REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

Volume 12, número 28 – 2019

ISSN 2359-2842

Spatial Geometry in High School: Contributions of the use of a Teaching and Learning Unit (UEA)

A Geometria Espacial no Ensino Médio: contribuições da utilização de uma Unidade de Ensino e Aprendizagem (UEA)

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ABSTRACT

This article presents a research carried out with high school students that aimed to investigate contributions of the use of a Teaching and Learning Unit through digital technologies for the development of spatial geometry concepts. Based on a qualitative approach, the study took as reference the Van Hiele model of development of geometric thinking. The results show that the use of digital technologies, especially of GeoGebra software, enabled the visualisation, construction and movement of different geometric objects, helping students to identify properties, analyse, conjecture and propose solutions. The analysis made it possible to perceive students' progress from the level of visualisation to the level of analysis, and indications of the transition from the level of analysis to the level of informal deduction.

KEYWORDS: Spatial Geometry, Van Hiele Model, Digital Technologies, GeoGebra Software.

RESUMO

O presente artigo destaca uma pesquisa realizada junto a estudantes do Ensino Médio que teve por objetivo investigar contribuições da utilização de uma Unidade de Ensino e Aprendizagem, com recurso às tecnologias digitais, para o desenvolvimento de conceitos da Geometria Espacial. A investigação, de base qualitativa, tomou como referência o modelo de desenvolvimento do pensamento geométrico de Van Hiele. Resultados apontam que o recurso às tecnologias digitais, particularmente a utilização do *software* GeoGebra, possibilitou a visualização, construção e movimentação de diferentes objetos geométricos permitindo identificar propriedades, analisar, conjecturar e propor soluções. A análise permitiu perceber um avanço dos estudantes do nível de visualização para o nível de análise e indícios da transição desse nível para o de dedução informal.

PALAVRAS-CHAVE: Geometria Espacial, Modelo de Van Hiele, Tecnologias Digitais, Software GeoGebra.

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Introduction

The teaching of geometry in Brazilian schools, at all levels, has undergone different stages in the last decades, according to Pires (2008). The author highlights three curriculum milestones that influenced the organisation of curricula regarding geometry: the so-called modern mathematics movement (1965 to 1980), the establishment of guidelines that sought to counteract the modern mathematics movement, led by state and municipal Secretariats (1980 to 1994), and, finally, the national project of curriculum reform, that refers to the National Curricular Parameters (PCN, in Portuguese) (as of 1995). However, we believe that, today, the discussions around the constitution of the National Curriculum Common Base – in Portuguese, BNCC - inaugurate a new period of reflections of the school curriculum in general, and particularly in relation to geometry.

The BNCC (BRASIL, 2018), as already highlighted in the PCN (BRASIL, 1998), points out how important it is that the geometric concepts are included in the mathematics curriculum, since the development of geometric thinking offers the students a type of view that allows them to understand, in an organized way, the world in which they live. According to the document, elementary education students should know how to make and how to validate conjectures, resorting to models, sketches, known facts, relationships and properties that may help to structure the thinking and the inductive and deductive reasoning, which can be enhanced by the development of the geometric concepts and their application in solving problem situations. Thus, the geometry developed throughout elementary school can and should be resumed in high school, since the skills expected for high school education "... will deepen and broaden the skills proposed in elementary school" (BRASIL, 2018, p. 522).

Despite the outstanding importance of geometry in the basic education, research works such as Pavanello's (1993), Pereira's (2001), Lorenzato's (2006) and Barbosa's (2011) indicate that the geometry teaching and learning is almost absent in basic education. The authors discuss the question, especially seeking to identify the causes that have led to the limited presence of geometry in the classrooms. These causes are somewhat related, according to the authors, to the influence of the so-called modern mathematical movement, and the perception that mathematics teachers themselves have difficulties with the subject.

Barbosa (2011) considers that teachers often do not approach geometry in the classroom because they themselves find difficulties with the content, since they may not have appropriated properly the matter during their own education process. Lorenzato (2006) emphasised the existence of a vicious circle, pointing out that a generation that did not study geometry does not know how to teach it.

On the other hand, there is a growing movement encouraging investigation, discussion and reflection on geometry to be worked in school, and how to conduct its teaching. Andrade and Nacarato (2004), based on research carried out in the proceedings of the Encontro Nacional de Educação Matemática/National Meeting of Mathematics Education from 1987 to 2001, present as an emerging trend in the research a work that involves geometry from an experimental perspective, developed in computational environments, which was corroborated by Petry's (2013) research, based on a similar analysis, although with a different methodology, in the period 2004-2010.

In this context, we agree with Valente (1999), when he affirms that the so-called traditional education has not been producing satisfactory results, mainly because it does not adapt teaching methods to the daily reality of the student and, in this context, the use of technology can create possibilities for working with mathematics, particularly with geometry, which is not present in other environments. All these issues, discussions, reflections and questions about the teaching and learning of geometry in basic education have led us to elaborate the question that prompted this research, referring to the possibility of using digital technologies in mathematics classes and their contributions for the teaching and learning of spatial geometry in high school.

Thus, this article brings results from the research carried out within the scope of a master's dissertation, whose objective was to investigate the possible contributions of the use of a Teaching and Learning Unit (in Portuguese UEA), using digital technologies, in the development of concepts of spatial geometry with a group of 3rd-grade high school students. It is an extension of the article "Contribuições de uma Unidade de Ensino e Aprendizagem (UEA) para o ensino de Geometria Espacial/Contributions of a Teaching and Learning Unit (UEA) for the teaching of spatial geometry" presented at the VII International Seminar on Research in Mathematics Education (SIPEM), in the year 2018, in Foz do Iguaçu, Paraná, Brazil.

Digital Technologies and the Teaching of Geometry

Recently, the use of digital technologies in mathematics education has influenced research regarding the development of students' geometric thinking, as much as teachers' posture and action. Kaiber, Vecchia and Scapin (2010) point out that the exploration of computational resources is necessary for education to fulfil its role, in a context where technology is increasingly present. The authors agree with the use of these resources because they provide students with interaction with a differentiated learning space, with access to different media such as softwares, videos, learning objects, chats, which, as already mentioned, are not present in other environments.

On the use of digital technologies, and especially on the so-called dynamic geometry softwares, Zulatto (2002) and Fernandes (2008) emphasised that the tools encourage an inquisitive attitude, allowing students to explore and manipulate objects, constituting a dynamic environment, which favours learning situations.

According to Pinto (2016), dynamic geometry softwares help to enhance learning, because through geometric constructions students can visually materialise the concepts. In this line of thinking, Fainguelernt (1999) stated that the use of the computer provides the student with simulations of situations, constructions of procedures, enabling him to analyse mistakes, correcting, resuming and adjusting them.

Agreeing with the authors, we consider that these softwares present resources with which the students can make geometric constructions - usually made with a ruler and a compass - while moving the objects, opening possibilities for the students to look at the same object from different perspectives, to experiment, to conjecture, to launch hypotheses and to test it by manipulating it, aiming at extracting its characteristics, properties and relationships. Besides, it is possible to carry out constructions that, through the traditional means, might be complex.

Thus, in order to provide students with the possibility of constructing geometric objects under different perspectives and forms of representation, modifying and transforming them, also allowing visual monitoring of the changes made, GeoGebra software was chosen for the research work.

However, we understand that providing a favorable environment for the construction of a geometric thinking throughout the school trajectory is not only linked to the use of resources for manipulation and experimentation with the use of digital or other technologies. A curriculum development perspective incorporating a decision-making on why to teach geometry, what to teach and how to teach considering the nature of the geometric objects and the specificities of the development of geometric thinking is mandatory. In this sense, Van Hiele model of development of geometric thinking can be conceived as a possibility of articulating what to teach and how to teach geometry, which is why we highlight aspects of this model.

Van Hiele Model and the Development of Spatial Geometry

Van Hiele's model of development of geometric thinking was built by Dina van Hiele-Geldof and Pierre van Hiele, in the Netherlands, in 1957 (CROWLEY, 1994). The model bases the work with spatial geometry developed in this research. Thus, it emphasises aspects of the model trying to establish or to point out characteristics of the levels of geometric thinking to be developed in the scope of the spatial geometric thinking. The properties and learning stages of the model were taken as in Van Hiele model, since we assume that they are guidelines that can be used to teach different subjects.

Next, the levels of the Van Hiele model are presented in Figure 1, with reference to the space geometry to be worked in high school. This approach was carried out with reference to the Van Hiele levels presented in Crowley (1994), and skills described in Hoffer (1981), namely: visual, verbal, graphic, logic and application skills.

Figure 1 - Spatial geometry from the perspective of the levels of comprehension of the Van Hiele model

Level/Descriptor	Spatial Geometric Thinking/Skills
Visualisation Perception of space as something that exists in the environment; geometric concepts seen as total entities, and not as entities that have components or attributes; identification of specific forms and their reproduction; learning a basic vocabulary.	• Identifies spatial geometric figures in objects or constructions in the environment and in representations. • Identifies geometric figures in space and planned. • Constructs geometric solids in cardboard, straw, or other materials. • Describes geometric figures using non-standard language (a cube looks like a box), for example.
Analysis From the observation and experimentation, the students begin to perceive characteristics of the geometric figures and to identify the properties; they recognise the figures by their parts. However, at this level, students cannot explain the relationships between properties and include classes.	• Identifies, classifies and compares solids according to their characteristics and properties. • Identifies and draws a solid in space, from an oral or written description of its properties. • Identifies the solid from different views. • Makes surface inferences from examples. • Uses appropriate vocabularies and symbols. • Solves geometric problems that require knowledge of the properties of solids in spatial and geometric relationships.

Informal deduction They can make interrelationships of properties between different figures; they are able to deduce properties and recognise classes of figures; they include classes and understand the meaning of definitions; they follow an informal test but cannot do it.	• Demonstrates understanding of the meaning of the concept, definitions, properties, characteristics of each spatial geometric figure. • Develops and uses definitions to describe solids. • Adds classes. • Presents informal arguments, from solid constructions or drawings. • Solves problems considering the properties and interrelationships between the figures. • Identifies information implicit in a particular spatial solid or in some information.
Formal deduction They can derive information from given information; make formal tests; master the deductive process; make interrelationships between axioms, postulates, definitions, theorems and demonstrations; are able to construct demonstrations of different forms.	• Identifies what is given and what should be proven in a situation or problem. • Verifies the relationships developed in the previous level (Informal Deduction). • Uses different demonstration techniques. • Compares different statements using one argument to accomplish another.
Rigor They can understand and use different axiomatic systems, as well as understand non-Euclidean geometry.	• Does formal tests. • Compares systems based on different axioms. • At this level, the non-Euclidean geometries are understood.

Source: Rodrigues; Kaiber (2017).

Based on the descriptors and skills indicated in the levels of the adapted model, the activities for the Teaching and Learning Unit proposed in the research were developed. These descriptors and abilities were also used to elaborate the research instruments, as well as reference for the analyses carried out.

Notwithstanding, the activities elaborated in the UEA did not go beyond the level of informal deduction of the model, but the skills proposed by Hoffer (1981) were emphasised whenever possible. Under the scope of the Van Hiele model beyond the levels, the learning stages were also considered in the planning of the work developed with the students.

Methodological Aspects

The research presented here is qualitative, but the use of quantitative data allowed us to highlight aspects of students' development, bringing important elements to the analysis. It was developed with a group of forty 3rd-grade high school students of Marechal Rondon State School[3], located in the municipality of Canoas, Rio Grande do Sul, in the first half of 2017.

[3] Research approved by the Ethics Committee in October 2016, number 59898416.9.0000.5349

The research group consisted of 60% female and 40% male students. The age ranged from 15 to 18 years, with 85% of students between the ages of 16 and 17 years old.

The investigation was organized in three stages. The first stage was to structure the UEA. It consisted of a set of activities to be developed in the classroom, using many digital technologies. The unit was organised in three axes: Geometry of Position, Primitive Notions and Basic Knowledge, and Polyhedra: Prism and Pyramid.

In this first stage, the following research instruments were also constituted, namely: Profile Questionnaire, Initial Research Instrument, Intermediate Research Instruments and the Final Research Instrument, all used throughout the investigation.

In the second stage, the UEA was applied to the participating research group, as well as the different research instruments. And in the third stage, the data were organised and analysed.

Analysis and Discussion of Results

At the beginning of the work, the Initial Research Instrument was used to identify aspects of the previous knowledge of the students participating in the research regarding the standard language, definitions of spatial and plane geometric objects, similarities and differences between them, relationship of position between lines, between planes and between plane and straight, as well as primitive notions.

For the analysis of the activities proposed in the research instruments, three categories were considered to enable us to evaluate the students' performance in activities and tasks, considering the Van Hiele model: unsatisfactory, percentage of correct answers less than 50%, satisfactory, percentage of correct answers between 50% and 70% and very satisfactory, percentage of correct answers greater or equal to 70%.

The Initial Instrument was composed of eight questions, classified according to the levels of the Van Hiele model (levels of visualisation, analysis and informal deduction), as highlighted in Figure 2, which shows the performance of the class in each question of the instrument.

Figure 2 - Performance of the class in the Initial Instrument

Performance of the Class (%)						
Question	Model level	Very Satisfactory	Satisfactory	Unsatisfactory	Did not answer	Total
1	Visualisation	100	0	0	0	100
2	Visualisation	62.5	22.5	15.0	0	100

3	Analysis	82.5	14.0	0	3.5	100
4	Analysis	27.5	31.0	18.5	23.0	100
5	Informal deduction	35.0	0	46.5	18.5	100
6	Informal deduction	53.0	0	37.5	9.5	100
7	Analysis	25.0	57.5	12.5	5.0	100
8	Informal deduction	10.0	32.5	37.5	20.0	100

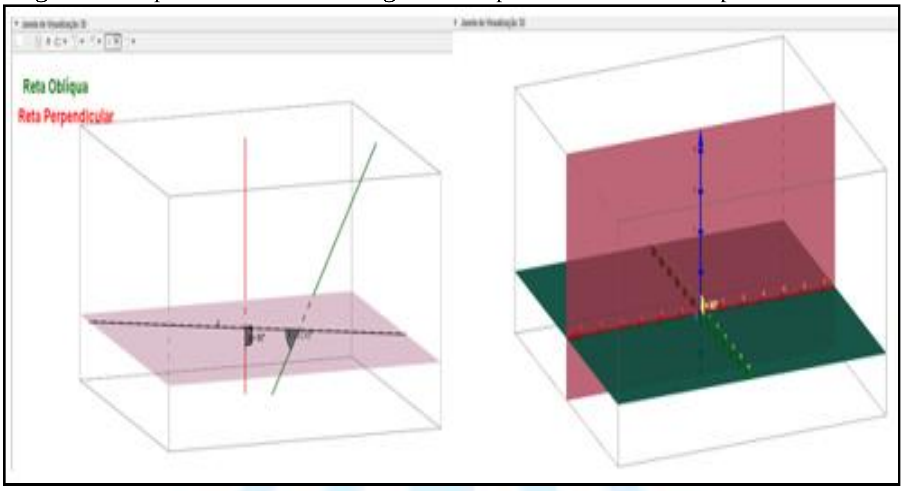
Source: Rodrigues (2018).

The analysis of this instrument enabled us to observe that the performance of the class in questions involving the level of visualisation was 92.5%, presenting a very satisfactory performance. This result is considered compatible with a performance of 3rd-grade high school students in relation to the contents of spatial geometry. Regarding the questions belonging to the level of analysis, the performance was around 79% (average of satisfactory and very satisfactory performances). On the other hand, in the questions that demanded knowledge of the skills related to the level of informal deduction, the students presented greater difficulty in understanding and solving the activities, and the average of the satisfactory performance (10.83%) and very satisfactory performance (32.66%) was around 21.7%, while about 15.5% of the students did not respond to relevant activities at this level.

Thus, from the results taken from this Initial Instrument, the materials of the Teaching and Learning Unit (UEA) that were in the process of elaboration were perfected and adequated. As already pointed out, the UEA was organized in three thematic axes: Geometry of Position, Primitive Notions and Basic Knowledge, and Polyhedra: Prism and Pyramid and, during its application, several activities were developed with the students, and certain activities were used as research instruments (in this case, they were part of the Intermediate Research Instruments). These tools have been developed and applied to monitor and analyse the development of students' knowledge throughout the application of the UEA. In the following, some of the activities selected to compose the so-called Intermediate Research Instruments are presented.

In the Geometry of Position axis, the relationships of position, in the plane and in the space, between straight lines, between planes and between straight lines and the plane were approached, trying that students understood these concepts. Throughout the application, GeoGebra software was used to provide situations where they could visualise, relate, modify, alter, compare the geometric relationships of position and thus conjecture on the characteristics and properties of the concept studied, as exemplified in Figure 3.

Figure 3 - Representation of some geometric positional relationships in GeoGebra



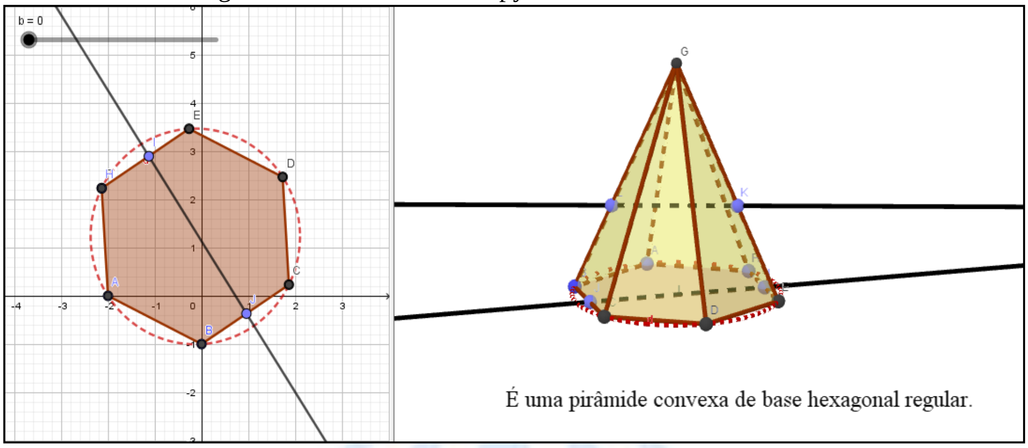
Source: Rodrigues (2018).

The results presented by the class were very positive, confirming what Giraldo, Caetano, Mattos (2012) emphasise regarding the use of a dynamic geometry software, because such use allows the students to interact with the objects dynamically based on the properties and relationships established, which enhances learning. The discussions raised by the solutions the students proposed allowed the presentation of syntheses of ideas that were being articulated for the formation of a certain concept and understanding of properties.

In the Primitive Notions and Basic Knowledge axis, students were offered a review of the basic concepts of plane geometry, also presenting the basic concepts of spatial geometry. In addition, students were asked to construct any geometric solid with the GeoGebra software, showing the characteristics and properties through treatments performed using the tools provided in the software and through observation.

The use of GeoGebra allowed the students to make geometric constructions that they do not usually make with their rulers and compasses. Thus, with the resources available, they could see one object from different perspectives, favouring experimenting, launching hypotheses and testing them, making conjectures, seeking to extract characteristics, properties and geometric relationships. Standing out in Figure 4 is the construction a group of students made and the analysis they carried out.

Figure 4 - Construction of a pyramid in GeoGebra software



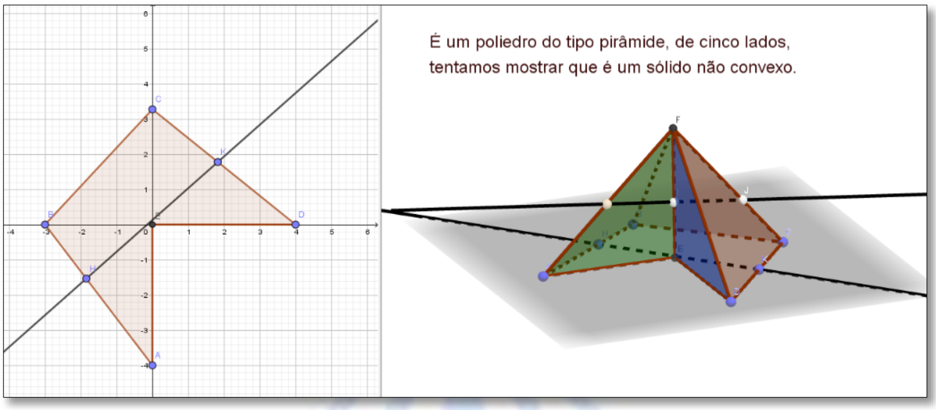
Source: Rodrigues and Kaiber (2018).

By observing the construction in the software, we could identify that the group of students had intuitive notions and even previous knowledge of concepts and properties that were important for the development of the geometric thinking. The construction of the circumference circumscribed in the polygon at the base of the pyramid, when presented to the large group, generated a productive and controversial discussion about regular polygons, as one of the participants argued that if a polygon was inscribed in a circumference, that would be a sufficient condition for it to be considered regular. On the other hand, to identify whether the constructed polygon was convex, the students chose to construct a line cutting the polygon, and from the movement of the line, they concluded visually on such attribute.

In this activity, it was also possible to perceive that the students mastered the skills indicated in the level of visualisation and analysis proposed by Van Hiele, since they presented knowledge related to properties and definitions of the represented geometric solid, which allowed to identify that they were developing skills related to the level of informal deduction of the model. From the constructions carried out and the analyses and discussions that followed, they were able to identify the characteristics and particularities of a regular polygon, which later led to the search for a definition for such object.

In the Polyhedra: Prism and Pyramid axis, the concept of Polyhedron (classification, elements, properties), as well as the concepts of Prism and Pyramid were worked. It is noteworthy that during the application of the UEA, different strategies and resources were used, such as constructing and planning solids with actual material, using different packages and models of acrylic solids and constructing with GeoGebra software, offering students different contexts. In Figure 5, an example of activity developed in the Polyhedra axis is presented.

Figure 5 - Activities proposed in the Polyhedra axis



Source: Rodrigues and Kaiber (2018).

The students presented skill and creativity when using the software in their constructions, however, they found difficulties with the concepts involved.

We can see in Figure 5 that the students chose to make representations of lines intercepting the polygon in the plane and in the faces of the polyhedron, as it had been done in the previous activity, trying to characterise the solid as non-convex. The group used various software tools to identify the characteristics of the object, making movements that allowed them to realise and identify, through that action, characteristics and properties. Through those procedures, they could identify that the object constructed was not a pyramid as "the base was a non-convex polygon", being called a "pyramid"-type polyhedron (which, in fact, is not a pyramid). The attempt to describe in natural language the object represented made possible the discussion about whether spatial figures have "sides" (rather than faces) and what would the edges be.

Thus, during the resolution of this activity, we verified that the students well-adapted and well-developed skills for spatial geometry at the level of visualisation of the Van Hiele model, considering that they performed the activity without great difficulties. However, students could not express themselves properly about important properties and notions to characterise the solid, evidencing, in this way, that they, in part, lacked knowledge, and did not have skills at the level of analysis and informal deduction of the model.

Regarding the accomplishment of these activities, we agree with Moraes (2014), who highlights the importance of the visualisation skill in geometry. According to the author, visualisation is an important process in teaching and learning, arguing that the development of visual thinking is quite complex. She suggests that through the use of the software, it is possible to propose a work that contributes to such development.

We also consider that the results obtained in the work with the Learning Unit were promising in face of the performance presented by the group in the Initial Instrument. The development of the UEA, together with the students' interaction with the GeoGebra software, has led to the development of the skills highlighted by Hoffer (1981), as well as enhanced knowledge. Corroborating Fernandes (2008), Kaiber, Vecchia and Scapin (2008) and Pinto (2016), we understand that the use of technology, in this case the use of dynamic geometry software, can be an ally to the students' cognitive development, since it allows to develop a work favouring different rhythms of learning, opening a space for them to seek solutions, analyse, build hypotheses and make conjectures on the issues and problems proposed, exploring both the visualisation and the movement provided by the software.

In Figure 6, we present a summary of the analyses produced during the development of the UEA, from the application of the Intermediate Research Instruments, in the different axes.

Figure 6 - Synthesis of the analyses of the Intermediate Research Instruments

Thematic Axes	**Analysis/Synthesis**
Geometry of Position	The students performed very satisfactorily regarding the visualisation and identification of positional relationships between straight lines and planes. Regarding the activities that required greater knowledge on the students' part, indicating nomenclature and characteristics of the geometric relationships, the performance was satisfactory.
Primitive Notions and Basic Knowledge	The activities were aimed at identifying knowledge and skills at the level of visualisation, analysis and informal deduction on this axis. At the visualisation level (level 1) of the Van Hiele model, the students presented very satisfactory performance, identifying the indicated geometric objects in both space and plane. At the level of analysis (level 2) and informal deduction (level 3), the students' performance was satisfactory, as the students reached partially the knowledge of each level.
Polyhedra: Prism and Pyramid	We sought to work on the skills of the level of analysis (level 2) and informal deduction (level 3) of the Van Hiele model. At both levels the students presented satisfactory performance.

Source: Rodrigues (2018).

In view of the work carried out in the UEA, we decided to build an assessment tool, called the Final Investigation Instrument, covering topics approached throughout the UEA, allowing an analysis of the possible advances made by the students involved in the research in relation to the concepts of spatial geometry. For the analysis of this instrument, we used the same criterion we adopted to analyse the Initial Instrument. It was composed of six questions that aimed to investigate students' knowledge on the topics developed. Thus, as in the initial analysis, we show the performance of the class when carrying out the activities of the Final Investigation Instrument (Figure 7), as well as the description and analysis of the responses.

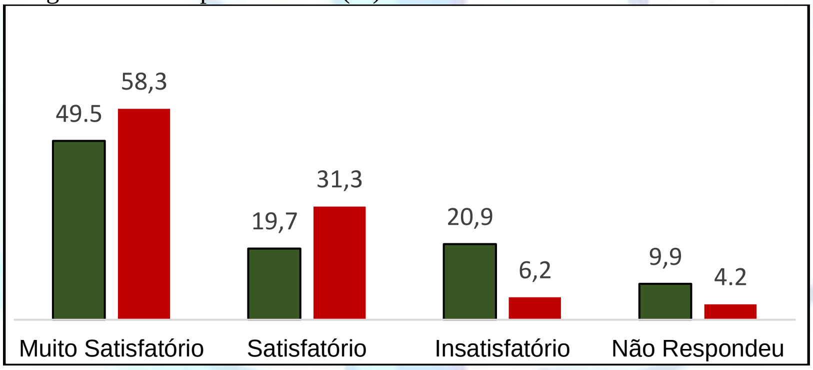
Figure 7 - Performance of the class in the Final Instrument

Performance of the Class (%)						
Question	Model level	Very Satisfactory	Satisfactory	Unsatisfactory	Did not answer	Total
1	Visualisation	75.0	25.0	0	0	100
2	Analysis	50.0	43.0	7.0	0	100
3	Informal Deduction	62.0	25.0	0	13.0	100
4	Informal Deduction	55.0	32.5	0	12.5	100
5	Informal Deduction	50.0	35.0	15.0	0	100
6	Analysis	57.5	27.5	15.0	0	100

Source: Rodrigues (2018).

By the analysis criterion, it can be observed that the group had a very good performance. At least half of the group presented a very satisfactory performance in all the questions, including in the questions classified as level of informal deduction of the Van Hiele model, unlike the result indicated in the analysis of the Initial Instrument. Looking for evidence that can enable to assess the evolvement of the UEA concerning the learning and the development of students' geometric thinking, we present in graph of Figure 8 the average of the performance of the class related to the Initial and Final Instruments.

Figure 8 - Class performance (%) from the Initial and Final Instruments



Source: Rodrigues (2018).

From the graph, it is possible to notice that the performance improved considerably, reaching 89.6% of satisfactory performance in the Final Instrument. On the other hand, unsatisfactory performance reached the Final with a percentage of only 6.2%.

We considered that the use of the GeoGebra software in the UEA contributed significantly to the appropriation of the concepts studied, considering that it has 2D and 3D windows and numerous tools that allow plane and spatial constructions based on properties of the object, also making possible to move the object, which greatly increases the opportunities to visualise, observe characteristics and conjecture about properties. Thus, we agreed with

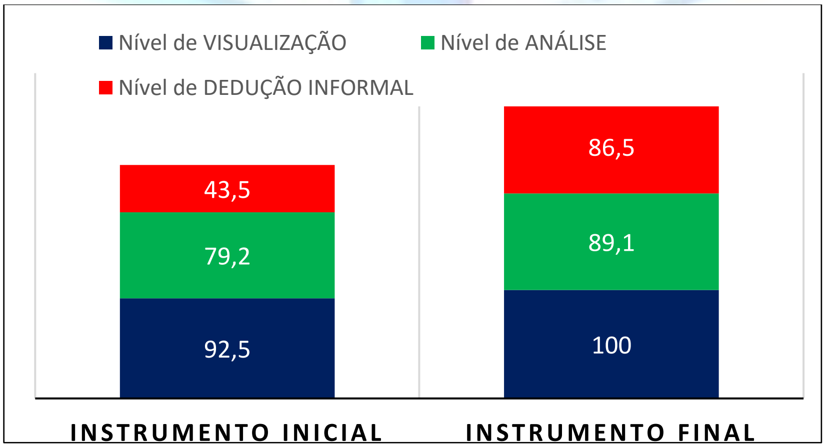
Borba (2011) when he states that visualisation is important for learning, since the several visual representations can help students to appropriate different concepts.

As to the development of the geometric thinking of those students, based on the Van Hiele model, now applied to spatial geometry, we observed that students articulated the issues very well at the level of visualisation of the model (level 1). Progress in the level of analysis (level 2) was also noted, as students demonstrated to master the standardised language as well as the ability to recognise characteristics and properties of geometric objects. At the level of informal deduction (level 3), the students showed to have apprehended definitions and concepts of geometric solids. Villiers (2010) states that the transition from the level of visualisation (level 1) to the level of analysis (level 2) involves reorganising the geometric thinking, being now not so abstract, but symbolic, since, at the level of analysis, the students master the standardised language. In the transition from the level of analysis (level 2) to the level of informal deduction (level 3), students can make connections between the new and the already consolidated concepts, besides recognising the properties of the geometric objects.

Thus, based on Van Hiele model, we observed the group's improvement from the level of visualisation (level 1) to the level of analysis (level 2), and significant evidence of the transition from the level of analysis (level 2) to the level of informal deduction (level 3). Therefore, the main objective of the UEA implementation was achieved, since it aimed to propose activities that could promote the construction of the concepts approached in class, through different strategies and activities. Therefore, the result was expressive.

In this context, we show in Figure 9 an analysis of the performance of students at the levels of visualisation, analysis and informal deduction of the Van Hiele model, based on a comparison between the instruments of Initial and Final investigation.

Figure 9 - Class performance (%) according to the levels of the Van Hiele Model



Source: Rodrigues (2018).

Thus, we considered that the students improved their performance in the three levels investigated, with emphasis to the progress they made at the level of informal deduction.

Final Considerations

The present research analysed the possible contributions of the use of a Teaching and Learning Unit (UEA) using digital technologies in the development of concepts of spatial geometry with high school students. As a positive point, we highlight the improvement of the students' knowledge throughout the development of the UEA.

The implementation of the UEA indicated that the group already presented the level of visualisation of the model regarding the contents developed. The group showed to have previous knowledge at the level of analysis, however it was possible to perceive that the students made progress throughout the application of the unit.

The greatest improvement occurred at the level of informal deduction of the model since, at the beginning of the investigation, the group did not have the necessary skills to work at this level. However, both pertinent geometric concepts and the abilities related to this level were developed during the work.

We must emphasise that GeoGebra software was an important tool in the development of this research, since it enabled the construction and visualisation of geometric solids, providing students with an environment where they could analyse, discuss and conjecture about the concepts related to geometry. From the data and analyses presented in this article, we consider that the results obtained in the investigation were satisfactory.

Furthermore, the research allowed the development of GEOE, an educational learning object for the concepts of spatial geometry. The GEOE emerged with the challenges faced by the group during the investigation, such as the lack of teaching materials as well as time management for developing the UEA. However, what was initially presented as a problem became, during the research work, a possibility to produce materials that could be used by the students in the classroom or even in other spaces. Thus, before the challenges and experiences lived during the application of the UEA, sought to develop an educational object that would be a support material for teachers' use in the classroom, and that students could use outside the school environment. In this way, the educational object GEOE (Spatial Geometry) was elaborated during this research work, being currently in phase of qualification and improvement.

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Submetido em Abril de 2019

Aprovado em Maio de 2019