REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

PERSPECTIVAS DA EDUCAÇÃO MATEMÁTICA

Volume 12, número 28 – 2019

ISSN 2359-2842

Development of Conceptual Fields Additive and Multiplicative in Teaching Negative Numbers: A Critical Analysis of Didactic Books

Desenvolvimento dos Campos Conceituais Aditivo e Multiplicativo no Ensino dos Números Negativos: Uma Análise Crítica de Livros Didáticos

Renata Arruda Barros[1]

Taíssa da Silva Lima Boaventura[2]

ABSTRACT

In this study, we propose a critical reflexive analysis in seventh-grade mathematics textbooks approved by the National Plan of the Didactic Book in order to investigate the light of Gerard Vergnaud's conceptual field theories and epistemological obstacles such as content integer numbers are addressed. The analysis is necessary in view of the difficulties encountered in the teaching of integer numbers in the second phase of elementary education. The theories studied are of great relevance, since they show how mathematical learning is given and what the difficulties are for the full understanding of this subject. We elaborated a bibliographical research on theories and, finally, the reflective analysis of the books.

KEYWORDS: Conceptual fields, epistemological obstacles, textbooks, integer numbers.

RESUMO

Nesse estudo, propomos uma análise crítica reflexiva em livros de matemática do sétimo ano do ensino fundamental aprovados pelo Plano Nacional do Livro Didático com a finalidade de investigar, à luz das teorias de campos conceituais de Gerard Vergnaud e obstáculos epistemológicos, como os conteúdos de números inteiros são abordados. A análise faz-se necessária diante das dificuldades encontradas no ensino aprendizagem de números inteiros na segunda fase do ensino fundamental. As teorias estudadas são de grande relevância, pois mostram como se dá o aprendizado matemático e quais as dificuldades para o pleno entendimento dessa matéria. Elaboramos uma pesquisa bibliográfica sobre as teorias e, por fim, a análise reflexiva dos livros.

PALAVRAS-CHAVE: Campos conceituais, obstáculos epistemológicos, livros didáticos, números inteiros.

[1] Instituto Federal do Rio de Janeiro/IFRJ Volta Redonda, RJ, Brasil. renata.barros@ifrj.edu.br
[2] Instituto Federal do Rio de Janeiro/IFRJVolta Redonda, RJ, Brasil. taissasilvalima@hotmail.com

Introduction

It is very common for teachers and students to find difficulties in teaching integer numbers. This content is addressed from the seventh year of elementary school and, most of the time, is seen as a challenge. The teaching guideline, National Curriculum Parameters - PCN (BRAZIL, 1997), expresses that teaching and learning integers numbers is not a simple task for students and teachers. A systematized work must be developed in order to minimize some difficulties encountered. The NCPs (BRASIL, 1997) also discuss the skills and competences that should be developed in the students.

> * to expand and construct new meanings for natural, integer and rational numbers from their use in the social context and from the analysis of some historical problems that motivated their construction;
> * solve problem situations involving natural, integers, and rational numbers and from them to enlarge and construct new meanings of addition, subtraction, multiplication, division, potentiation and root extraction;
> * identify, interpret and use different representations of natural, rational and integer numbers, indicated by different notations, linking them to mathematical and non-mathematical contexts;
> * select and use calculation procedures (exact or approximate, mental or written) depending on the proposed problem-situation (BRASIL, 1997, p.64).

Faced these demands and the difficulties encountered, it is necessary to reflect on how the textbooks, approved by the National Textbook Plan (PNLD/2010), deal with the contents of integer numbers.

It is well known that books are currently used by teachers and students as teaching materials and occupy a prominent place in most classrooms. Arruda and Moretti (2002) emphasize that the regularization of this resource comes from the Comenius era with its Didactic Magna, where a single book was proposed as a reference to the student.

According to Schubring (2003), the books already existed before the technology was invented to print them. Over time, people have sought ways to record their cultures in writing, and have sought a way to preserve them.

It represents not only a pedagogical support material or didactic resource, but a cultural element of systematized, collective study, capable of assisting in the construction of knowledge, based on teacher interactions and mediations, choices and critical-reflexive analyzes.

With all these attributes and importance in the school context, it is important to analyze how this didactic material approaches some contents. In this context, this research is restricted to the content of whole numbers (worked from the 7th year of elementary school), since it is pointed out by the guiding document itself as an obstacle for both teachers and students (BRASIL, 1997). In this way, we opted to analyze how this content is approached in textbooks, from the theories of conceptual fields and epistemological obstacles.

It is noteworthy that the choice of Gérard Vergnaud's theory of conceptual fields took place, since this psychological theory will help to understand the way in which knowledge is given and how it must be worked, in order to guarantee better learning.

On the other hand, the theory of epistemological obstacles was chosen because it will help to understand the origin of the difficulties encountered in the process of learning integer numbers in order to facilitate reflection on the processes necessary to overcome these difficulties.

Still the following research questions are raised: how are epistemological obstacles to integer numbers treated in textbooks? How do the situations proposed for the teaching of integer numbers in textbooks fit the theory of conceptual fields?

How far does the focus of the approach overly focused on formulas and procedures?

The research hypothesis is that the didactic book can restrict the teacher's performance in the full development of the conceptual field by disregarding the epistemological obstacles present in the construction of the integer number concept.

Thus, the general objective of the research is to analyze how textbooks present the concept of integers, the light of conceptual field theory and epistemological obstacles.

The aim is to verify if: the textbook approach on the contents of integer numbers complies with the recommendations proposed by the official documents: the National Textbook Plan (PNLD) and the National Curriculum Parameters (NCP); if the focus of the textbook approach on the contents of integer numbers is directed to repetitions of formulas and procedures. Also: evaluate how situations are proposed by textbooks; how the books deal with the historical questions connected with the process of constructing whole numbers and if the textbooks deal with the epistemological obstacles present in the process of constructing whole numbers.

Theory of conceptual fields

Gérard Vergnaud, emeritus director of studies at the National Center for Scientific Research (CNRS) in Paris, is the forerunner of the psychological theory of conceptual fields.

Theory helps to understand how children construct mathematical knowledge and how they learn. The researcher is guides by Piaget's theories, and makes references to it. The studies are based on both Piaget and Vygotsky since Vergnaud considers the ideas of adaptation, unbalance, rebalancing (which are part of the concept of scheme) and gives importance to social interactions and language in the process of understanding the conceptual fields by students

> Vergnaud defines conceptual fields as being, in the first place, a set of situations which domain requires, in turn, the domain of several concepts of different natures. (MOREIRA, 2002, p. 3)

For the researcher, every mathematical concept is inserted in a conceptual field. The conceptual field, in another hand, is composed of several situations of different types.

> Conceptual field is for him an informal and heterogeneous set of problems, situations, concepts, relations, structures, contents and operations of thought, connected to one another and probably intertwined during the acquisition process (MOREIRA, 2002, p. 8).

Knowledge, for the author, is build from different problems that arise. These problems are conceptualized as "situations." A concept is not formed with a single situation, it depends on time, experience and maturity. For the researcher, the conceptual field is seen as a unit of study to give meaning to the difficulties observed in the conceptualization of the real.O saber, para

> Vergnaud takes as a premise that knowledge is organized in conceptual fields which domain, on the part of the subject, occurs over a long period of time, through experience, maturity and learning (MOREIRA, 2002, p. 8)

Conceptual fields are learned over a long period of time, this does not occur in small lapses of hours, days, weeks, months, or years. It is not a short time that the child understands a new concept. The theory itself guarantees that:

> The construction and appropriation of all the properties of a concept or all aspects of a situation is a very breathtaking process that extends over the years, sometimes a dozen years, with analogies and misunderstandings between situations, between conceptions, between procedures, between signifiers (MOREIRA, 2002, p. 9).

We have as an example the time taken to understand the process of adding the integers. If we think of a situation that seems simple to an adult, we can see how many concepts are involved and how much the learner needs to have acquired from concepts to successfully solve the proposed situation. To solve "Beatriz had 6 candies and got 5 from her mother. How many candies does she have? ", We can observe a number of surrounding concepts, such as: addition, notion of time (had / have); counting and decimal system. As elucidated, these concepts require a time interval and, for subjects to master, new properties and problems must be studied.

It is important to emphasize that the difficulties that appear in the domain of these fields can not be simply circumvented, ignored or forgotten, this does not guarantee learning, they must be met and faced.

Vergnaud, in his theory, still uses some terms like: concept, situation, scheme and operative invariants.

He begins by defining concept as a triplet of three sets, C = (S, I, R), where:

> S S is a set of situations that give meaning to the concept;
> I is a set of invariants (objects, properties and relations) on which the operability of the concept rests, or the set of operative invariants associated with the concept, or the set of invariants that can be recognized and used by the subjects to analyze and dominate the situations of the first set;
> R is a set of symbolic representations (natural language, graphs and diagrams, formal sentences, etc.) that can be used to indicate and represent these invariants and thus represent the situations and procedures to deal with them (MOREIRA, 2002, p. 10).

Here we can understand the meanings of concept. It is a set of situations that may arise, whether they are didactic or just everyday episodes - we then have S; to solve this new situation the student needs to evoke knowledge and all this knowledge used is called I. R is every way to represent this evoked knowledge.

Gérard Vergnaudain present situations (S) as tasks:

> The concept of situation aplied by Vergnaud is not that of a didactic situation, but that of task, and that every complex situation can be analyzed as a combination of tasks, for which it is important to know their own natures and difficulties. The difficulty of a task is neither the sum nor the product of the different subtasks involved, but it is clear that performance in each subtask affects overall performance (MOREIRA, 2002, p. 11).

These tasks are the situations experienced by the children and, as already said, may be didactic or not. From them, human conceptions are being structured and shaped. Hence there is a need to provide learners with situations that stimulate them in order to generate new knowledge. When the teacher asks, for example, for the student to solve the situation exemplified "Beatriz had 6 candies and won 5 from her mother. How many candies does she have? ", The child is facing a situation. The same thing happens when she needs to verify that the change she has received is correct. All these situations experienced, when dominated or modified by the students, generate new conceptions. From many situations, concepts gain meaning.

To solve a task situation, the student will need to organize actions that allow him to finish successfully what was presented to him. This organization is what Vergnaud defines as schemas. It is the schemes that give meaning to situations:

> A scheme is a universal that is efficient for a whole range of situations and can generate different sequences of action, information gathering and control, depending on the characteristics of each particular situation. It is not behavior that is invariant, but the organization of behavior (MOREIRA, 2002, p. 12).

We can understand schemes as the actions used by the subject. They can be gestural, such as counting objects, counting fingers and toes, plotting diagrams, graphs, and can also be verbal.

It is valid to point out that all this organization of actions is subjective and varies from individual to individual. In a proposed task, each student will choose a scheme and use it to reach a resolution, and it can be evoked simultaneously with each new situation. We take, for example, a common classroom situation. A teacher proposes to the students a situation:

Felipe is reading a book that has 80 pages, so far he has read 64 of these pages. How many pages are left for the boy to finish reading?

In the class, everybody with the same age range, several different resolutions may appear as solution to the situation. A student can use subtraction (80 - 64); another student can count in the fingers how many pages were missing to complete 80 (64 + 16). These ways of organizing thought for resolution, we take as schemas and they vary from subject to subject.

The scheme is directly linked to how each one reacts to new tasks. And for the organization of the same, many "ingredients" are used. Goals and anticipations, rules of action such as "if, then”, permeate the individual’s mind in order to seek information for their action.

The more schemes the individual has, the more able to solve the tasks he will be. His cognitive development will occur as his repertoire of schemes is broader. If the subject does not have all the necessary skills to solve the situation, there will be a time of reflection and exploration, with the attempt to use several schemes. If the scheme is ineffective, it will be modified.

It is important for the learner to be in contact with many types of situation so that he structures his thoughts and creates schemes to solve the challenges that are proposed to them. In this context, there is a reflection: Are the situations presented in the books enough to develop many conceptions, thoughts and meanings? It is believed that most situations experienced by the students will be those proposed by the textbooks.

Here, it is up to the teacher to help the student to appropriate several schemes because the greater the number of schemes, the easier it will be to deal with the situations presented to him. Care must be taken, however, that these do not become stereotypes, they must be meaningful to those who use them. There is no question of the importance of avoiding the memorization and repetition of procedures, which can create stereotypes rather than mathematically valid and consistent schematics for those who use it.

In this context, it is valid to reflect if the materials used by teachers and students also escape this repetition. Textbooks, for example, should not elucidate and value the culture of memorizing and repeating formulas and procedures. This support instrument should, in fact, present a vast repertoire of situations that prepare the student to solve many problems and at the same time, that these situations have meaning for the reader.

Understanding the meaning of concept, situation and scheme, we can speak of the operative invariants, which are defined as the knowledge contained in the schemes. It is through it that the student solves the situation. It is divided into: concept-in-action and theorem-in-action:

> The terms "concept-in-action" and "theorem-in-action" are called the knowledge contained in the schemas. They can also be referred to by the more comprehensive term "operative invariants" (1993, p. 4). Scheme is the organization of conduct for a certain class of situations; theorems-in-action and concepts-in-action are operational invariants, so they are essential components of the schemas and determine the differences between them. Theorems-in-action and concepts-in-action are the concepts used in the schemes. The first can be understood as a proposition / proposition considered as true, while the second is a category of thought considered relevant. (MOREIRA, 2002, p. 14).

To solve a problem, for example, the individual needs to understand the concepts involved in it and even implicitly, create a theorem in his mind. Taking the example used: "Beatriz owed R$ 15,00 in the market near her house. She paid a portion of that debt and continued with a negative balance of eight reais. What is the amount paid?", We observe the number of concepts-in-action involved in this understanding as: addition, notion of time, counting, decimal system, initial and final state. There is still the opposite reasoning to find the final situation, the individual should add 8 units to the 15 existing negative units. For this, a theorem-action must be used. All this thought is entirely connected. Concepts are "ingredients" of theorems and theorems give meaning to the contents.

We must consider that, mosto f times, students are not able to explain or even express in natural language the theorems and concepts in action.

> Most of these concepts and theorems-in-action remain totally implicit, but they can also be explicit or become explicit and there is the teaching process: to help the student to construct explicit and scientifically accepted concepts and theorems, based on implicit knowledge. It is in this sense that concepts-in-action and theorems-in-action can progressively become true scientific concepts and theorems, but this can take a long time. In this situation, it is up to the teaching to help the student construct those concepts that are implicit in their thoughts in a way that can be explained and recognized as scientific theorems (MOREIRA, 2002, p. 15).

The teacher has the task of mediating, since he is the one who helps in the process that characterizes the domain of the conceptual field by the student. It is also his work to help students in the acquisition of new schemes and operative invariants so that the student is able to successfully solve many situations which they are submitted. The student should always be in contact with fruitful situations so that the concept is meaningful. In this sense, we intend to evaluate how textbooks can help teachers to play this role.

The conceptual field of additive structures is the set of situations which domain requires an addition, a subtraction, or a combination of those operations. (MOREIRA, 2002, p. 9). It is notable that there is no field for subtraction since subtracting is nothing more than adding a positive number to a negative number. We then realize that addition and subtraction are part of a single conceptual field. Together, addition and subtraction form organized systems that must be studied.

We took, for example, the following situation: Marina had a few cards, won 15 in a game and got 35. How many cards did she have? Note that the same situation can be solved by using addition or subtraction. The student can establish as a scheme to operate: 35 - 15 and succeed in solving the problem, just as he can also use addition and count from 1 to 1 from number 15, also having the desired solution, 20.

In the additive field, the tasks and situations encountered are classified as: composition, transformation and comparison.

In the composition two parts are presented and one has to find the whole, or else, knowing the whole to discover one of the parts. For example, in a class, there are 15 boys and 13 girls. How many children are there in all?

In transformation, there is a change in the initial state, this change can be a loss or a gain. For example, Pedro had 37 balls, but lost 12. How many balls does he have now?

I another hand in the comparison we find the confrontation between two quantities, as in the following example: Paulo has 13 cars and Carlos has 7 more than him. How many cars does Carlos have?

All these situations refer to the additive field. As already mentioned, individuals need to present several schemes and operative invariants that will enable them to solve these different situations that may arise

The conceptual field of multiplicative structures is the set of situations which domain requires multiplication, division or a combination of such operations (MOREIRA, 2002, p. 9). Together, multiplication and division form organized systems that must be studied.

> Vergnaud defines conceptual field as being, first, a set of situations which domain requires, in turn, the domain of several concepts of different natures. For example, the conceptual field of multiplicative structures consists of all situations that can be analyzed as problems of simple and multiple proportions for which multiplication, division or a combination of these operations is usually required. Several types of mathematical concepts are involved in the situations that constitute the conceptual field of multiplicative structures and in the thought necessary to master such situations (MOREIRA, 2002, p. 9).

In the following situation, we can see a problem involving multiplication and division: Eight children brought 16 soft drinks to Carolina's birthday. If all the children took the same amount of drink, how many bottles did each take? In solving the problem, the student can operate: 16: 8 and obtain the result: each child took 2 bottles, or still operate: 2 x 8 = 16 and obtain the desired option: 2.

In the multiplicative field, the tasks are organized into: proportionality, rectangular and combinatorial organization.

Problems involving proportionality present the idea of regularity between elements. For example, at Carolina's birthday party, each child took 2 soft drinks. In all, 8 children attended the party. How many soft drinks were there? There is a regularity in the task, A is for B to the same extent as C is for D.

In rectangular multiplication, tasks involve thoughts of row X column. For example, in a hall we have 5 rows with 4 seats in each. How many seats are there in this room? Or: A hall has 20 chairs, with 4 of them in each row. How many rows are there in total?

Combinatorial tasks involve combinations of elements. For example, a girl has 2 skirts and 3 blouses of different colors. In how many ways can it be arranged by combining skirts and blouses? The elements are combined with each other.

The National Curriculum Parameters (NCP, 1997) recommend that situations in the Conceptual Field of Multiplicative Structures be explored from the first years of schooling and they should be worked through the whole process in a way that allows the student to solve problems each time more complex. However care must be taken that these tasks have meaning. The need to escape from memorization and repetition is indisputable.

Epistemological obstacles present in the process of constructing integer numbers

NCPs (BRASIL, 1997) show how numbers can be approached and point out how they can provide situations that lead to learning. By the end of the sixth year of schooling students are required to be able to solve all tasks involving transformation, composition and comparison. Already in the seventh year, learners come in contact with the integer numbers, a new learning.

For masters and learners, explaining and understanding all the peculiarities of this new set is not an easy task.

This challenge and the present difficulties may be linked to epistemological obstacles. Bacherlad's thoughts in his theory of learning are a way to understand how difficulties arise and how they can be faced.

According to Pommer (2010), the thoughts of epistemological obstacles were introduced by Gaston Bachelard; his idea was spread and later used by Guy Brousseau. He defined in 1976 that:

> An obstacle of epistemological origin is truly constitutive of knowledge, it is the one from which you can not escape and can be found in principle in the history of the concept (IGLIORI, 2010, p. 97).

Epistemological obstacles are understood as historical inheritances that can not be avoided, they are due to resistances arising from knowledge itself and are part of the construction of knowledge, being found in the history of development and evolution of concepts. The epistemological obstacle is linked to the process of historical construction and student learning.

> The notion of an epistemological obstacle as being that obstacle linked to the resistance of an ill-adapted knowledge, (...) being a means of interpreting some recurring and non-random errors committed by the students, when they are taught some topics of mathematics. (IGLIORI, 2010, p. 99).

From this perspective, the idea of analyzing mathematical knowledge and trying to understand how the process of constructing this knowledge takes place is a way to understand and overcome the obstacles that will appear.

More specifically, we will study the set of integers and the obstacles encountered in all their structuring. Igliori (2010) analyzed these obstacles in the study of negative numbers and pointed out:

> The notion of number, for example, has been elaborated in a process of coping with obstacles. This is the case, for example, of the conceptualization of negative numbers, of the introduction of the number zero, of the awareness of the existence of an irrational number, of the imaginary number (IGLIORI, 2010, p. 104).

These concepts were not intuitive to the human being and for that reason was an obstacle that devastated the mathematical world. Igliori (2010) points out that the construction of the negative numbers was a surprising slowness (1500 years, from Diofante to our days).

Boyer (1974) points out that for the Greeks, "numbers" always meant finding natural positive integers. The concept of negative numbers will appear in China in 300 b.C in a rudimentary way in everyday activities, and yet the negative responses were not accepted as solutions of equations. It is perceived that the concept of negative number was already a challenge at that time.

After the Chinese, it is believed that the indus worked with the negative numbers and had a symbol to represent the debts.

In Arab culture, the equations that generated a negative integer result were simply abandoned, so the negative numbers were slow to be accepted in the West.

In 1484 a negative number appeared related to the result of an equation. The usual "+" and "-" symbols were designed in 1489 in an arithmetic book. But, only in the nineteenth century, the negatives were pointed out as an enlargement of natural numbers and became part of the hierarchy of integer numbers.

Faced with the history of integer numbers, it is evident that the human being throughout the construction of the concept, faced challenges to accept the existence of this set of numbers and many years went by until it was actually integrated the numerical hierarchy.

Research methodology

A qualitative bibliographical research was carried out with the objective of reflectively analyzing the contents of integers of the seventh year of elementary school in light of two theories: theory of conceptual fields and epistemological obstacles.

Important points about Vergnaud's theory and, secondly, the theory of social and historical epistemological obstacles were investigated. In another step, the 7th grade math books were analyzed to see how the integer-numbers subject is approached. It was also investigated how the situations are proposed to work the concept of integers. Each analysis was wrote down and compared with another textbook. A questionnaire was made to guide the research and ensure that all books were evaluated with the same look. The questions are: Do books address epistemological obstacles? Does the book address historical aspects related to negative numbers? Does the book, in discussing the concepts of negative numbers, present different situations? Does the book present a number of situations in its activities? Does the book, when addressing the concepts connected to the operation of negative integers, present different situations? Does the book list the multiplication/division of negative numbers to aspects considered important in Vergnoud's conceptual field theory? Does the book relate the addition/subtraction of negative numbers to aspects considered important in Vergnoud's conceptual field theory?

From the answer of these questions, a comparative analysis of the results was made.

The research was based on theories of conceptual fields and epistemological obstacles, since they help in the understanding of how mathematical learning happens and the obstacles to its assimilation.

The importance of this research is believed, since the content of integer numbers is seen as an obstacle for teachers and students.

The books surveyed were drawn among the seventh-grade mathematics textbooks approved by the National Plan of the Didactic Book for the year 2018. The reflective analyzes were done in three of these books, since it is believed that this quantity will allow a comparison of the materials.

Analysis of textbooks

The first work analyzed was the textbook of the 7th year of elementary school: Discovering and applying mathematics:

The book presents a chapter with fifteen pages intended for the content of positive numbers and negative numbers, divided into: positive numbers and negative numbers; adding positive numbers and negative numbers and subtracting positive and negative numbers.

On the first pages of the chapter, the author presents a numerical line of natural numbers in order to explore the students' prior knowledge. From this line, the content of integers is addressed. There is no direct exposition of the concept of positive and negative numbers. A sequence of activities involving altitude, balances and debits, distances and temperatures are presented.

There are several practical situations in content in the book, however, there is no historical mention of the integer-number construction process or any other historical fact that refers to the content. However, the epistemological obstacle to acceptance of the idea of negative quantity is worked out from the situations already mentioned.

The next pages show the operations with the integers. The presentation is made using the number line associated with the notion of direction. There is no direct exposition of formulas for solving addition and subtraction, but there are situations that induce the student to discover them. There are situations of composition in addition activities and transformation situations in subtraction and there are also exercises of direct application of calculation procedures.

The addition is presented, at first, by the exploitation of bank balances, then it is shown in the numerical line and is worked with notion of direction (right and left). There are two situations in which the student must complete a table by adding the integers. For this, one must consider each positive number as a balance and each negative number as a debt. In this topic, as in the previous one, the historical issues were not addressed at any time.

The subtraction is approached from the idea of addition, then it is completed: to subtract two integers, simply add to the first the opposite of the second number. The situations are in very reduced number, one situation addresses goals of one championship and another, bank statement. There are, once again, no situations that address the historical issues.

At no time, whether in addition or subtraction, was there direct exposure of formulas to operations with integers.

The book does not present concepts or situations involving multiplication and division of integer numbers.

The second work analyzed is the textbook of the 7th year of elementary school: Mathematics in the Right Measure:

The book has a chapter for integer numbers divided into action on positive numbers and negative numbers, action on addition and subtraction of integers, multiplication and division of integers, and mathematical challenges.

There are, in the beginning, several situations to approach the concepts of integer numbers: temperature, elevations, elevators and Christian calendar. There is also the historical presentation of the construction of the set of negative numbers. The text presents a chronology about the integers and mentions several mathematicians who contributed positively to the construction of this numerical set.

The operation of the addition is presented by making reference to already studied concepts (natural numbers) and to the use of balls in green and red colors to represent negative quantities. These quantities are nullified when summed since each color represents a positive number and a negative one. Several situations are addressed to work on the concept of addition.

The subtraction is indicated in the same way as the addition, using colored balls representing the positive and negative numbers. It is found in the material a topic with practical actions to make sense of subtraction.

There is also a topic that relates to content already studied in previous years. The author presents addition and subtraction as reverse operations as well as in the set of natural numbers. He further emphasizes that subtracting a negative number is like adding its opposite.

The multiplication is approached in a historical context that mentions the history of the construction of the integers. The author uses, in the first moment, the idea of adding equal parcels to address the content and the idea of commutativity. He presents the product 3. (-4) as: (-4) + (-4) + (-4) + (-4) = -12

At the end of this approach, the author presents the rule of signals: The product will be positive if two numbers have equal signs and will be negative if they have different signs.

The third work analyzed is the textbook of the 7th year of elementary school: Project Teláris:

The book presents a chapter with forty-six pages devoted to the content of integer numbers, divided into: introduction; exploring the idea of positive numbers and negative numbers; the set of integers; comparison of integer numbers; operations with integers; numeric expressions with integers and Cartesian coordinates with integers.

On the first pages, the author presents several situations so that the reader can construct the concept of integers. Work is performed with situations involving temperature, altitude, and time zone. There is also a practical activity that can be done in the classroom in order to give meaning to the concepts involved in the process of learning the numerical field of integers. There are several situations in the book, however, in this first contact with the integers there is no historical reference to the process of constructing integer numbers or any other historical fact that refers to the content. However, the epistemological obstacle to acceptance of the idea of negative quantity is worked out from the situations already mentioned.

The next pages show the operations with the integers. The presentation is made using the number line associated with the notion of direction. There is no direct exposition of formulas for solving addition and subtraction, but there are situations that induce the student to discover them. There are situations of composition in addition activities and transformation situations in subtraction and there are also exercises of direct application of calculation procedure.

The addition is shown in the numerical line and the subtraction by the calculation of the inverse operation as follows: the result of a subtraction of integers can be obtained by adding the first number to the opposite of the second. Thus, the book uses the concept of symmetric additive to introduce the concept of subtraction of integers. Again, at no time were the historical issues addressed.

In the approach of multiplication and division, there is a great amount of activities in which the direct application of procedures is privileged. The author begins with the concept of multiplication attached to a multiplicative table to present the present regularity. The student must reproduce the table and after deducting the procedures (in multiplication, if a factor is zero, the result is zero; the product of two numbers with equal signs always have positive sign, the product of two numbers with different signs always has negative signals). There are, in sequence, a number of activities to be solved by applying these procedures.

The division is presented as inverse multiplication operation. To do, for example (-12): (+ 3), the student should have the following thought: what is the number that multiplied by +3 results in -12? After this small presentation of the concept of multiplication and division, students should continue with activities aimed at applying the procedures taught and learned.

Contrary to what happens with addition and subtraction, the book does not present situations that give meaning to the concept of multiplication and division. At no time does the book mention, either in content or activity, relationships between multiplication and division operations and the conceptual aspects of the multiplicative field (eg proportional thinking).

Just as in the introduction, addition and subtraction, multiplication and division do not present historical questions related to content.

Final considerations

From the analysis of these three didactic works, one can see how the integer content is approached and how the situations are presented.

The three books separate a chapter, varying the number of pages, to deal with the contents of integer numbers. In these chapters, we find subtopics for addition, subtraction, multiplication, and division operations. The three works begin the chapter with differentiated situations for the concept of integer to be understood. At this stage, it is noted that there is a concern in presenting several situations. There is an approach aimed at overcoming the epistemological obstacles linked to the acceptance of the idea of negative quantity and it also attends the theory of conceptual fields.

All authors work with several situations to expose and construct the concept. There is work with: altitudes, balances and banking strata, distances, temperatures, Christian calendar and time zone. With this, one realizes that there is a good approach in the concept construction issue. There are several situations that also help in the understanding and acceptance of the negative quantity. However, the historical approaches to the process of constructing integer numbers are still scarce, only one of the works analyzed was of concern to the question.

After approaching the concept, the authors treat addition and subtraction operations.

In this topic, they also present diverse and different situations. There are approaches of operations on the numerical line associated with distance. Some authors consider, as in additive conceptual field theory, addition and subtraction as a single algebraic operation and a single field, and thus do not separate them into two topics. It is possible to perceive a good number of situations in the approach to additive operations, which is important for the development of the additive conceptual field, because the greater the number of situations, the greater the number of schemes the student can develop and, solve complex situations.

However, after the initial approach, in the intended part of the activities, the books end up turning to formulas / procedures, which can lead the student to abandon the schemes that began to develop and adopt stereotypes of resolution. The historical issues facing the epistemological obstacles are once again abandoned.

After the approaches towards addition and subtraction, the authors work on multiplication and division. Contrary to what happens in the previous chapter, books do not present situations that give meaning to the concept of multiplication and division. The books do not mention at any time, either in content or in activity, relationships between multiplication and division operations and the conceptual aspects of the multiplicative conceptual field (eg proportional thinking). The concern is to present procedures for the student to perform the calculations. In the first moments, the rules of signs are already presented. The issues facing the epistemological obstacles are not worked, either in the historical questions or in the acceptance of the concept. It is observed that there is scarcity in the various situations and in the approach of the concept. Thus, it is believed that the approach of textbooks, in what concerns the multiplication of integers, propitiates the construction of schemes devoid of meaning.

Referências

ARRUDA, J. P. de; MORETTI, M. T. **Cidadania e Matemática: um olhar sobre os livros didáticos para as séries iniciais do Ensino Fundamental**. Contrapontos, n. 6, p. 423-437, setembro/dezembro de 2002.

BACHELARD, G. **A formação do espírito científico: contribuição para uma psicanálise do conhecimento**. Tradução Estela dos Santos Abreu. Rio de Janeiro: Contraponto, 1947.

BRASIL. **Parâmetros Curriculares Nacionais do Ensino Fundamental**. Brasília: MEC,1997.

BRASIL. Guia de livros didáticos PNLD 2010: matemática. MEC, 2010.

BOYER, C. B. **História da matemática**. Trad. Elza F. Gomide. São Paulo: Edgard Blücher, 1974.

CENTURIÓN , M.; JAKUBOVIC, J. **Matemática na medida certa** – Matemática- 7º ano, São Paulo: Leya, 2015.

DANTE, L. R. **Projeto Teláris** – Matemática – 7º ano. São Paulo: Ática, 2016.

IGLIORI, S. B. C. Obstáculo epistemológico e Educação Matemática. In: Silvia D. A. Machado. (Org.). **Didática da Matemática: Uma Introdução**. 1 ed. São Paulo: EDUC, 2010, v. 1, p. 89-11.

MAZZIEIRO, A. dos S.; MACHADO, P. A. F. **Descobrindo e aplicando a matemática**, 7º ano: livro do professor. Belo Horizonte: Dimensão, 2015.

MOREIRA, M. A. A teoria dos campos conceituais de Vergnaud, o ensino de ciências e a pesquisa nesta área. **Investigações em Ensino de Ciências** (UFRGS), Porto Alegre, v. 7, n.1, 2002.

POMMER, W. **Diversas abordagens das regras se sinais nas operações elementares em Z**. Disponível: http://scholar.google.com.br/. Acesso em: 02 abr. 2012.

SCHUBRING, G. **Análise histórica do livro didático de matemática: notas de aula**. Tradução: Maria Laura Magalhães Gomes. Campinas: Autores Associados, 2003.

Secretaria de Estado da Educação do Rio de Janeiro – SEEDUC/RJ. **Conexão Professor**. Disponível em: http://www.conexaoprofessor.rj.gov.br/curriculo_identificacao.asp.

Submetido em Abril de 2019
Aprovado em Maio de 2019