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Situation of teaching involving the exponential function: meaning attributed by teachers

Situações de ensino envolvendo a função exponencial: sentidos atribuídos pelos professores

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ABSTRACT

This work presents the results of a master's thesis from UFPR. The research developed had as one of its objectives to recognize the meanings attributed by the teachers to exponential function, considering the analysis and elaboration of teaching situations. The methodology presented is guided by the assumptions of the activity theory, cultural history and teaching activity (AOE in Portuguese). It is concretized in the analysis of data collected from the interaction with teachers of the public school system in the "Pedagogical workshop of Mathematics" (OPM in Portuguese) of the Federal Technological University of Paraná (UTFPR), through audio/video recordings. From the results obtained in the research, we highlighted the transformation of the meanings attributed by the teachers to the exponential function during the planning and implementation of the learning triggering situations collectively elaborated. Considering the Historical and logical movement of the exponential function and using the teaching activity as a theoretical-methodological basis, thus being a possibility for the teacher to organize the teaching of the exponential function.

KEYWORDS: Teaching Exponential Function, Pedagogical Workshop of Mathematics (OPM), Teaching Oriented Activity (AOE), Learning triggering situations.

RESUMO

O presente artigo apresenta resultados de uma pesquisa em nível de mestrado que teve como um de seus objetivos reconhecer os sentidos atribuídos pelos professores à função exponencial, considerando a análise e elaboração de situações de ensino. A metodologia apresentada orienta-se pelos pressupostos da teoria da atividade, histórico-cultural e Atividade Orientadora de Ensino (AOE), e concretiza-se nas análises de dados coletados da interação com professores da rede pública no projeto de extensão "Oficina Pedagógica de Matemática" (OPM) da Universidade Tecnológica Federal do Paraná (UTFPR), através de gravações de áudio/vídeo. Dos resultados obtidos na pesquisa, destaca-se a transformação dos sentidos atribuídos pelos professores à função exponencial durante o planejamento e concretização das situações desencadeadoras de aprendizagem elaboradas coletivamente, considerando o movimento histórico e lógico da função exponencial e recorrendo à Atividade Orientadora de

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Ensino como base teórico-metodológica, podendo assim, ser uma possibilidade para o professor organizar o ensino da função exponencial.
PALAVRAS-CHAVE: Ensino de Função Exponencial, Oficina Pedagógica de Matemática, Atividade Orientadora de Ensino, Situações Desencadeadoras de Aprendizagem.

Introduction

The teaching of the function's concept is fundamental for Mathematics and Science in a whole. However, researches such as those of Silva (2014) and Pereira (2010) reports that the teaching and learning process has become a major concern for teacher and researchers due to students' distresses in understanding such concept, such as is also reported in the article by Trindade (1999). In the concept of exponential function specific case, it is observed the difficulty of students to recognize situations of exponential growth, frequently considering the emphasis in teaching for the substitution of numerical values in variables, establishing the law of function from values previously defined or even the mechanically construction of graphs.

It is a fact that this exponential, empirically-based approach to teaching, based on symbolic and apparent registers, does not support the appropriation of exponential function's concept by students in their theoretical form, and students do not understand the necessity of this function related to the phenomena of reality, nor in the human experience in its different epochs. Although there are several teaching researches like those of Pereira (2015), which emphasize the scope of the infinite applications of this function, that overpasses many areas of knowledge like Archeology, Architecture, Biology, Economy, Demography, among others.

Researches such as those of Pereira (2010) also present exponential and logarithmic functions' methodological approach in a conceptual and graphic perspective in High School and elaborate a sequence of activities using the software Winplot, and referenced in Polya[4] (1995) to solve problems and Miranda and Laudares (2009)[5] regarding focus on conceptual understanding.

Other researches such as Silva (2015) that focuses on the process of teaching and learning the exponential function by High School students in Brazil, or Matos (2014) who carried out an exponential and logarithmic functions' study, with the purpose to serve as a

[4]POLYA, G. **A Arte de Resolver Problemas.** Rio de Janeiro: Editora Interciência, 1995.
[5]MIRANDA, D. F.; LAUDARES, J. B. Informatização no Ensino da Matemática: Investindo no Ambiente de Aprendizagem. **Zetetiké**, Campinas – SP, v. 15, n. 27, jan., jun., p. 71-88, 2007.

support material to enrich and energize the classes. They reveal possibilities for the exponential function teaching, without reinforcing the manipulation of its technical aspects; however, these results are not manifested in classroom teaching yet.

The classroom practice evidences what the literature states regarding the difficulty of students to learn the concepts of exponential function, as well as the difficulty of teachers to maximize the teaching of this concept. The search for elements to overcome this difficulty triggered the need for the master's research (SILVA, 2018) that aimed to analyze the teaching situations of the exponential function considering its historical and logical movement.

For this work presentation, it was defined from the developed research to recognize the meanings attributed by the teachers to the exponential function, while elaborating learning triggering situations, resorting to the Teaching Counselor Activity (AOE) as a theoretical-methodological basis. The data were compiled in an extension project with professors from the public school system entitled *"Oficina Pedagógica de Matemática"* (Pedagogical Workshop of Mathematics) during the year 2016. The meetings of this project involved studies on the historical-cultural theory; activity theory; the teaching activity and the teaching of exponential function.

The first results of this work were presented at the VII SPIPEM (Seminário Internacional de Pesquisa em Educação Matemática) in 2019, and were expanded and reformulated here.

Theoretical Basis

The concepts related to the exponential function are considered scientific, represented by symbolic systems that mediate the action of man, with objects and phenomena. According to Vygotsky (2001), these concepts develop themselves in the subject by the path of teaching, intentionally and consciously, and by a deductive process. They do not relate directly to the object, but are mediated by other concepts.

It is understood that such concepts can be formed by the theoretical path of thinking (DAVIDOV, 1982), which differs from the empirical by operating with the concepts themselves that arise in the psychic activity of the subject and can be represented by different semiotic systems. Sousa and Moura (2016) based on Davidov (1982) emphasizes that traditional didactics do not consider the logical-historical movement of concepts development,

and thus do not reveal the essence of concepts reflected in teaching objects. The term "essence" for this theoretical line can be understood from the following statement:

> The development of the object's study, in turn, creates the indispensable premises for a deeper understanding of its essence, for, enriched with the knowledge of the history of the object, we must once more resume the definition of its essence, correct, to complete and develop the concepts that express it (KOPNIN, 1978, p. 186).

According to Kopnin (1978), "the logical reflects not only the history of the object itself but also the history of its knowledge" (p.186). Thus, the unity between the historical-logical movements reveals not only the external and perceptible nexus of the concepts but also the internal nexus.

The external nexus relates to the formal language of the concept, stripped of the human work that generated them, of the contradictions, unlike the internal ties that are permeated with history. The external nexus are explained in the classroom, from the symbolic aspects contained in the concepts. It is as if the symbols had a life of their own; speaking for themselves (SOUSA; MOURA, 2016). These concepts are presented, in their last stage of rigor, from experiments or memorizations, without considering the historical syntheses that led them to such a condition of systematization and rigor, highlighting the conceptual product and not the process of its constitution, or the needs that generated it.

External nexus in the exponential function's case are considered as the graphical and algebraic representation of function, normally identified and recognized by students, but which do not reveal their essential aspects or the necessity of such concepts in human experience. Internal nexus may include interest studies that historically reveal the need for a knowledge to organize its calculus.

According to Eves (1995) there are evidence of problems with interest dating from 1700 BC on a Louvre's board, which the Babylonians solved using linear interpolation. Another internal nexus can be considered by studies on arithmetic and geometric progression. According to Struik (1992) several mathematicians of the seventeenth century 'had been confronted with the possibility of coordinating arithmetical and geometric progressions, especially with regard to favor work with the complicated trigonometric tables' (p. 152-153). Such idea was appropriated by Napier to identify that in a geometric progression in which the numbers are powers, it is observed that the exponents form an arithmetic progression.

Shall then be considered the importance of understanding the historical and logical movement of the exponential function, in search of elements to organize its teaching and

analyze the teaching situations trying to overcome its external aspects, mechanics revealed in its apparent form. As educational implications, if the teacher organizes his teaching activity, relying only on the logical aspect of definitions and syntheses' form, he will lose all the rich historical process of constructing the concept. In the same way, if the teacher bases his activity only on the historical aspects, he will lose the logical aspect of the concept's evolution, being able to characterize the historical aspect as curiosity, as it happens in several textbooks (FRAGA, 2016).

A possibility of organizing the education to overcome the appearances revealed by the external links and considering the historical and logical movement of the concepts and the internal links of the exponential function presents itself with the *Atividade Orientadora de Ensino* (MOURA, 2010). This is based on the assumptions of the theory of activity (LEONTIEV, 1983) and on cultural historical theory and is characterized as "[...] that which structures teaching in order to allow subjects to interact, mediated by content negotiating meanings, with the objective of collectively solving a problem situation "(MOURA, 2001, p. 155).

Moura's Teaching Counselor Activity (AOE) has defined elements for knowledge's organization, among them stands out the teacher's intentionality; the collective process in search of possible paths to solve the problem; the analysis and synthesis of the resolutions found; the theory and practice's unit (MOURA; SFORNI; ARAÚJO, 2011). It also consider that the relationship between teaching and learning's activity allows the appropriation of concepts, and it is necessary to recognize the conceptual nexuses revealed through the logical and historical movement.

Targeting the concept's appropriation of exponential function by the student, the AOE considers that the student's need can be revealed by the triggering situation of learning (which can be a game, a virtual story or a daily situation) and has as its essence the human need of knowledge's construction. In this sense,

> The main objective of this is to provide the student the need to take ownership of the concept so that his actions are conducted in the search for the solution of a problem that mobilizes him for learning activity - the knowledge's appropriation (MOURA et al., 2010, p. 221).

The "game" feature is something much mentioned in the teaching and learning processes' discussion of Mathematics. The game should place the student in the face of a

problem situation similar to that experienced by the human being when dealing with concepts of mathematics (MOURA et al., 2010).

However, the "emergent situation" is the one that emerges from everyday life, making an educational practice possible by placing the student in the face of the need to experience significant problems' solutions to them.

In turn, the "virtual history is defined as:

> [...] a narrative that allows the student to be involved in the solution of a problem as if it were part of a collective that seeks to solve it for the purpose of satisfying a certain need, similar to what may have happened in a certain historical moment of humanity (MOURA et al., 2010, p. 105).

It is emphasized that Teaching Counselor Activity is a process that involves the teacher and the student and not an object (MOURA et al., 2010), and is constituted as a unit between teaching (teacher activity), and learning (student activity) (MOURA et al., 2010). The summary of this process is presented below (Figure 1).

Figure 1 - Relationship Between Teaching Activity And Learning Activity

Source: Moraes (2008, p. 116)

The AOE's elements and the historical and logical movement supported the work carried out in the extension project "*Oficina Pedagógica de Matemática*" (OPM), from which data were collected to carry out this research, making it possible to reveal the meanings attributed by teachers to the exponential function while analyzing teaching situations and elaborating situations that triggers learning.

Conditions, actions and methodological operations: The work with the teachers in the Pedagogical Workshop of Mathematics

The Pedagogical Workshop of Mathematics (OPM) is organized as an extension project for teachers (public school system) offered by the Departamento Acadêmico de Matemática da Universidade Tecnológica Federal do Paraná (UTFPR - Curitiba). The general objective of OPM is to "promote among teachers' university, teachers of the basic education school system, the articulation theory / practice (praxis) that ground their actions within the teaching activity of mathematics". In the year of 2016, the data of this research were obtained from the development of the OPM that had the theme defined as "Exponential Function". The OPM 2016 started with 23 participants and ended with 8 participants, who attended 27 meetings held weekly at UTFPR facilities, in addition to subgroup meetings and intervention in the school environment.

It was debated in the project meetings, concepts of activity theory and teaching activity. From the fifth meeting, the participants were organized into two subgroups to continue with historical studies on the exponential function and curricular document studies, as well as to elaborate the situation that triggers the learning for intervention in a classroom. One of the subgroups elaborated a situation involving the understanding of the exponential function related to the water's boiling temperature and another group associated the situation with the measurement of an earthquake's intensity. The fact that they collectively elaborated the situations, discussing the teaching organization, made the participants incorporate not only practical aspects, but also theoretical ones that helped them in the senses' change related to exponential function.

The expression "sense" attributed by teachers refers to personal understanding, which involves relationships that relates to the exponential function's concept in their personal formation. What differs from the "meaning" of the exponential function that already has in its

relatively stable nucleus of understanding of this social and historical concept. As Oliveira (1997) differentiates sense of meaning:

> Meaning itself refers to the system of objective relations formed in the process of word development, consisting of a relatively stable nucleus of word comprehension shared by all people using it. Sense, meanwhile, refers to the meaning of the word to an individual, composed of relations that relates to the context of word's use and to the individual's affective experiences (OLIVEIRA, 1997, p. 50).

The participants' discussion was conducted considering the questions presented at the third meeting of the project and foreclosed at the last meeting: What is the relevance of teaching exponential function content? What are the main difficulties students have to learn this concept? What teaching resources and/or methodology do you use to teach exponential function?

All OPM's material was audio and video recorded. In order to identify the lines in the transcriptions, the following was used: Participant identification symbol, meeting number (E number in Portuguese); Identification of the audio or video number (A or V number); speech time beginning. For example, the recording (E1, V3, 2:30) indicates the third video of the first encounter and the beginning of the speech happening at two minutes and thirty seconds within that video. The questionnaire's registration will be indicated by Q (delivered by participants in E3).

The analysis presented here was essentially based on the records of the third (E3) and the last one (E27), highlighting the teachers' statements P1 and P8 in the transformation of their personal senses attributed to the exponential function, but we will also quote the teachers' statements: P4, P5, P6, P9, PO1, PO3 and AP1.

P1	Bachelor and graduated in Mathematics with specialization in Educational Law, she taught in high school classes (Mathematics and Physics) in the school system of Curitiba
P8	Licentiate in Mathematics and Pedagogy with specialization in Mathematics Teaching, and taught in classes of primary education in the Prefeitura Municipal de Curitiba
P3	Master's degree in Mathematics and Physics with specializations in Higher Education, Special and Inclusive Education, Psychopedagogy and Special Education teaching, with emphasis on Higher Skills and Giftedness and served in the Departamento de Educação Básica (Mathematics Team) of the Secretaria de Estadual e Educação do Paraná
P4	Licentiate in Mathematics with specializations in Psychopedagogy and Special Inclusive Education, at the time of the research, she taught at a periphery college in the school system of Curitiba, in classes of Mathematics Elementary Education
P5	Licentiate in Mathematics with specializations in Psychopedagogy, Epistemology and Mathematics pedagogical practices, Mathematical Literacy and Special Inclusive Education, at the moment she taught in a college of the periphery in the school system of Curitiba, in classes of Elementary and High School in Mathematics
P6	Graduated in Pedagogy at the time of the research, she worked as a pedagogical teacher at a traditional college in the Curitiba school system
P9	Master of UFPR at the moment of the research, graduated in Mathematics, professor of mathematics in the school system of Curitiba, teaches to classes of Elementary School

P01	Doctor and Master in Mathematics, Licenciated in Mathematics, teacher at an college education institution and organizer of the extension project Oficina Pedagógica de Matemática
P03	Doctor and Master in Mathematics, Licenciated in Mathematics, teacher at an college education institution and organizer of the extension project Oficina Pedagógica de Matemática
AP1	A student of the High School's 1st year in a Curitiba's traditional college, at the time he was in a Scientific Initiation's scholarship of High School

The analysis of the other meeting's speeches were also considered to help the understanding of the movement of planning and the sense that these subjects attribute to the exponential function during the process of elaboration of the situations triggering the learning. According to Leontiev (1983), the meaning is tied to the question of motive.

> Accordingly, what I really am aware, the way I become aware, and the sense that I have for consciousness is determined by the reason for the activity within which my action is embodied. "Therefore, the question of meaning is always a question about the motive" (LEONTIEV, 1983, p. 230, free translation).

In this way, it is intended to recognize the personal sense attributed to the exponential function as well as the reasons and needs of teaching this function considering these professors in teaching activity.

Revealing the meanings attributed by teachers to the exponential function

Discussions at the third meeting of the project were conducted looking for what participants understood about exponential function and its teaching. The data collected evinced that teachers consider the importance of teaching exponential function, and the need for application in the student's daily life,

> P8: what goes through my head is like this, the exponential function has much importance in everyday life even in the question of financial mathematics bank interest. Today everyone is indebted, right, this thing to bring to our everyday, has the biology part , there's the computer part and the high school student, they really the ace in their sleeves to decide what they want to do and what they're going to do, so I think it's important [...] (E3, V1 , 5:50).
> P9: [...] I think it is important to work the exponential function. It has a relation to the logarithm, it has all those things and also has the relation to the exponential function with application in other areas, like chemistry's biology in other areas, even if the student does not like it, see no use at all, it's important that he knows it

The P9 also responded writing about not seeing direct applicability in the student's life; however, she reported the importance of learning this concept for her formation.

> P9: In the student's daily life? Perhaps not, but one must think of ways to show the usefulness of these functions and in what areas they can be used, thinking that the

> student will continue his studies, it is interesting to see that many concepts/contents studied have utility and are used in other areas (Q).

The teacher P8 also considers as one of the students difficulties with learning exponential function the mathematical vocabulary: "I think, the terms, mathematical terms, for some students in general, they are very impoverished in mathematical vocabulary" (P8, E3, V2, 25:20). P6 commented on the importance of contextualization in favor of knowledge, to the detriment of meaningless formulas.

> P6: It is important the course proposal to reach out to them and explain why. Why do you have to learn this? The reason? Where are you going to use this? And sometimes the mathematics seems to be a seven headed beast for many people, and sometimes it is well at the base itself, at the base is where there they begin with a lot of difficulty and from there comes these formulas, with these situations and little experience and hence he does not hear this previous explanation, why, where, when [...]. Explain why I have to, because I have to go through it (E3, V1, 12:43).

In the course of the discussions, P5 does not recognize the importance of teaching the exponential function, putting the blame, in this section, on the school structure and the teachers.

> P5: [...] but I, particularly, do not see much relevance to the exponential function, why? Because we have a short time, the content of the first year is a lot for you to teach, our students today, I'm talking about the state students, they have no basis in math, they do not have basic math, maybe it's because of this teacher's exchange, some have more commitment, some less, and so it goes. So we do not have time to teach it, and when we get there in the first year, it comes with that idea of elementary school, that woohoo, you know, the student speaks, I didn't have class and a teacher helped to pass through the council , then they already come with this thing (E3, V1, 14:39).

Participants consider the need to explain why they teach exponential function, and associate this teaching with the training of exponentiation and nth root operations.

> P5: [...] we have content's excess and there is a lot of stuff there that the student will not use and we have to teach because it has all that plan, and we know it has, and when you get into the exponential function, I think, the only thing, I, so I wanted to do this to they understand a little better. The only relevance that I think it has, we work quite a part of the nth root and exponentiation with our student, and they do not have that basis (E3, V1, 15:25).
> P3: In the exponential function, we find the power's operation directly related and with this, the student have contact since the fundamental - final years. It is one of the teaching factors of relevance of E. Func. In the High School is to give/provide moment of applicability and deepening of the power's operation (Q).
> P3: Conceptual difficulties of the operation involved as: relationship of equality; decomposition into prime factors; decimal number system (in relation to decimal number decomposition); the number written in decimal form (Q).

Teacher P5 recognizes her own difficulty in teaching the exponential function. "Because I also have doubts about it, I know that I go there, I do power, I do this, I do that" (E3, V1, 16:45).

Several teachers commented that they do an exponentiation review. "Revising power, so they have a better basis to easily understand the exponential function" (P4, Q). P5 answered that she would do a review of exponentiation, nth root. She also commented that she would not go into this subject because she did not find it relevant.

> P5: Reviewing power, nth root, textbook. However, I honestly do not go into the matter very much. And until then, I did not see much relevance in the subject. For this reason, I will take this course to get a better understanding of the subject and to learn "(Q).

In addition, this teacher possibly does not believe that the student will learn the exponential function, as reported below:

> P5: We cannot forget that our students are coming from high school, first that they does not know arithmetic tables. Second, they does not know how to do multiplication and division. They does not know how to do division account with decimal number, so you can imagine what he will think when you get there in the exponential (E3, V2, 07:27).

Unfortunately, it was not possible to follow this teacher's process, since she did not attend the meetings anymore.

One of the teachers (P4) revealed conceptual difficulties considering the exponential function as a continuity of the linear function.

> P4: [...] I do not have yet experience in high school, I have never worked with this content, but because it is a simple function's continuity, it is a continuation, right? (E3, V1, 28: 8).

This teacher is seeking knowledge of the exponential function to improve her teaching and overcome her limitation.

The teacher P1, in this encounter, reveals conceptual approximations to the exponential function, although not systematized.

> P1: [...] I have had experience in the high school, and I see that the exponential function is one of the most important things we have. One of the most beautiful in my point of view to teach [...] I believe that everything that exists a great variation, a variation that does not come momentary, it is not, that thing like the PA. Oh, I already know what the next value will be, because that is a constant, right. Then the exponential function, it comes exactly to explain everything that has a variation and I cannot automatically design it, for example, if I have a colony of bacteria at first, or

> virus, then one of them divides itself into several cells, and then it divides, divides, divides, arriving at a certain moment that we cannot anymore. It has to have an analysis to verify, so does the population, so if I want to know, what will be the population of a given city, 10 years from now, what is the purpose of this, to know the population 10 years hence. [...] So then, we have the exponential function, since nature, when I have the nth root's questions, the wave's questions, so I have an earthquake, that the base of the calculation of the Richter's scale is the basis of the logarithm, and if I do not know the exponential I will not understand the logarithm [...] (E3, V2, 00:42).

We verify that she recognizes the form of exponential growth as different from linear growth, although she does not express herself clearly and connects the exponential function to application situations. It also highlights the students' difficulty in analyzing charts and the use of tricks to change the basis.

Teacher P1 described her methodology in teaching exponential functions:

> P1: I used to take a few leaflets there, from stores, from cars' promotions, from these things. That year, I did practice with a teacher, but then I could not find the work, I think he kept it. So, even then, I vaguely remember some situations, it was really cool, it was with biology, and from there it also involved a part that I cannot explain, that it was a matter of half-life, that I remember, from there he spoke, he stayed, there was an issue explained from the expert, from Carbon 12 (E3, V3, 17:27).

From the reports of the OPM participants, we can see that it is important for them to teach the exponential function, which only one participant disagreeing with it because she do not have enough knowledge about this function. Therefore, we can identify that it is a necessity to teach the concept of exponential function and that teachers were placed in the process of activity, because they are possibly concerned with satisfying this need to teach, that is, a motive, with the purpose of teaching this function. Leontiev (1994) thus defines the activity:

> By activity, we designate the processes psychologically characterized by what the process as a whole directs itself (its object); always coinciding with the objective that stimulates the subject to perform this activity, that is, the motive (LEONTIEV, 1994, p. 68).

With the progress of the extension project and during the meetings, the teachers P1 and P8 were part of the subgroup that elaborated the situation triggering the learning 'Earthquake'. Further details of this process can be found in Panossian et al. (2017).

In one of this group's discussions, the members considered that the choice of problem situations in textbooks is not an easy task, and that often does not reveal the essence of the concept of exponential function, as they could perceive in a situation of population growth that involved application of the algorithm, replacing the value of the variable in the exponent. The

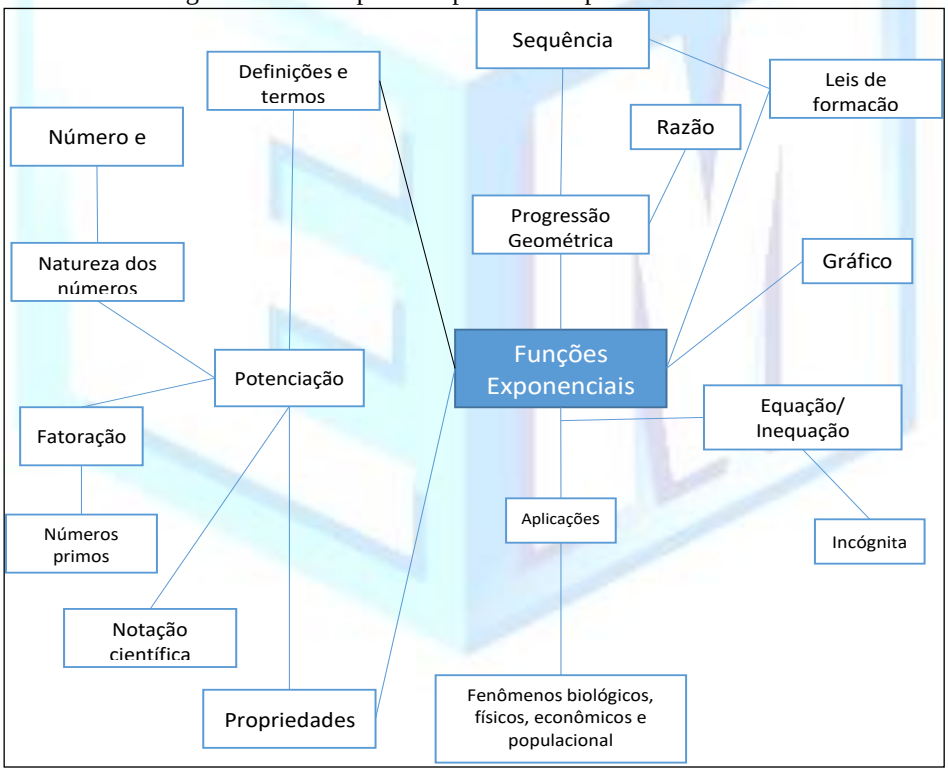
work in this subgroup articulated the mathematical concept of exponential function, with other areas of knowledge such as History, Geography and Physics.

This subgroup also discussed the obligatory nature of a course in the mathematics' history in the mathematics licentiate course, according to the following dialog:

> P1: At University, I had a little of mathematics' history. In fact, it did not have to be optative. (E9, A1, 34:00).
> P9: We sin a lot, because we want to give the subjective, subjective, and we do not contextualize the business, we do not say where it comes from and where it goes [...] (E9, A1, 35:00).
> PO1: Because you did not have it. (E9, A1, 35:02).
> P9: If I told a little about the history of it, maybe they would be motivated to find out together, there (E9, A1, 35:07).

At this moment the subgroup revealed awareness of the need for the historical and logical movement of the exponential function for the teaching organization. The subgroup also assembled the conceptual map/scheme of the exponential function, recorded in Figure 02 below:

Figure 02 - Conceptual Map Of The Exponential Function



Source: Research Data (2016)

In order to organize the development of the actions within the school, the members of this subgroup studied the inclusion of arithmetic progression and geometric progression (as internal nexus) in the exercises. Considering that, the exponential function is a relation that

associates an arithmetic progression with a geometric progression, and consequently a relationship that transforms sums into products and vice versa. They also aimed at students to understand that the exponential function does not have a linear behavior, in addition to considering different scales used to measure the intensity of an earthquake. The Richter's magnitude scale was used for students to identify that a magnitude 4 earthquake is potentially 10 times more intense than a magnitude 3 earthquake. They planned the conditions to apply the exercises with whole and decimal exponents, and converting the mathematical expression magnitude of an earthquake, from logarithms to exponential, through tables using the power properties. They discussed collectively what students would need to calculate the magnitude of the earthquake, using the records collected on the seismograph of each student that was elaborated in the physics classes, also discussing the energy's calculation of some released earthquakes previously presented.

It was possible to recognize that the AOE as a theoretical-methodological basis helped the teachers to organize the teaching of exponential functions. The movement of planning and replanning of their actions demanded studies from the teachers on the historical movement of this function and the human needs that generated it, to the definition of actions and operations to put them into practice in the classroom.

At the seventh meeting, P8 reflects on "The importance of collective planning, exchange of experience, exactly what we are doing here, okay" (E7, V1, 12:29), it is assumed that the human being is formed in actions developed collectively and internalized. According to Moura et al (2010) explanation:

> [...] the man's psychic development is accomplished through what Vygotsky called the process of internalization. According to this author, intrapsychic relations (individual activity) are formed from interpsychic relations (collective activity). It is in this movement from the social to the individual that is given the appropriation of concepts and significations, that is, the appropriation of the experience of humanity (MOURA et al, 2010, p. 208).

Moura et al. (2010) understand that "school as a privileged social place for the appropriation of historically produced knowledge, necessarily involves assuming that the teacher's action must be organized intentionally for this purpose" (MOURA et al., 2010). Then, by organizing his teaching, the teacher promotes ownership of the knowledge to the student.

The P1 in this same encounter narrates her change in relation to the teaching of exponential function, recognizing the intentionality associated with this function's historical understanding:

> P1: Seek an intentionality for the exponential function, the why, how? I have difficulty, actually from the own history of the exponential function, of how it showed up. Before, a few years ago, for a power, the equation and exponential function is almost all the same, which modified there, that now instead of having a real number I was using a variable in the exponent and finished ... right there. However, the difference of the equation is the signal that changes ... (E7, V2, 15:21).

In the interval between the eighth and ninth meeting, this teacher sent a message to everyone:

> Hello, guys! I would like to share with you some ideas - nothing remarkable, but that remained (still are) messing with my fragile neurons! I enjoyed the holiday and I have been reading some very interesting things, in my point of view. I found that the logarithms (Napier) arose before the exponential independent function, and in an independent way, which greatly aided the navigations and the whole historical process of the aristocratic turning towards the bourgeoisie in power, resulting in the commercial mathematics interest (17th century). (In this period, the logarithms had the function of facilitating the calculations of addition-multiplication, subtraction-division, which they were calculated through trigonometry). Around 1680, Jacob Bernoulli, analyzing a situation of compound interest, perceived the possibility of working the same situation through power, a fact that led him to study the strings suspended by two extremes (forming the idea of exponential, but does not publishing it). His brother Johann Bernoulli, after Jacob's death, published in 1697, in Principia Calculiexponentialum, various calculations of exponential function. Only long after that someone realizes that the inverse function of the logarithm is the exponential function (they arise from the same needs, variations occurring in nature, needs to facilitate navigations and commercial transactions of the period). Well, maybe for the group this is not relevant, or nothing new, but I was very happy for the personal discovery and wanted to share, because I always imagined that from a certain commercial or maritime situation [...]. There was some of this situation (1,05) raised to the power of x = 2 and they did not have the answer and they were searching for solutions through the initial knowledge of Napier, making the logarithm appear as an inverse function after the exponential function. Guys, I'm sorry for the ignorance in the area, but I never looked for the exponential function's history, and the same fragility I found only now in the textbooks, where they do not mention anything about it, showing only the history of the logarithm.

It is observed in this teacher P1 her movement in teaching activity, studying in depth the historical and logical movement of the exponential function, and being aware of the needs to study this concept in the course of time.

In order to confirm this personal development of P1, we present her dialogue with PO1 about what she thinks about the exponential function and the help of OPM in organizing her teaching, and about the internal nexus of the function: PA and PG, which when studying, found:

> P1: I think so, it is very relevant, because, I think I have already commented, there everything have an evaluation, then, because it is the PA and PG that we saw in the exercises, right? It had its own logarithm, I think; this gives a better foundation for him to understand that I have come out of the exponential and goes to Log [...]. Moreover, since the logarithm has a series of applications, then the exponential function is directly related. So, it is of extreme importance, it cannot be dissociated

> beyond the power to apply in mathematics, in physics, in biology. In the half life's case, we picked from the book the geography's statistics, which is the population part, the financial part, entering into the technical subjects of financial mathematics and the statistics itself, so I mean, it encompasses a lot, which I think makes it a primordial thing for the subject, which cannot be withdrawn (E27, V1, 05:20).
> PO1: [...] we finish the linear function, starts the quadratic function, the exponential function begins, the exponential function ends, but this relationship is not established for me. It is not like the project wanted to do this, but this was something. I believe the idea of the Earthquake, I think it had to pull the relations between the greatness and the relationship of PA with PG, that we also works, what we do not work together, works separately, box of PA and PG and the box of exponential function and we do not articulate (E27, V1, 14:41).
> P1: It only articulates when, for example, there is a new book that we saw, teacher. I think it was Dante, when we saw the interaction of PG. It is one of the rare books that have this in it. It is a flaw I do not remember seeing this analysis in the graduation course (E27, V1, 16:22).
> PO1: I did not see it either, even teaching, so I never thought about making the relationship. By the way, I never did the relationship, we do not teach exponential function, because yes, it's in the end part of functions, you wants more instead of faster, well, right, until then you already give equation, the graph, is it like that, so be it. That is what you already have in the book, we took care of it, but actually did not, right, and we do not really care. Moreover, we really need all the previous contents of power, of power's property, right (E27, V1, 16:51).

We observed in this report that the P1 has another more organized view of the exponential functions' teaching.

Teachers also commented at the last meeting about the planned teaching situations and implemented in the second half of 2016:

> PO3: Acho que a situação desencadeadora é aquela que a gente consegue agregar um maior número possível de conceitos, dentro da disciplina, dentro daquele conteúdo, e ver a relação com ela aplicação no dia a dia. Pelo menos, foi isso que eu entendi, então foi o que ficou marcado, né, então vou ver, a não vou pegar aquela situação, mas acho que até é mais tranquila de trabalhar do aluno aprender, agora vou ter que ver de um olhar diferente, vendo essas condições, né, quais os conteúdos que vai ser abordados, quais os conceitos que eu tenho que retomar, pra ver para eu preparar a minha aula, é nesse sentido que eu percebi, assim qual é a objetividade, né, qual objetivo que eu tenho que atingir com ela. Eu não sei gravei coisa errado (E27, V2, 04:11).
> P8: A P1 estava falando que lembrando assim, não sei se de repente iguala, ou não, aquela relação de estudo que tem, naquela Universidade Noroeste do Estado do Rio Grande do Sul, tem essa questão assim, de você estudar a colocar um assunto um tema que está atrelado de organização do ensino, mas é uma coisa que de certa maneira, a gente, não é que falha, ainda que não chegou o momento de estudar, mas aqui talvez seja o momento, é de pensar essa situação de aprendizagem, fazendo esse trabalho interdisciplinar, não só desencadeado o conceito matemático, Ok. O nosso foco foi, eu acho que a gente precisa ter um foco, porque se não você vai querer estudar tudo ao mesmo tempo, e aquela história, o isolado precisa ser recortado, porque a gente não consegue estudar tudo ao mesmo tempo, por isso a gente precisa fazer o recorte. A gente fez o recorte da função exponencial, dentro da matemática, como a gente poderia ter feito um recorte mais aprofundado, eu sei que você fez, mas gente não estudo tanto aqui, isso ficou, mas a seu cargo, no conceito da física, né (E27, V2, 05:43).

In this same dialogue, P8 refers to the concept of the isolate from Caraça (1951), as a reality cut and was studied during the OPM.

At this meeting, participants also took up the discussion about the use of the graph in the teaching of exponential functions, PO3 considers that you can use software, but that the it makes the student understand the concept, as he says:

> PO3: I do not see problem in making use of Excel, or other software to make the graph, but to explore, to make the student understand, what is that line, what is the ordered pair, x and the value that the function assumes in that x, the student understand this (E27, V2, 01:26).

For the AP1 the importance of doing the chart manually, because the student learns the concept doing.

> AP1: It's because, you only have the graph, there's the Geogebra, we put it on the computer, it's the graph, it's very different when you make the graph, you understand a lot better, it's not that thing, the way I see it there, but when you build it you have the concept (E27, V2, 02:19).

The participants reported on their experiences during the course of the year within the OPM and they took it to them.

> P8: Even personal growth, right? Not only [...] (E27, V2, 27:59).
> P1: Another thing, as you can see, is that in the part of basic mathematics, and the exponential function itself, is not something crazy, right? The exponential function, logarithm, it has nothing to do with it, but let us go, it's not good for anything, it's not used for anything, it's actually that we were able to show them that no, there is a variety of things that can be used. And another thing that can be put, it is that the student in the school, not only use what he will need, he is not in a good technical training there. He did not see himself pushing the button, he just need to generate that button. He is there to have a greater horizon, which is equal to the texts that the madam (PO1) brought, right? It was part of the study meant to show all that learned over time, and have a critical point of view, and to take this knowledge, not just epistemological of the concept, I want to learn it because I am going to use it (E27, V1, 09:38). [...] I felt a maturation in the "S" class, right, by several factors that happened in the "T" class. It is a group that will take time to mature, but even so, what they have achieved, they did it, it's already a step, but then it had all the other external factors that contributed to the non-maturing, right, in that period. But, it had a gigantic importance for them to see it, what is applied, see it in a different way, I think, that we, at least shook "T". And the "S" I think he was wonderful, the change that he had, the first moment, so far, was a disinterested class, it was a class that did nowhere, did not produce anything, and suddenly, they are asking, I think it is very valid (E27, V1, 11:13). [...] but, it is also by knowledge, teacher, so, I had an idea of the function of exponential history, and then when I started to study the subject, it was totally the opposite of what I believed. Then I thought that I learned a lot for me, but I cannot teach it, to explain this, that is a tragedy, right? I learned for me as a person, I satisfied myself, I came here not for the title, I came most of all to learn. I ended up liking it, it was what I was looking for, it was a dull thing that arrives and stays, and it is the same as a course. Once, I went to 2 UFPR mathematics courses in, it was interesting, but for those who were looking for that ENEM pattern exercises, but I did not see it (E27,

> V2, 28:28). [...] it was more a technical thing, for me a click snapped to seek, it was different from here, here it seems to value more human side, so, I like those (E27, V2, 29:35). [...] I do not know if it is my impression, but I think so, it considers the person a lot, the knowledge, which led me to study (E27, V2, 29:52).

OPM participants concluded that when the concepts are related to the subjects of the student's daily life, they more easily take advantage of the concept of exponential functions for their life, thus, an organized learning situation of the AOE allows another methodological organization to this function's teaching. See the participants' dialogue:

> PO3: It is because sometimes he can solve a problem using something simple, which he has already seen in elementary school, but using this other knowledge, exponential function or logarithm, solves it in a simpler form, right. To understand that, the construction of this concept, it came to help facilitate how to solve some problems that demanded immense calculations, right, came to facilitate, it is important in this regard. By day-to-day application of the student, what is meant by the day-to-day application, the experience that the student is having. I do not think to show him, where he uses that function in everyday life, what the student of the high school does in day to day, you cannot immediately identify, here has an exponential function, here does not, or does he use it. But, he understands that this is what will ease for him, if he follows a career in the social field, many of the problems for him to solve will be needed to have that a mathematical knowledge. Or, if he goes for the math and sciences, or everything else, or to a medical area, he will also need it, but in that sense. Not to apply day by day, in the little things of what he is doing, but it is to show that depending on the path followed from the profession, he will be much more useful (E27, V1, 07:17).
> P1: Another thing, as you can see, is that in the part of basic mathematics, and the exponential function itself, is not something crazy, right? The exponential function, logarithm, it has nothing to do with it, but let us go, it's not good for anything, it's not used for anything, it's actually that we were able to show them that no, there is a variety of things that can be used. And another thing that can be put, it is that the student in the school, not only use what he will need, he is not in a good technical training there. He did not see himself pushing the button, he just need to generate that button. He is there to have a greater horizon, which is equal to the texts that the madam (PO1) brought, right? It was part of the study meant to show all that learned over time, and have a critical point of view, and to take this knowledge, not just epistemological of the concept, I want to learn it because I am going to use it (E27, V1, 09:38).
> AL1: I think that this blurred part with the person's experience, for example, if since the beginning of the elementary school's education, you are accustomed for example to see a teacher only identify it, he gives a formula, and only that, and done, mechanical, but it is very different when you understand why it was done like that. From the moment the person asks why, she learns a lot more, because she leaves that thing, there is only I will do it this way, she ends up memorizing it (E27, V3, 10:00).

It is considered the change of direction attributed to the exponential function resulting from the intentional movement to elaborate the triggering situation of learning that allows the surpassing of appearances, and make students and teachers to appropriate the mathematical concept, the first to learn and the second to teach. It was evidenced in this transformation the teaching of the exponential function in the speeches and in the actions of the teachers P1 and

P8, by their involvement in the triggering situation of learning, as well as in the planning and study of this function's historical and logical movement.

Final remarks

The purpose of this work was to recognize the meanings attributed to the exponential function by teachers in teaching activity, considering the principles of Teaching Oriented Activity. For that, we accompanied a group of teachers during the extension project Pedagogical Workshop of Mathematics (OPM), presenting primarily the change of two teachers who participated in the process.

As demonstrated by analyses, it was possible to observe that when the teacher is in activity, and their needs are connected to the objective, the planning of the teaching situation surpasses the simple choice of a textbook situation. The teacher becomes aware of the need to research the concept, from its history (historical and logical movement) seeking its essence (nexus). This movement of planning and collective research potentiates its own formation, and adds value to what is being taught, in this case the exponential function.

In this regard, it is possible to overcome the empirical thinking that favors technical aspects associated with the exponential function, such as substitution of numerical values in the variable, mechanical construction of graphs, or memorized formulas, through the appropriate choice of teaching situations. The fact that the teachers' group created their own situation considering the historical and logical movement of the concept implied changes in meaning and performance in their teaching process. This movement recognizes the elements of the Teaching Oriented Activity according to Moura (2001):

> The teaching oriented activity has a need: to teach; it has actions: it defines mode or procedures of how to put the knowledge in game in the educative space; and elects auxiliary teaching tools: methodological resources appropriate to each objective and action (book, chalk, computer, abacus, etc.). And, finally, the processes of analysis and synthesis, throughout the activity, are moments of permanent evaluation for those who teach and learn. (MOURA, 2001, p. 155).

The teacher's need is evidenced by teaching scientific knowledge, in this case exponential functions. Specifically the teachers P1 and P8 recognize the essence of the exponential function with their internal links PA and PG and incorporate it into the planning of the teaching 'earthquake' situation, which motivated students and teachers in the appropriation of this scientific concept.

Therefore, teaching beyond appearances is possible, and involves teaching organization, through research on the essence of the concept, that is, its internal links in its historical and logical movement. In addition, with the possibility of elaborating learning-triggering situations, so that students take ownership of the concept in a conscious and meaningful way.

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