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PERIODIC PHENOMENA: a didactic sequence for the introduction of trigonometric functions

FENÔMENOS PERIÓDICOS: uma sequência didática para a introdução de funções trigonométricas

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ABSTRACT

This investigation is an excerpt from a master's thesis, which aimed to demonstrate the activities that were used to trigger the learning of the concepts of trigonometric functions. We used the modeling of a phenomenon so that the students perceived the regularities present when realizing the horizontal and vertical projections of a ball attached to a bicycle wheel. This research is qualitative in nature, in which for its development we carry out participant observation and documentary analysis. The theoretical references used include studies on the theory of meaningful learning and mathematical modeling. In general, as results, we identify a new perspective for the introduction of the concept of trigonometric functions using mathematical modeling. It should be noted that this work is an extension of a text already sent to the SIPEM.

KEYWORDS: Mathematics education, mathematical modeling, trigonometric functions, and meaningful learning theory.

RESUMO

Esta investigação é um excerto de uma dissertação concluída, que teve como objetivo demonstrar as atividades que foram utilizadas para desencadear a aprendizagem dos conceitos de funções trigonométricas. Utilizamos a modelação de um fenômeno para que os alunos percebessem as regularidades presentes ao realizarem as projeções horizontais e verticais de uma bolinha presa a uma roda de bicicleta. Essa pesquisa é de natureza qualitativa, em que para o seu desenvolvimento realizamos a observação participante e a análise documental. Os referenciais teóricos utilizados contemplam os estudos sobre a teoria da aprendizagem significativa e a modelagem matemática. De modo geral, como resultados, identificamos uma nova perspectiva para a introdução do conceito de funções trigonométricas utilizando a modelagem matemática. Cabe destacar que esse trabalho é uma ampliação de um texto já enviado ao SIPEM.

PALAVRAS-CHAVE: Educação Matemática, Modelagem Matemática, Funções Trigonométricas e Aprendizagem Significativa.

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Introduction

The present work is a cut of the master's thesis, in which a didactic sequence was proposed with the use of mathematical modeling for 25 students of the 3rd year of High School. This sequence aimed to allow students to relate periodic phenomena to trigonometric functions, especially the sine and cosine functions. In order to support the analysis and activities, the assumptions of Meaningful Learning Theory were used.

In the master's thesis 13 (thirteen) activities were presented, in which the objective was to work on the period and the trigonometric functions concepts. For being a clipping of the master’s thesis, we will present one of these activities in this article. This activitie is related to the movement of a bicycle wheel with it stopped, that is, the wheel turns but the bike does not leave the place. We chose this activity because it is possible to demonstrate the purpose of the master's thesis which was to answer the following question: "Are the teaching strategies guided by the modeling and referenced by the Meaningful Learning Theory potentiate students' learning regarding the trigonometric functions sine and cosine?"

In relation to trigonometry from our practice, we realize that the students feel many difficulties to assimilate the concepts of trigonometric functions, that is, they do not relate these functions to periodic movements like the waves of the sea, the index of rains and in our case the wheel of a bicycle, in this sense, we used the phenomenon of the movement of a ball attached to the wheel of a bicycle, in which this movement had the objective to organize the preexisting concepts in the cognitive structure of the students, that is, the modeling of these phenomena would be the organizer prior knowledge of the students.

Regarding the teaching of trigonometry in the National Curricular Parameters of Mathematics (NCP), it is stated that:

> [...] the relationship between learning mathematics and the development of skills and competencies and Trigonometry, provided that their study is linked to the implications, avoiding excessive investment in the algebraic calculation of identities and equations to emphasize the important aspects of trigonometric functions and analysis of their graphs. Particularly for the individual who will not pursue his studies in the so-called exact career, what should be ensured are the applications of Trigonometry in solving problems involving measurements, especially the calculation of inaccessible distances, and in the construction of models that correspond to periodic phenomena. In this sense, a project involving also physics can be a great opportunity for meaningful learning (BRASIL, 2000, p. 44).

In this sense, the NCP presents the importance of the students having contacts with the trigonometric functions, especially their graphs and, to relate them to the periodic phenomena present in nature.

The National Curricular Common Base (BNCC) in relation to trigonometry, mentions the importance of:

> Identify the fundamental characteristics of sine and cosine functions (periodicity, domain, image) by comparing representations in trigonometric cycles and Cartesian planes, with or without the support of digital technologies (BRASIL, 2017, p. 105).

BNCC adds that for the study of the trigonometric functions digital technologies can be used and that it is necessary to make comparisons between the representation of the trigonometric cycle to understand the periodicity, domain, an image of this type of function.

In this sense, we believe that our study provides contributions to the study of trigonometric functions since we use mathematical modeling to understand the regularities and the periodicity existing in the movement of a bicycle wheel and relating to the periodic functions.

Research methodology

This research has a qualitative nature, in which we seek to interpret the phenomenon under study, observing it in depth in order to analyze the process as a whole, based on the studies of Goldenberg (2007).

To obtain the data, we use participant observation. In this research model, the researcher inserts himself into the researched environment seeking the most reliable actions that appear in the middle of the investigation. For Wilkinson (1995): (i) it allows the entrance to certain events that would be privative and to which a strange observer would not have access; ii) allows observation not only of behavior, but also of attitudes, opinions, feelings, and overcome the problem of the observer effect.

For Lorenzato and Fiorentini (2012, p.108) the "participant observation" is a strategy that involves not only direct observation but the whole set of methodological techniques (including interviews, consultation of materials, etc.), assuming a great involvement of the researcher in the studied situation.

The records of the observations were carried out with the help of a field diary, this is an instrument in which the observer records the actions observed when he was inserted in the researched medium, on which we rely on the studies of Fiorentini and Lorenzato (2012, p.118 -119).

In our research we recorded what was observed in the field diary shortly after each meeting, we did not make the records at the time of observation because we understood that this could focus the participants in carrying out the activities.

In the research, we also used the data analysis seeking to relate the data obtained with the theory used. Lüdke and André (1986, p.38) point out that "documentary analysis can constitute a valuable technique for approaching qualitative data, either by complementing the information obtained by other techniques or by revealing new aspects of a theme or problem."

For our analysis, we used the student response protocols, the field diary records, and recordings that were performed at each meeting. We sought to interpret these records in the light of meaningful learning theory.

Theoretical framework

In this section, we present the theoretical framework used to develop the research: Meaningful Learning Theory (MLT) and Mathematical Modeling.

The MLT is a constructivist theory that it assumes that for the student to learn a concept, it must be linked to some knowledge already established in its cognitive structure. According to Santos (2014), the theory of Ausubel is a constructivist theory, because it has as a characteristic to explain how the new knowledge structured in previous knowledge of the student is formed. Thus, in this theory, the apprentice is a subject, protagonist of his own learning process, someone who will produce the transformation that generates self-knowledge.

This theory focuses mainly on the learning that occurs within the classroom. Ausubel, Novak, and Hanesian (1980) point out that this theory provides a foundation for teachers to discover more efficient methods for teaching. Like this:

> The theory aims at learning that occurs in the classroom, so this theory tries to give subsidies to teachers to create a better learning environment for students, not neglecting that evaluation is the responsibility of the teacher (AUSUBEL; NOVAK; HANESIAN, 1980, p.3).

For Ausubel et al. (1980), this construction, by the apprentice, does not take place by itself and in the cognitive emptiness, but from situations in which it can act on the object of its knowledge, think about it and seek the answers of its experience.

Thus, this author classifies two types of learning: i) mechanics, which is learning in which there is no logical and clear dialogue between new ideas and those already existing in the cognitive structure of the subject, that is, there are a non-substantive interaction and literal relationship between the new knowledge and subsuming[3] concepts of the cognitive structure of the learner and ii) the meaningful, in which the subject manages to establish a substantive and non-arbitrary relationship between the new knowledge and the knowledge that he already has.

Therefore, the challenge of the teacher in this theory is to propose situations in which the student can put his knowledge at stake and learn meaningfully the new knowledge.

We understand that Mathematical Modeling can be understood as a good teaching strategy since it presupposes the teaching of mathematical concepts from the students' knowledge, so we understand that modeling is perfect support for the theory of meaningful learning.

Modeling can be understood as a teaching strategy that allows the student to approach mathematical contents from phenomena of their reality and aims to explain mathematically everyday situations, from the most different areas of Science, with the purpose of educating mathematically. It allows a reversal of the "common model" of teaching, since, through modeling, problems are first selected and mathematical contents emerge in order to solve them (BURAK, 1987, 1992).

In Bassanezi's view (2015), Modeling is the art of transforming problems from reality into mathematical problems and solves them by interpreting their solutions in real-world language.

In the teaching practice, we perceive that Modeling has as the main characteristic, to lead the student to assimilate mathematical knowledge from real situations. However, there are different conceptions on how to apply Modeling in teaching. For Bassanezi (2015) and Burak (1992), students should choose the generative themes and the teacher from these

[3] Subsuming is the basis for meaningful learning, ie their relevant knowledge, already established in the structure, on which the foreground will be anchored.

choices should help the students to find the mathematical solutions to the chosen problem. For Beltrão (2009) and Sadovsky (2010), the choice of the subject by the students can be difficult as the school has a program to follow.

According to Burak (1992) and Bassanezi (2015), the choice of the theme that will generate the construction of a mathematical model must be attributed to the students. For this, the set of prior knowledge should guide the way forward in this construction process. In contrast, Beltrão (2009), assuming aspects of his practice, indicates that prior knowledge, the deadline previously established to build the course program and the requirements of the institution are obstacles to fruition of the guidelines to leave the student's responsibility to the choice of the theme in the Modeling process.

Barbosa (2001), after conducting a mapping of the researches that dealt with mathematical modeling, describes that there are three ways of performing the modeling for teaching in these ways the author presents as cases 1, 2 and 3, in these cases he presents the teacher's involvement in choose the themes to be modeled.

Barbosa (2001, p. 8-9) presents these cases as follows:

> Case 1. The teacher presents a description of a problem situation, with the information necessary to solve it and the problem formulated, and students are responsible for the resolution process.
> Case 2. The teacher brings to the classroom a problem from another area of reality, and it is up to the students to collect the necessary information to solve it.
> Case 3. From non-mathematical subjects, students formulate and solve problems. They are also responsible for collecting information and simplifying problem situations.

Although there is this difference between the attribution of the choice of the phenomenon, there is convergence to the understanding that the Modeling has as characteristic, to promote that the student seeks the solutions of the problems from their previous knowledge, mobilizing different knowledge to create strategies of resolution, evaluation and reflection on the problem studied.

Developed activities

Ausubel, Novak, and Hanesian (1980) indicate that for learning to be meaningful the new knowledge must start from the more general concepts to the more specific ones, in that sense realizing a progressive differentiation and integrative reconciliation. Thus the activity started from more general ideas about periodic movements, the movement of a bicycle wheel, to more specific knowledge the sine and cosine functions.

Moreira and Buchweitz (1993) point out that in meaningful learning the student must have previous knowledge to anchor the new knowledge, so we assume in this work that the bicycle movement is already known empirically by the students.

In relation to modeling, we used for this work case 2 presented by Barbosa (2001) in which the teacher presents the phenomenon and the students collect the data making the necessary resolutions.

Before starting the activity we present the students with a bicycle and show the movement that we wanted to study, that is, the movement of a tennis ball attached to the wheel, in which the wheel moves, but the bicycle does not leave the place, for this we raise the rear wheel and turn the pedal. Next, we present the activity in which the students would validate or not their ideas, we do not use the bicycle for these measurements.

The activity carried a text about cycling in which it was highlighted accessories that would be placed on the bike to make it different, in the case presented was a ball attached to the wheel (Figure 1).

Figure 1- Activity Statement

O ciclismo traz benefícios físicos e emocionais, contribuindo muito para a qualidade de vida. "Como atividade aeróbica, gera perda de peso, ajuda a equilibrar a pressão e os níveis de triglicérides. Também trabalha equilíbrio e confiança, além de relaxar e combater o estresse. Praticada com bom senso e na medida da forma física de cada um, a atividade quase não tem restrições", além disso, existem pessoas que adoram decorar suas bicicletas, seja com adesivos ou outros acessórios colocados na roda ou em outras partes da "bike".

Felipe, um aluno do 2° série do ensino médio adora andar de bicicleta e em sua roda ele costuma colocar uma bolinha de tênis como mostra a figura.

Quando o mesmo estava aprendendo funções periódicas na escola, decidiu fazer uma representação do movimento dessa bolinha em relação ao eixo vertical e horizontal tomando por referência os ângulos.

Obs.: Considere o tamanho total do raio da roda da *bike* como 1 (unitário), assim cada subdivisão (será dividida em 10 partes) do transferidor equivale a 0,1.

Source: COSTA (2017 p.55).

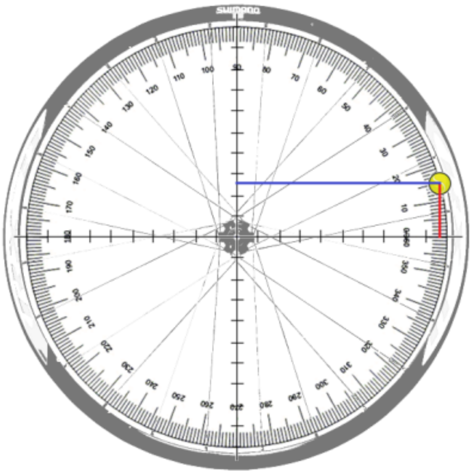
The purpose of the statement was to introduce the ideas that would be worked out in the three subsequent activities and also to "activate" the students' knowledge about cycling.

Next, an activity is presented to the students, in which they would check the horizontal and vertical projections of a ball attached to the radius of a bicycle; the ray was subdivided into ten parts so that the students could establish the sine and cosine trigonometric ratios (Figure 2).

In this activity the students looked for the data in the activity itself and perceived the regularities present in the filling of the table, in this way, they modeled the worked periodic phenomenon.

Figure 2 - Table of trigonometric ratios

1. Complete a tabela a seguir, usando a imagem acima: (Utilize a régua)



Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96					
Vertical	0,26					

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal						
Vertical						

Ângulo	195°	210°	225°	240°	255°	270°
Horizontal						
Vertical						

Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal						
Vertical						

Source: COSTA (2017 p.56-57)

The activity does not yet have the names sine and cosine, because the intention is that the student uses the concepts he already knows in the case of this activity horizontal and vertical distances (more general knowledge) to later introduce more specific knowledge (sine function and cosine).

In carrying out this activity students placed the "ball" at the indicated angle and performed an approximate measurement of the "fraction" (figure 3) of the value obtained in horizontal and vertical. In this construction, they verified that the movement of the ball along the angles established patterns. In this sense, they realized that the movement of the ball on the bicycle was always the same and that the movement is repetitive (periodic).

The modeling for this activity was not intended to allow students to arrive at a mathematical formula, the goal was for them to realize that the movement has patterns, so the teacher could later systematize sine and cosine functions.

Bassanezi (2015) indicates that much more than modeling a phenomenon the student must realize that mathematics is present in several situations of his daily life, in this sense the modeling can be conceived as a trigger for learning new mathematical concepts in our case trigonometric functions.

This activity seeks to relate the ideas of cycling to the construction of a vertical projection table (sine) and horizontal projection (cosine) of a ball using the size of the radius. The movement that we are dealing here is the movement of the ball with the bicycle stopped. The activity aims to investigate what the students know about cycling to build a new concept. It agrees with Miras (2006) that introduces:

> [...] prior knowledge is the foundation of the construction of new meanings. A more meaningful learning is the more meaningful relationships the learner is able to establish between what he already knows, his previous knowledge and the new content presented to him as an object of learning ... if we put ourselves in the perspective of the student, in the logic of the constructivist conception, it is possible to affirm that there can always be previous knowledge regarding new content to be learned, otherwise it would not be possible to attribute an initial meaning to the new knowledge. (MIRAS, 2006, p. 61-62).

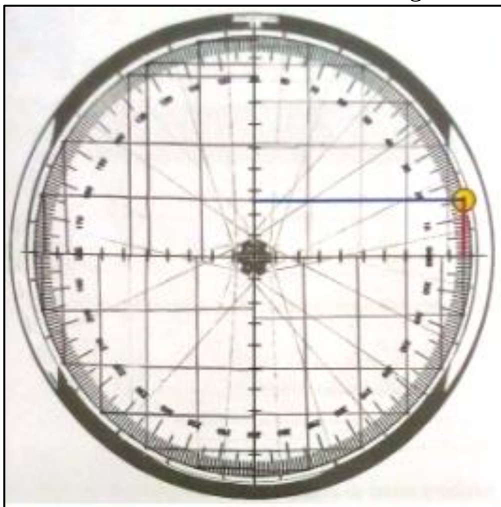
It is also emphasized that for Ausubel, Novak, and Hanesian (1980) the essence of Meaningful Learning Theory is that new ideas must be related to some relevant aspect of the student's cognitive structure, as an example, they highlight images, symbols, and propositions. In doing so the authors believe that students will be predisposed to learn, another important aspect of meaningful learning.

Analysis of the presented activity

The activities were carried out with a group of 25 students from the 3rd grade of the High School of a public school in São Paulo. They were organized into groups of 5 students. The activity proposal was the application of a didactic sequence with the use of modeling in 8 lessons of 50 minutes. The purpose of the application was to use several tools to enhance the learning of the contents of trigonometric functions, that is, we used concrete materials such as bicycle, ruler and protractor and technology GeoBrain.

We will present the responses of the groups (in Figures 3, 4, 5, 6 and 7) in front of the activity presented in the previous section since they show how the students performed when solving them.

Figure 3 - Student Protocol - Group 1



1. Complete a tabela a seguir, usando a imagem acima: (Utilize a régua)

Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96	0,89	0,71	0,49	0,23	0
Vertical	0,26	0,5	0,7	0,87	0,95	1

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal	-0,26	-0,52	-0,71	-0,89	-0,96	-1
Vertical	0,96	0,89	0,71	0,52	0,26	0

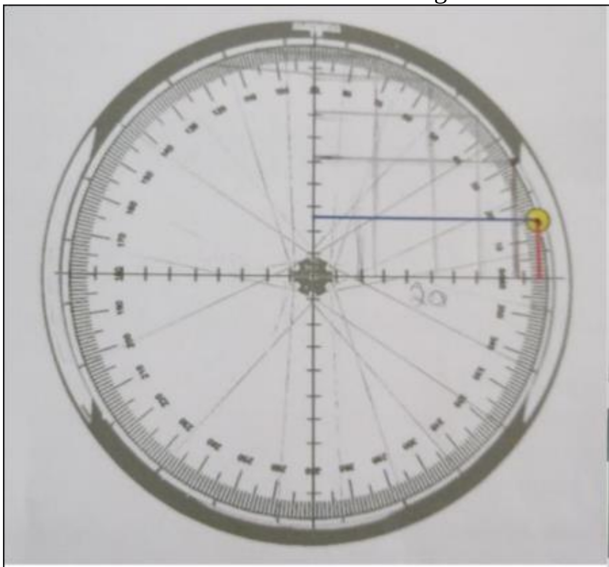
Ângulo	195°	210°	225°	240°	255°	270°
Horizontal	-0,96	-0,89	-0,71	-0,52	-0,26	0
Vertical	-0,26	-0,52	-0,71	-0,89	-0,96	-1

Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal	0,26	0,52	0,71	0,89	0,96	1
Vertical	-0,96	-0,89	-0,52	-0,26	-0,26	0

Source: Researcher's Authorship

Although the group was dispersed during the activity, they were able to correctly answer all the blank fields of the tables, but it is worth noting that the students did not perceive the regularity in the quadrants, being necessary that they fill all angles of the table.

Figure 4: Student Protocol - Group 2



Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96	0,89	0,71	0,52	0,26	0
Vertical	0,26	0,52	0,71	0,89	0,96	1

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal	-0,26	-0,52	-0,71	-0,89	-0,96	-1
Vertical	0,96	0,89	0,71	0,52	0,26	0

Ângulo	195°	210°	225°	240°	255°	270°
Horizontal	-0,96	-0,89	-0,71	-0,52	-0,26	0
Vertical	-0,26	-0,52	-0,71	-0,89	-0,96	-1

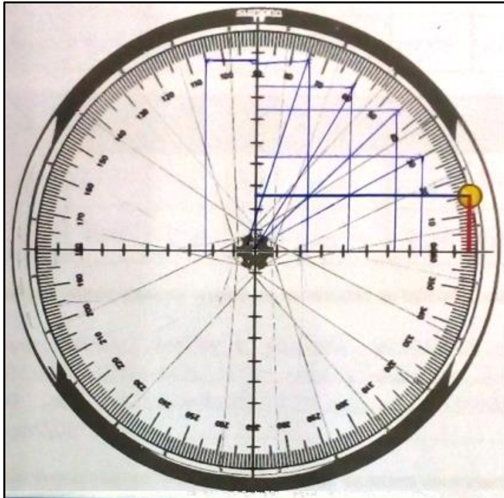
Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal	0,26	0,52	0,71	0,89	0,96	1
Vertical	-0,96	-0,89	-0,72	-0,52	-0,26	0

Source: COSTA (2017 p. 90)

The protocol of group two demonstrates that they were able to respond to what was proposed and that they still realized that the values are repeated so the group did not feel the need to take notes in the four quadrants of the activity.

Aragão (1976) describes that when the learning is meaningful the learner is able to put their knowledge into play without the help of concrete materials.

Figure 5 - Student Protocol - Group 3



1. Complete a tabela a seguir, usando a imagem acima: (Utilize a régua)

Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96	0,74	0,62	0,41	0,22	0,0
Vertical	0,26	0,43	0,65	0,75	0,85	0,8

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal	-0,22	-0,41	-0,62	-0,74	-0,96	0,8
Vertical	0,85	0,75	0,65	0,43	0,26	0,0

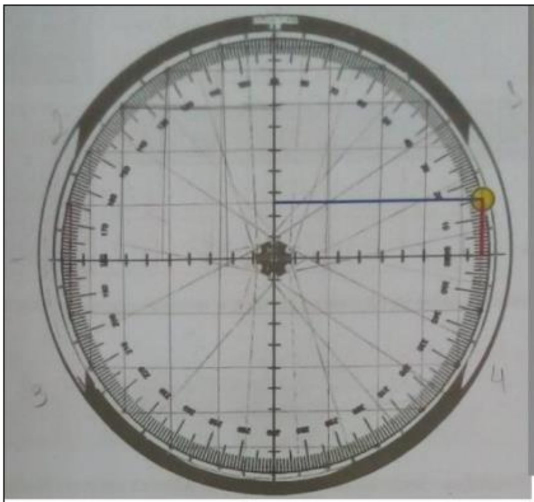
Ângulo	195°	210°	225°	240°	255°	270°
Horizontal	-0,96	-0,74	-0,62	-0,41	-0,22	0,0
Vertical	-0,26	-0,43	-0,65	-0,75	-0,85	-0,8

Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal	0,22	0,41	0,62	0,74	0,96	0,8
Vertical	-0,85	-0,75	-0,65	-0,43	-0,26	0,0

Source: Researcher's Authorship

Group 3 seemed to have an understanding of the regularity of the movement, however, by placing the points in the wrong positions in relation to the example, it filled the table erroneously. The activity for them was meaningful because they perceived regularity, but the errors in indicating the points led to incorrect answers.

Figure 6: Student Protocol - Group 4



1. Complete a tabela a seguir, usando a imagem acima: (Utilize a régua)

Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96	0,88	0,7	0,49	0,24	0
Vertical	0,26	0,5	0,7	0,86	0,95	1

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal	-0,21	-0,51	-0,71	-0,87	-0,97	-1
Vertical	0,95	0,86	0,7	0,5	0,26	0

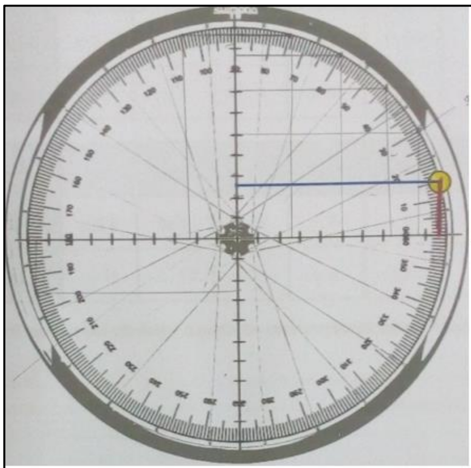
Ângulo	195°	210°	225°	240°	255°	270°
Horizontal	-0,97	-0,87	-0,71	-0,51	-0,21	0
Vertical	-0,25	0,5	-0,71	-0,88	-0,99	-1

Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal	0,24	0,49	0,7	0,88	0,96	1
Vertical	-0,99	-0,88	-0,71	-0,5	-0,25	0

Source: COSTA (2017 p. 92).

The group 4 protocol also presents the correct responses to the activity, but different from the previously presented group did not perceive the regularity present in the quadrants, needing to verify all the values in each quadrant. In the process of meaningful learning the learner puts his knowledge into play, here we see that they present an understanding of the quadrants, but still do not realize at that moment that there is regularity, in this sense the teacher should use the above stated by the students to establish the relationships.

Figure 7: Student Protocol - Group 5



1. Complete a tabela a seguir, usando a imagem acima: (Utilize a régua)

Ângulo	15°	30°	45°	60°	75°	90°
Horizontal	0,96	0,88	0,73	0,5	0,26	0
Vertical	0,26	0,5	0,7	0,88	0,96	1

Ângulo	105°	120°	135°	150°	165°	180°
Horizontal	-0,26	-0,5	-0,7	-0,88	-0,96	-1
Vertical	0,96	0,88	0,73	0,5	0,26	0

Ângulo	195°	210°	225°	240°	255°	270°
Horizontal	-0,96	-0,88	-0,73	-0,5	-0,26	0
Vertical	-0,26	-0,5	-0,7	-0,88	-0,96	-1

Ângulo	285°	300°	315°	330°	345°	360°/ 0°
Horizontal	0,26	0,5	0,7	0,88	0,96	1
Vertical	-0,96	-0,88	-0,73	-0,5	-0,26	0

Source: COSTA (2017 p. 93).

Group 5 also seemed to have an understanding of the regularity of the movement and correctly recorded the values in the table, this group was the most active during the activities, its members constantly discussed searching the best ways for the answers.

Here we present the activity proposed for this article, in which the main objective was to verify if the students established relationships between their previous knowledge and the new knowledge that would be structured from the data pointed out in the table. Groups 2 and 4 succeeded in establishing the required relationships.

Conclusions

In this work we aimed to demonstrate an activity that was used to trigger the learning of the concepts of trigonometric functions, here we used the modeling of a phenomenon, the movement of the tennis ball attached to a bicycle wheel, so that the students perceived the present regularities when making the horizontal and vertical projections of a ball attached to a bicycle wheel.

In addition to realizing that the movements are periodic and that the increase of the ray did not interfere in the values of the projections since they are ratios between horizontal or vertical length and radius. Thus we believe that this work gives a new perspective for the introduction of the concept of trigonometric functions since it is part of the knowledge present in the cognitive structure of the student and foments subsidies for the introduction of new concepts.

As future perspectives, this work can be used by several teachers in the site www.ensinomatematica.com.br, and from this use, can derive research on the implementation of Modeling in the teaching of periodic functions in different contexts of teaching. This research is important for mathematics education, since it offers a new perspective for the teaching of periodic functions, being subsidized by Modeling and Theory of Meaningful Learning.

The use of methodology and theory ensured that this study did not start from a cognitive vacuum of the author, but from a systematic study of several researchers.

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