possibilities offered by this resource to work and raise other types of questions, for example: Where can we put the mirror to obtain something we do not see? With this type of experience, it is not only easier to deepen the notion of repetition, but also the recognition of the value of the axis of symmetry (WALTER, 1985).

Dos Santos and De Melo (2012) point to paper folding as another interesting context to evoke symmetry. In this same line Thaqi, Giménez, Aljimi (2015) expose an experience using paper folding in teacher training. In effect, bend with successive parallel axes, serves to

visualize that given an initial figure (cropped or drawn) when unfolding the sheet of paper (that looks like an accordion), it allows to see the repetitions, between which some figures appear upside down, and others in the same sense. This is also shown in movement analysis tasks with letters. However, the motor problem that underlies this type of activity should be considered when developing it with young children. What is usually done as an activity in children's education classes by folding a sheet, almost always in half, to "verify" if a figure is symmetrical, does not support the view on the properties of this notion, especially if it is not It is done with an exploratory approach and accompanied by good questions.

A phenomenon associated with cultural aspects rarely used in classes with children aged 3 to 6 years is the use of rollers or cylindrical seals that formerly used different indigenous communities for body painting or textile printing. This activity is where you can recognize more clearly a module that repeats itself. Of course, if it becomes clear that what appears is a translation. In the case of symmetry, it can only be recognized if the figure engraved on the roller is symmetrical.

Figure 3 - Different rollers/ cylindrical rollers used by indigenous communities in Colombia

Source: The authors

In the case of the roller on the left side in Figure 3, a vertical symmetry can be recognized if the roller is taken and used horizontally. And a horizontal symmetry can be observed if the roller is used vertically. We think that this is an interesting context, especially in Latin America, where in addition to approaching the study of some traditions of indigenous communities, we can analyze their designs and the mathematics involved in them.

Analyzing a professional task with future kindergarten teachers

Following Ponte Segurado and Cunha (2003), a professional task is proposed to analyze and observe the content of two school experiences that are narrated in an article

format. The first of these narratives is an explanatory dossier of a school experience, which is prepared by a teacher with 20 years of experience in early childhood education. In the second narrative the teacher who develops the experience shares the writing with a researcher in mathematical education.

An ethnographic case study is carried out, which focuses on content analysis on the written productions of a group of future teachers. A methodology similar to that used by Atherton et al. (2018), in effect, in this case a descriptive analysis is carried out, in which the researchers recognize characteristics of the answers given by the participants to a professional task. This analysis is carried out as a first step to an explanatory analysis that is carried out later and that is not detailed in this article.

From the point of view of training, it is intended to promote in future teachers their competence in didactic analysis, particularly in aspects related to the recognition and analysis of good teaching practices. It is intended that they identify (in the two narratives) key ideas about the teaching and learning situations developed, and that they value their potential.

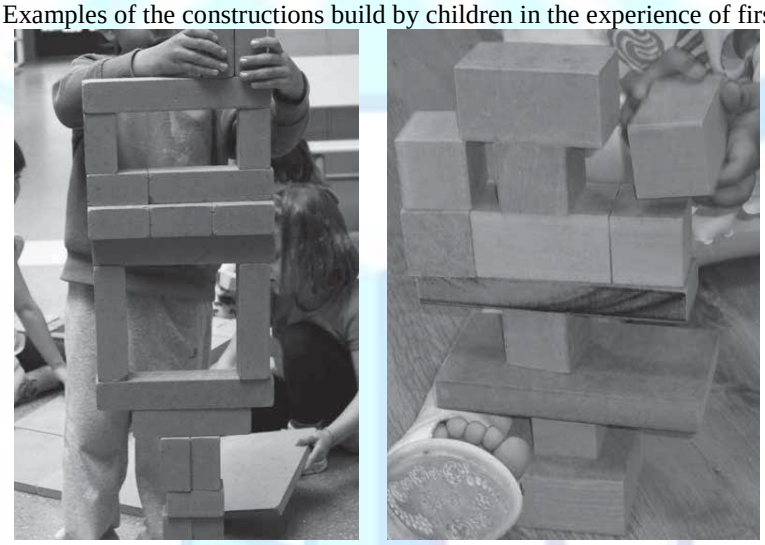
These narratives were selected because each of them describes a story that recognizes a certain difficulty or learning challenge; the training purposes of the authors are evident, and the initial challenge is explained. When presented in an article format, the selected narratives allow us to "enter" a class practice that has been analyzed and in which important information is highlighted. In addition, the two experiences described are a source of learning of what we want that future teachers can also do in their professional life.

Data are collected with a group of 46 future teachers of the subject: Mathematics, Experimental Sciences and Education of the Degree in Early Childhood Education of the University of Barcelona. This subject corresponds to the second year of training and is of a multidisciplinary nature.

The objective of the professional task is to recognize the specialized mathematical knowledge of future teachers on the transformations. We are interested in identifying if the participants recognize the value of the phenomenon associated with the mathematical ideas that are presented to the students of Early Childhood Education in each of the experiences. It is proposed to future teachers to read the two narratives in order to observe their ability to recognize how certain contexts and tasks contribute to the development of the notion of transformation.

The first narrative entitled: "*Spontaneous appearance of symmetrical constructions during free play in Early Childhood Education*" (DE CASTRO, 2012) describes a school experience developed with children from 2 to 6 years old. In this proposal, based on free exploration activities, equilibrium and construction situations are studied in which the intuitive recognition of symmetry is identified. In fact, a construction activity is explained with the Froebel woods, made with groups of children organized in different age ranges (2 to 3 years, 3 to 4 years, 4 to 5 years and 5 to 6 years). In this story the evolution of the different school activities is shown and the actions carried out by the children in each of them is described.

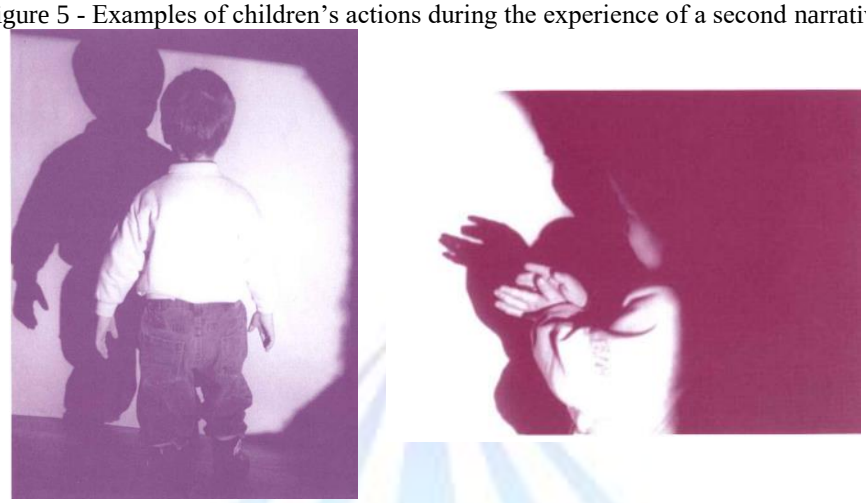
Figure 4 - Examples of the constructions build by children in the experience of first narrative



Source: De Castro (2012)

The second narrative called "The shadows in Early Childhood Education 0-6" (DOLCI, 1984) describes a school experience developed with children aged 5-6 years, in which they experiment with shadows (produced by the sun and a lamp) and evokes the notion of homothetic transformation or projection. In this narrative a description of what happens in the process is made and the practical possibilities of this type of activities are highlighted.

Figure 5 - Examples of children's actions during the experience of a second narrative



Source: Dolci (1984)

We consider that to be able to observe and study this type of narrative in the initial formation of teachers of early childhood education is relevant, since it shows, as it is possible to approach in initial ages, daily situations in which phenomena of the physical world and transformations are studied.

To guide the reflection on the narratives, the future teachers are asked to answer by pairs, a questionnaire that is structured in four fundamental questions. These questions are presented below:

- Before starting the analysis of school experiences, look for what mathematical properties are associated with the notions of symmetry and projection. Subsequently, identify which of these properties are worked in each of the two experiences. Justify your answer with solid arguments.
- What physical-natural phenomena are put into play in the experiences described in each of the articles to work with children in early childhood education the idea of transformations? Explain some phenomenon that is not mentioned in the two experiences, and that could be taken into account for the work with symmetry and projections.
- Explain what aspects of contextualization are observed in the tasks developed in each of the experiences.
- How would you justify a colleague who, with these activities, promotes geometric reasoning?

Among the properties expected to be observed and / or used in the case of symmetry, we find that the symmetries are inverse movements (they change the orientation of the figures and the rigid movements maintain the metric properties of the figures). Our hypothesis is that there is a lack of connection between the personal conception of future teachers about transformations and the institutional conception, which contributes, as stated by Blanco,

Godino and Diego-Mantecón (2018), to the fact that make a good association between the real context and the mathematical object addressed.

Some findings from the professional task

Below we show some of the results obtained in the study organized based on the questions proposed in the professional task. It begins by discussing some aspects of the content and properties of the transformations recognized by future teachers. Then some of the interpretations they make of the contexts are shown. Likewise, key elements of the phenomena - mathematical object relationships identified are described; and, finally, the elements that future teachers refer to when they discuss the potential of geometric reasoning of experiences are described.

a) Content and properties of transformations

We consider it important to mention that future teachers recognize their weaknesses in the mathematical knowledge involved in the experiences. In effect, some of them requested explanations for a task they considered difficult. Initially they decide to search the internet (on pages such as Wikipedia) and other sources of information to "refresh" their mathematical ideas or search for those they did not know. This is evidenced in comments like the following:

"We have strong gaps in many aspects of mathematics as it is the subject of transformations. We realized that we do not know about the properties of symmetry. We never hear about homothety. We did not understand many things we read on the Internet. We did not know if we were responding well to some questions you asked us. We were clear that we liked the experiences when reading them, but we had never analyzed such ideas in Infantil "(Couple 2).

In some cases they expressed that they were being asked too much theory, and that what was being done was far from the practice that they saw developing in schools.

Regarding the properties of the transformations, 40% of couples correctly describe that the shadow retains the number of sides of the figures but changes the sizes. The fact of using material such as cubes or similar pieces of wood can casually create symmetry simply because the child puts one piece on top of another. The materials used influence the fact of creating symmetry, equal pieces, wooden blocks, symmetrical shapes, etc. Another example

would be a construction made of equal pieces of wood. In many constructions we can observe bilateral and rectangular symmetry, and that leads us to think that it is not a casual but intentional symmetry. In effect, the future teachers mention "We found many pieces placed respecting the symmetry" or "there are too many elements respecting the symmetry as to be considered accidental" (Pair 2).

One of the pairs of future teachers is able to establish correspondences between the properties of symmetry (which they found and listed) and some aspects described in the narratives. Thus, in the case of narrative 1 they state: "*For example, when children play freely with the cubes, they divide a figure with a triangle, whose base is a square, thus forming two equal squares (property 1 of which in the symmetry, a straight line can divide the figures or objects into two equal figures); in figure 8, the center of the square is the center of the symmetry (property 10), and in it, we can see how the center of the figure is between point A and A '(property 4 which says that the point O , center of symmetry, is between any point A and its symmetric A 'at the same distance.The symmetric of O is the same)* "(Pair 4)

On the other hand, regarding the experience on shadows (narrative 2) they recognize the property of conservation of the form. This is evident in comments such as: "*By themselves they are able to recognize that, approaching or moving away from the point of light (projector), their shadow changes in size and therefore shadows can represent projections that do not represent reality, they can see a giant hand but in reality it is only a hand of normal size* "(Couple 22).

b) Interpreting the idea of context

Analyzing the explanations of future teachers we can say that they mostly associate the context with the "place" in which the activities occur. There are very few allusions to the context as a scenario that allows the mathematical object to be concretized. In effect we find expressions such as "*It is done in the classroom, so it already contextualizes children in a known and limited space*" (Pareja10). Also statements in which contextualization is interpreted as a methodological process of management in the classroom, for example, "*In the second activity, we do not see contextualization, but a process of decontextualisation and disinformation, since at no time the teacher makes an approach of the activity* "(Couple11).

Some couple alludes to the fact that the context allows the recognition of properties. Thus, for example, they point out that *"... horizontal and vertical stacks influence the*

properties of symmetry. For example, in the constructions of horizontal stacks, the tendency to add equal pieces to one side and another side of a structure, evokes the equality with respect to the axis "(Couple 13), Confuse the processes of contextualization and connection, in effect, to the Inquiring about aspects of the context identified in the school experiences mention: "*They are associated to connect previous knowledge with new ones. In this way, you can see that there are not very large separations in the various blocks of content* "(Pair 8). On the other hand, some pairs of future teachers, allude (although not directly) that the interdisciplinary is mediated by the contexts. This can be evidenced in responses of the type: "*In addition, these two experiences relate everyday situations, social or cultural, with mathematical knowledge*" (Couple 7). Other future teachers try to specify the mathematics that is promoted with these contexts: "*appear symmetry, proportion and geometry*" (Couple 7). But in most explanations, it is not explicitly explained how the meaning of these mathematical notions is treated in relation to the phenomenon studied in each experience. Rather, the meaning is juxtaposed to the context without relating them.

c) Relation between phenomena and mathematical objects

One of the relationships identified by future teachers is that which associates the notion of symmetry with equilibrium, for example, with respect to the first narrative, they comment: "*An important phenomenon that appears clearly at the moment when students begin to make vertical constructions. It is striking that they themselves place their hands so that when they are removed, they do not move and, therefore, the construction does not fall.*

In the case of the second narrative, they identify that in the experiences with shadows the idea of transformation is worked on, for example they say, "*In the projection with the shadows, the symmetry works very significantly, because every movement that the child makes it is projected by the shadow* "(Couple 1); "*When children put themselves in front of the light in the dark, the shadow is the symmetrical image of the child" (Couple 5) and "the children see exactly their shadow reflected, and see the symmetry in a sagittal plane*" (Couple 7)

Other future teachers make affirmations although erroneous as a consequence of not establishing a good observation of the physical phenomenon, let's see for example the following statement: "*In the article of the shadows, it can be clearly observed that bilateral symmetry is worked, that is, the reflection that occurs on a wall, in this case, when the children are placed in front and due to the light, their shadows are reflected on the white wall. For example, when the teacher tells them to touch the head of their shadow, there is a*

movement that is immediately reflected in the shadow and in which the mathematical concept is worked on in the human body "(Couple 17).

Although at times of the training has been discussed with the future teachers on the notions of similarity and homothetic, when doing the analysis of the narratives do not get to associate these mathematical objects with the shadow phenomenon. Another evidence of this type of error, we find when they say that *"... at the time that a child is placed in front of the projector and the wall we understand that the center of his body is symmetrical with each extremity of his own both in real body and in the shadow* "(Couple 8).

The difficulties to establish relationships between the phenomenon and mathematical object is also evident when referring to the first narrative. For example, some future teachers talk about proportion where there is not, is the case of the Couple 5 that states, "*To the Once the symmetry is worked, the proportion is also worked, that is, in all the constructions that are shown in the article, the children place geometric figures of different measures, proportions, to build their idea, but as they grow, they realize that for there to be a proportion, they must place the pieces that are the same size, so that a balance is produced and their construction does not fall ".*

This type of comments makes us think about the need to experiment with the future teachers the activities (carried out with the children) described in the narratives, to analyze and discuss from the own experience the equivocal statements. It also makes us reflect on the need to discuss the colloquial language that is sometimes used when talking about mathematics or vice versa.

d) About the potential of reasoning

When future teachers are asked to explain why these narratives promote the development of geometric reasoning, in general terms are alluded to expressions that evoke the observation of objects and are not alluded to processes of reasoning and proof. For example, it is stated: "*with shadows we can identify an object, recognize it among others, find differences when comparing it with other objects, characterize it with its own features, find properties, etc*." (Couple 11). In other cases, statements are made but with arguments little related to reasoning, for example: "*In addition, the analysis, organization and systematization of spatial knowledge gives rise to geometry, and all this is enhanced by these activities*" (Couple 15).

Some future teachers refer to the cognitive value of the tasks, but without emphasizing the reasoning as requested. In the activity with shadows, they recognize that the experimental process allows not only to identify the phenomenon, but also helps the children to create a habit of observation, to know the world around them, to become familiar with their own body and to identify changes and explain them. An example of this are the following comments: "*Children discover the shadows of the way in which the teachers show it to them and understand it according to the activities they do and their demonstrations*" (Couple 7) and "*In addition, by moving or setting aside they have been able to observe a change in the shape of their body in relation to their shadow and they themselves have been able to position themselves so that their body can be seen completely on paper*" (Couple 8)

Final remarks

The professional task developed with the future teachers addresses a relevant mathematical topic that usually is not developed in the initial stages, therefore having experiences that allow to recognize the intuitive knowledge that children have in this respect, is fundamental for the planning and implementation of proposals of school education focused on the teaching of transformations.

The selected narratives have allowed to have different types of examples to approximate the future teachers to the comprehensions of children about different geometric notions. Likewise, the fact that the experiences are based on manipulative and exploration activities is important, since they present a set of rich activities that promote the development of geometric reasoning and incorporate a globalized and competently approach, as proposed in the curricula current What we consider is fundamental for the initial training of teachers.

The professional task has allowed future teachers to notice that in the observation and analysis of everyday phenomena can be recognized key aspects for the study of mathematical notions. However, as has been shown in the previous section, future teachers have difficulties to establish adequate relationships between concrete actions of school experiences and arguments or reasoning of a geometric type. For example, the participants of the study neither perceive in the second narrative, that the action of moving away or approaching the wall to see what happens with the shadow, is evoking a property of the transformation. Nor do they interpret that in the first narrative when a student in the construction game puts pieces (of the

same characteristics) on one side and another, is exhibiting his intuition about the relationship symmetry and equilibrium, where he assumes that for the latter to be maintained, it is necessary to locate equal parts on both sides of an axis (immaterial).

In the productions of future teachers we can see that when they allude to the mathematical elements involved in the narratives, in several cases, they use an erroneous lexicon and terminology, some of these uses are influenced by the searches they make of terms in dictionaries and not in mathematics or mathematics didactics books. This also leads us to think about the need to discuss with our students about the relevance of the sources, since although authors and texts are suggested throughout the course, they prefer to make other consultations to solve more "immediately" your doubts.

From the two analyzed narratives, the future teachers recognize the value of the didactic proposals developed in each case. However, they fail to distinguish the level of depth in the reflection in each of them. In very few cases do they allude to one of the narratives being more descriptive and that the other, in addition to explaining the experience, analyzes and reflects on what happened.

The results found reaffirm our idea that it is key in initial training that future teachers of early childhood education know interesting geometric experiences and activities. They should also have the possibility of having examples of good class analysis, made by research teachers who reflect on their own practice. It is clear that if we want to overcome the limited idea of considering that the importance of a context lies only in its motivating effect, we must place special emphasis on this notion and spend more time discussing and deepening the contextualization process and how it can be addressed in the class from the early ages.

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