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Concepts and theorems-in-action of seventh-year students in rectangular configuration problems

Conceitos e teoremas-em-ação de estudantes do sétimo ano em problemas de configuração rectangular

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ABSTRACT

In this article we analyse the performance and strategies of students of the 7th grade of elementary school in the resolution of situations of the multiplicative field identified as situations of rectangular configuration. We classify the levels of reasoning and describe the concepts and theorems-in-action employed by them. These are terms used in the conceptual field theory to designate the knowledge present in the schemes mobilized by students in a problem situation. We collected the data by applying a test for students of a public school in Rio de Janeiro, and by interviewing them. The results point to students' difficulties in dealing with such situations. The error rates are very high and rise when the situation requires a division for its solution. However, we recognise that, even if students are wrong, they have knowledge that can serve as a basis for overcoming misconceptions and constructing concepts associated with the operations.

KEYWORDS: Conceptual Field Theory. Multiplicative Field. Rectangular Configuration. Elementary School.

RESUMO

Neste artigo analisamos o desempenho e as estratégias de estudantes do 7º ano do Ensino Fundamental na resolução de situações do campo multiplicativo identificadas como situações de configuração retangular. Classificamos os níveis de raciocínio e descrevemos os conceitos e teoremas-em-ação empregados por eles. Estes são termos empregados na Teoria dos Campos Conceituais para designar os conhecimentos presentes nos esquemas mobilizados pelos estudantes numa situação problema. Coletamos os dados por meio da aplicação de um teste para estudantes de uma escola pública do Rio de Janeiro, e entrevista com os mesmos. Os resultados apontam para as dificuldades dos estudantes ao lidarem com tais situações. Os índices de erros são altíssimos e se elevam quando a situação requer uma divisão para sua solução. Todavia reconhecemos que, mesmo errando, os estudantes possuem conhecimentos que podem servir de base para a superação de ideias equivocadas e construção de conceitos associados às operações.

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PALAVRAS-CHAVE: Teoria dos Campos Conceituais. Campo Multiplicativo. Configuração Retangular. Ensino Fundamental.

Introduction

The ideas discussed in this article stem from the research presented in GT2 (Mathematics Education in the final years of Elementary and High-School Education) of the VII International Symposium on Research in Mathematics Education, under the title *O desempenho de estudantes de sétimo ano do ensino fundamental em situações de configuração retangular/The performance of seventh-grade elementary students in situations of rectangular configuration*. To deepen the reflections on that research, in this paper we describe and analyse the strategies employed by 7th-grade students to deal with problems belonging to the multiplicative field, specifically those involving the rectangular configuration, seeking to unveil concepts and theorems-in-action (VERGNAUD, 1990). For that, we chose to apply a test composed of 14 problem situations of the multiplicative field in a 7th-grade class from a school in the northern zone of Rio de Janeiro. Besides the rectangular configuration, the situations proposed in the test are also attached to the axes: simple proportion, double proportion, multiple proportion, and multiplicative comparison. It is a classification of the situations of the multiplicative field elaborated by Magina, Merlini and Santos (2012), in the light of the conceptual field theory.

The data collected in this study interact with other studies, such as those developed by Magina, Merlini and Santos (2014), Souza (2015), Milagre (2017), Luna (2017) and Barbosa and Oliveira (2018). The first study, like this one, focused on the strategies employed by students in the 3rd and 5th grades of elementary school, but in problems of simple proportion, and pointed to a limited evolution of students' competence in dealing with multiplicative problems. The last study, also focused on students, investigated the strategies of 7th graders to deal with problems related to the product of measures, and offered a classification for such strategies. The last three research works, focused on teacher training, revealed that the teachers were unfamiliar with the possibility of classifying multiplicative situations, leading them to favour only one or two types of situation. Thus, they emphasise the need for continuing education for teachers who teach mathematics to review their teaching practices in the multiplicative field.

As Magina, Santos and Merlini (2014) and Barbosa and Oliveira (2018), who had the students as subjects of their research, we prioritised in this study the performance of the 7th-grade students in situations of rectangular configuration. Resuming Barbosa and Oliveira's investigation (2018), we used the classification of strategies proposed by these authors, but we proceeded to identify the knowledge present in the mental schemes employed by the students, the said concepts and theorems-in-action (VERGNAUD, 1990).

The rectangular configuration is one of the classes of the product axis of measures both in the classification of multiplicative structures proposed by Vergnaud (1990) and in the classification proposed by Magina, Merlini and Santos (2012). It encompasses problems in which continuous or discrete data is spread over rows and columns. The problems that involve the calculation of areas are examples. It is a class little explored by elementary school teachers, as indicated by the studies we mentioned earlier, and difficult for students to understand. Like all the research we have mentioned, we are grounded on Gérard Vergnaud's theory of conceptual fields.

Conceptual Field Theory

The conceptual field theory was developed in the 1970s by the French psychologist Gérard Vergnaud. For him, a conceptual field is "a set of situations whose treatment implies schemes, concepts and theorems in close relationship, as well as linguistic and symbolic representations that can be used to symbolise them" (VERGNAUD, 1990, p. 147). Mastering a conceptual field does not occur in two months, not even in a few years. On the contrary, new problems and new properties must be studied over several years for the student to master them entirely. From this perspective, a concept cannot be reduced to a definition, especially if we are interested in its teaching and learning.

It is through the situations to be solved that a concept acquires meaning for the child. According to Vergnaud (1990), a concept is associated with the triad *Situations (S), Invariants (I) and Representations (R)*, where S, I and R are sets defined as follows: S is the set of situations that make the concepts meaningful (combination of tasks), (I) is the set of invariants (objects, properties and knowledge contained in the strategies used to deal with situations) and R is the set of symbolic representations that can be used to characterize and represent these invariants, therefore, representing situations and procedures.

It should be clarified that Vergnaud (1990) defines situation as task. In each conceptual field, there is a wide variety of situations and children's knowledge is then shaped by the situations they encounter and progressively master. In summary, faced with a new situation, the individual adapts his or her previous knowledge and develops new, and increasingly complex, skills. Thus, revealing the influences of the Piagetian theory are the situations that give meaning to the concept. The understanding of a concept does not emerge just from one type of situation, and a simple situation always encompasses more than one concept.

Following the triad supporting the concept, we have the invariants. According to Magina *et al*. (2001, p.12):

> Invariants are essential cognitive components of schemes. They can be implicit or explicit. They are implicit when linked to students' action schemes. In this case, although the students are not aware of the invariants they are using, they can be recognised in terms of objects and properties (of the problem) and relations and procedures (done by the student). Invariants are explicit when linked to a conception. In this case, they are expressed by words and / or other symbolic representations.

There is no problem solving without putting into play the operative invariants (which are the hidden part of the conceptualisation) and the symbolic representations. It is also important to highlight that there is mathematical knowledge involved in invariants. This knowledge is implicit in students' actions in dealing with situations and, most of the time, they fail to explain them. They are knowledge that Vergnaud (1990) names *knowledge-in-action* and, because they make sense for the students, from them, the teacher can begin the process of conceptualisation. Hence, the relevance of the research such as the one we present in this article. In this way, Barbosa and Oliveira (2018, p.3) state:

> When we observe the strategies employed by students to deal with certain problem situations of the multiplicative field, inferring the knowledge involved in them, we offer elements that can guide the work of the teacher and the elaboration of teaching interventions that promote the learning of this field.

In short, a teacher's work should, knowing the schemes mobilized by the students, favour the revision and construction of new schemes. Thus, Vergnaud calls *scheme* the invariant organization of the behaviour for a given class of situations (1990, p. 136; 1993, p. 2; 1994. p. 53; 1996, p. 201; 1998, p. 168) and states that it is in the schemes that the knowledge-in-action of the subject must be investigated, i.e., the cognitive elements that make the action of the subject to be operative. There is much implicit in the schemes. Many schemes can be evoked successively, and even simultaneously, in a new situation for the subject (1990, p.140). The conducts in a given situation rests on the initial repertoire of schemes that the individual has, and the cognitive development can be interpreted as consisting mainly of the development of a vast repertoire of schemes affecting very different spheres of human activity.

The expressions *concept-in-action* and *theorem-in-action* designate the knowledge contained in the schemes. Vergnaud also designated them by the more global expression: *operational invariants*. More specifically, Vergnaud (1990) divides the operational invariants fundamentally into two logical types: propositions and propositional functions. Invariant-type propositions are likely to be true or false. Theorems-in-action are invariants of this type. To clarify this logical type, Vergnaud (1990) gives, as an example, what children between 5 and 7 years of age discovered. They realised that it is not necessary to count everything to get the cardinal of $A \cup B$ if A and B have already been counted. This knowledge can be expressed by a theorem-in-action:

$$Card(A \cup B) = Card(A) + Card(B) \quad \text{provided that } A \cap B = ø.$$

Vergnaud (1990) proposes yet another example in recalling the moment when the child understands that in a trade situation, if the quantity of objects is multiplied by 2, 3, 4, 5, 10, 100 or a simple number, then the price is 2, 3, 4, 5, 10, 100 times greater. For him, this knowledge can be expressed by the theorem-in-action

$$f(nx) = nf(x)$$

It is worth mentioning, however, that different situations involving a same concept may present varying degrees of difficulty, because they require different theorems-in-action from the child in their resolution. Limiting this idea to the learning of operations, Franchi (1999, pp. 159-160) adds:

> Research in the field has widely found that the type of mathematical operation mobilised in the problem-solving process does not constitute the essential factor of difficulty for children. Those factors are in the order of magnitude and nature of the - natural, rational etc.- numbers, in the textual structure, in the type of numerical referents (km, km/h, m); but are essentially situated in the thinking operations necessary to establish relevant relationships between the data of the problem. There may be a large gap in the students' mastery of two situations involving the same mathematical operations and different variables.

The following problems presented by Magina (2002) allow us to exemplify Franchi's statement:

Problem A: Márcio invited three friends to his birthday party. For each friend he wants to give 5 marbles. How many marbles does he need to buy?

Problem B: Carlos will be celebrating his birthday. Every friend who comes to his party will get 3 balloons. He bought 18 balloons. How many friends can he invite?

The relevant concepts are the same for both situations, but situation B is much more difficult for seven or eight-year-olds because it involves reasoning back and finding the initial state. Such reasoning depends on a strong theorem-in-action:

$$I = T^{-1}(F),$$

where I is the initial state, F is the final state, T is the direct transformation, and T^{-1} is the inverse transformation.

Moreira (2002) discusses an example that may further elucidate ideas about concepts-in-action and theorems-on-action. He suggests considering the following situation proposed by Vergnaud (1994, p. 49) for 13-year-old students: *Flour consumption is, on average, 3.5 kg per week for ten people. How much flour is needed for fifty people for 28 days? One student's response: 5 times more people, 4 times more days, 20 times more flour; hence 3.5 x 20 = 70 kg*.

Citing Vergnaud, Moreira (2002) states that it is impossible to understand this reasoning disregarding the following theorem implicit in the student's mind: $f(n_1x_1, n_2x_2) = n_1n_2\, f(x_1,x_2)$, i.e., Consumption (5 x 10, 4 x 7) = 5 x 4 Consumption (10, 7). And he adds:

> Naturally, this theorem works because the ratios of 50 people to 10 people and 28 days to 7 days are simple and obvious. It would not be so easily applied to other numerical values. Therefore, its scope of application is limited. Nevertheless, it is a theorem that can be expressed, for example, in words: Consumption is proportional to the number of people when the number of days is kept constant; and is proportional to the number of days when the number of people is kept constant. It can also be expressed by the formula C = k.P.D., where C is the consumption, P is the number of people, D is the number of days and k is the consumption per person per day. (MOREIRA, 2002, p.11)

It becomes clear that the ways of expressing the reasoning discussed above are different and, as far as cognition is concerned, they present different levels of difficulty. On this, Moreira (2002, p.11) concludes that those *different ways of expressing the same reasoning are not cognitively equivalent. The second one is more difficult. They are complementary ways of explaining the same mathematical structure implicit in different levels of abstraction.*

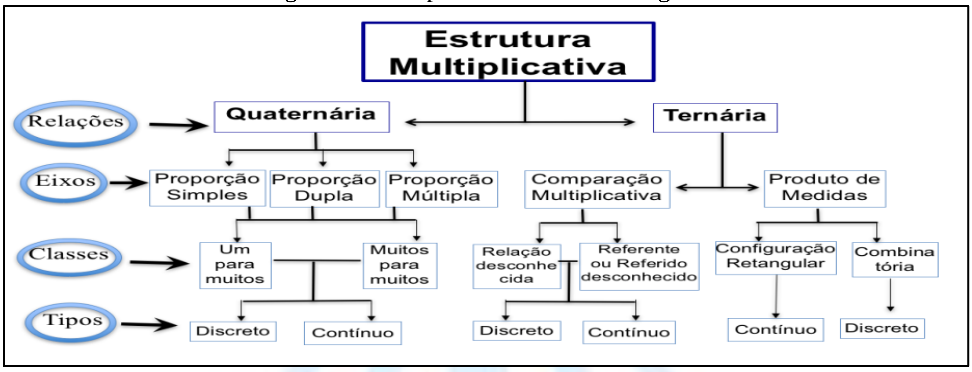
Turning now to propositional function type invariants, it is correct to say that, according to the theory of conceptual fields, they are not susceptible to being true or false, but they constitute the "bricks" indispensable to the construction of propositions. For example, the concepts of cardinal and collection, of initial state, transformation, and quantified relationship are indispensable to the conceptualisation of additive and multiplicative structures. Vergnaud (1990) insists, however, that these concepts are rarely explained by students, even though they are constructed by them in action. He also emphasises that the relationship between propositional function and proposition, and consequently between theorem-in-action and concept-in-action, is a dialectical relationship: *there is no proposition without propositional function and there is no propositional function without proposition. In the same way, theorem-in-action and concept-in-action are constructed in close interaction* (VERGNAUD, 1990, p.164).

The Conceptual Field of the Multiplicative Structures

The conceptual field of multiplicative structures or multiplicative conceptual field is the complex of situations that encompass, simultaneously, the various multiplications and/or divisions with theorems that support those situations. As Luna (2017, p. 51) states, we can cite in this set "simple proportion and multiple proportion, direct and inverse scalar relationship, quotient and product of dimensions, linear combination and linear application, fraction, relationship, rational number, multiple and divisor etc.".

Because it is a conceptual field, its appreciation and treatment encompass several types of specific symbolic concepts, procedures, and representations. Vergnaud (1990) classified the multiplicative relationships into two categories that encompass multiplication and division, terming them as *ternary relations* and *quaternary relations*. The ternary relations link three quantities that may be of a different nature. Quaternary relations, however, involve four quantities, two of which are of the same class and the other two belong to another class. Magina, Merlini and Santos (2012) constructed the table below in the light of Vergnaud's classification.

Figure 1- Multiplicative structure diagram



Source - Magina, Santos and Merlini, published in SANTOS, 2015, p.105.

In this paper, we emphasise the rectangular configuration, which, as can be seen in the table, is a class of the product of measures axis, which, in turn, corresponds to a type of ternary relation. In the product of measures, one quantity is the product of two others, in the same numerical and dimensional plane. An example of the rectangular configuration can be experienced in situations where the dimensions of a rectangle are given, and its area is requested. Assuming that the dimensions are in centimetres, the size of the area will be the square centimetre and the number corresponding to it will be obtained by the product of the dimensions.

The Method

To reach our objective, we applied a diagnostic test for 3 hours in a 7th-grade class (with 40 students) from a school in the western zone of Rio de Janeiro. Next, we informally interviewed them to have them explain their records and procedures. The test is composed of 14 problem situations belonging to the multiplicative conceptual field. Of these, we focused on the two situations involving the rectangular configuration: one in which factors are given and the unknown is the product (factor-factor); and another in which one factor and the product are given and the unknown is the other factor (factor-product), which can be obtained by a division. We present the situations in Chart 1 below:

Chart 1 - Situations of the diagnostic instrument involving the rectangular and combinatorial configuration classes

Status	Statement	Class	Operation
Q5	Ruth wants to change the floor of her bedroom. This room is 3 m wide and 6 m long. How many square metres of floor does Ruth need to buy?	Rectangular configuration (factor-factor)	Multiplication
Q7	Vera's garden area is rectangular and has 24 m^2. The width is 4 m. What is the length, in metres, of this garden?	Rectangular configuration (factor-product)	Division

Source: Research data (2014).

First, we corrected the tests, and, for these questions, we quantified the wrong answers, the right answers and types of strategies the students employed. Next, we focused on the students' strategies in Q5 and Q7, seeking to establish a classification for these strategies based on the classification Barbosa and Oliveira (2018) established. Next, we interviewed the students. We present here a qualitative analysis based on the crossing of the students' records in the test with their speeches in the interviews, which were recorded and transcribed. Thus, our analysis fits into Goldenberg's (1999) conception of qualitative research. According to this author, in this type of research the researcher does not worry about quantifying the investigated group, but rather about the in-depth understanding of the reality of each individual, group, organization or institution, their trajectories and subjectivities.

Data analysis

We aimed to analyse all the strategies used by the students, both for wrong and right answers, and grouped them into categories according to their levels of complexity. As the studies of Barbosa and Oliveira (2018) also turned to the strategies employed by 7th-graders in the same problem situations, the categorisation presented here guided our analysis, and our data led us to three levels of complexity, namely: incomprehensible (level 1), additive (level 2) and multiplicative thinking (level 3). Below, we introduce them, describing them and observing their incidence number.

At level 1, or incomprehensible level, are "the answers in which the student did not explain on paper the operation used to solve the problem or, when he/she did, we could not identify the reasoning used" (MAGINA, SANTOS & MERLINI, 2014, p. 9). Thus, part of this level included the strategies in which the student made a meaningless drawing for his or her resolution, repeated one of the numbers contained in the problem statement, or may have chosen other mathematical concepts other than the four fundamental operations, such as fractions and simplification of fractions, without being able to understand the reason for such. At this level, students' responses are invariably wrong. Chart 2 shows the number of strategies ranked at level 1 per question.

Chart 2 - Quantitative of strategies ranked at level 1 per question

Level 1 - Incomprehensible	
Question	Incidence
Q5	4
Q7	8

Source: Prepared by the author.

Of the 72 non-null responses given to questions Q5 and Q7, 12 belonged to this level, being 4 of Q5 and 8 of Q7. If we compare question 5 with 7, we observed that this strategy was mostly present in Q7. We expected to find a configuration close to this chart’s, because, as Barbosa and Oliveira (2018) indicated, Q7 presents a degree of difficulty greater than Q5. This result is also in line with Franchi's ideas (1999), when he states that situations in which the final state is given and the transformation for students to discover the initial state are more difficult for them. It should also be noted that not even the students who produced such a result were able to explain their records and strategies when they were interviewed.

Level 2, or level of additive thinking, encompasses strategies that involved an addition, a subtraction or any combination of these operations. Like Barbosa and Oliveira (2018), we found two distinct strategies of action at this level, which generated two sub-levels: addition or subtraction of the numbers present in the statement (2A) and calculation of the perimeter instead of the area (2B).

Also, at this level, students' responses are invariably wrong. Their incidences are shown in Chart 3:

Chart 3 - Quantitative of strategies classified in levels 2A and 2B

Level 2 Additive			
		Q5	Q7
2A	Addition of data	10	4
	Subtraction of data	2	3
2B	Perimeter calculation	3	-

Source: Prepared by the author.

As we can see, although Q5 has several right answers well greater than Q7, the strategy 2A was quite used by the students in Q5, which surprised us. We cannot forget that this is a matter of immediate application of the concept of rectangle area, and that this should have already been mastered by 7th graders. However, as in Barbosa and Oliveira's study (2018), we were also surprised that the 3 students who subtracted the data in Q7 were among those who added the data in Q5. The observation of these data led us to hypothesise that the students recognised the difference between Q5 and Q7 and then concluded that the operation that would solve Q7 should be the inverse operation of the one that would solve Q5. However, as they still thought additively, they resorted to the subtraction, which is the addition inverse operation. Our hypothesis was confirmed when we interviewed the students. The transcript of the following interview confirms us:

> Researcher: Why did you subtract?
> Student: Because I knew I had to do the opposite. If in question 5 I added, in question 7 I thought I had to subtract.

It is important to note that there was in the scheme employed by the student a series of mathematical knowledge, the so-called theorems-in-action and concepts-in-action. The recognition of the reversibility between addition and subtraction and the mastery of the algorithms of these operations are examples of this knowledge. The first, which may be true or false, is a theorem-in-action. The second, founding the theorem-in-action, corresponds to a concept-in-action. In a broader sense, we can also say that the association of the problem situation with an operation, which may also be true or false, can also be understood as a theorem-in-action. This theorem is evidently false, since addition and subtraction as they were employed do not solve problems of rectangular configuration. However, as the research works of Souza (2015), Milagre (2017) and Luna (2017) suggest, the teacher can use this knowledge and propose new reflections and arguments so that ideas are discarded, and students start associating situations Q5 and Q7 to multiplication and division.

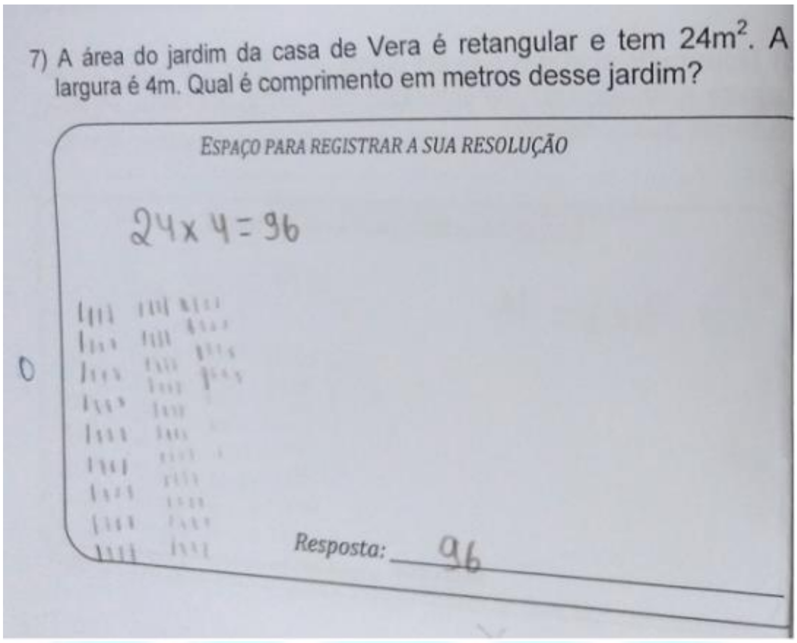
With respect to Q5, we kept the statement as in Barbosa and Oliveira (2018). Barbosa and Oliveira (2018) pointed out that, for a rectangle of 3 units long and 6 units wide, the perimeter and area correspond to the same number, but with different units. Fortunately, in this question, all the students left their calculations registered and knew how to explain their strategies in the interviews, which allowed us to differentiate those who calculated the area from those who calculated the perimeter.

In Magina, Santos and Merlini's studies (2014, p.12), at level 3, or level of transition from the additive to the multiplicative thinking, the strategy used by students "consisted of forming groups of the same quantity. It is a question of adding several times the same amount, whether it is represented by grouped icons (IIII IIIIIIII = 12), or numerically (4 + 4 + 4 = 12)". According to these authors:

> Such strategy approaches multiplicative thinking, but is anchored in the additive reasoning, that is, forming groups of the same quantity to perform the addition operation. When the representation is pictorial, it is well demarcated by the groups drawn; when the representation is numerical, the strategy is explicitly the sum of equal parts. (MAGINA, SANTOS, MERLINI, 2014, p. 13)

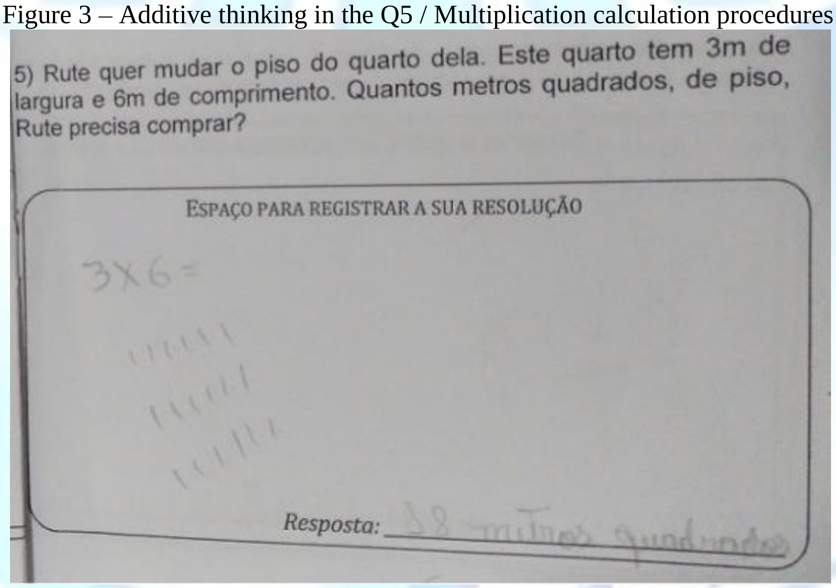
Considering these notions, Barbosa and Oliveira (2018) affirm that the product of measurements is one of the elements of rupture between the additive and multiplicative fields. Thus, in this study, as well as in the last one we have mentioned, there is no transition from the additive to the multiplicative thinking and level 3 is already the multiplicative level. However, it should be mentioned that in the calculation procedures presented by three students of level 3, we can still find the influence of the additive reasoning. The examples in Figures 2, 3 and 4 illustrate correct and incorrect responses that reveal this influence:

Figure 2 - Additive thinking in Q7 / Multiplication calculation procedures



Source: Research data.

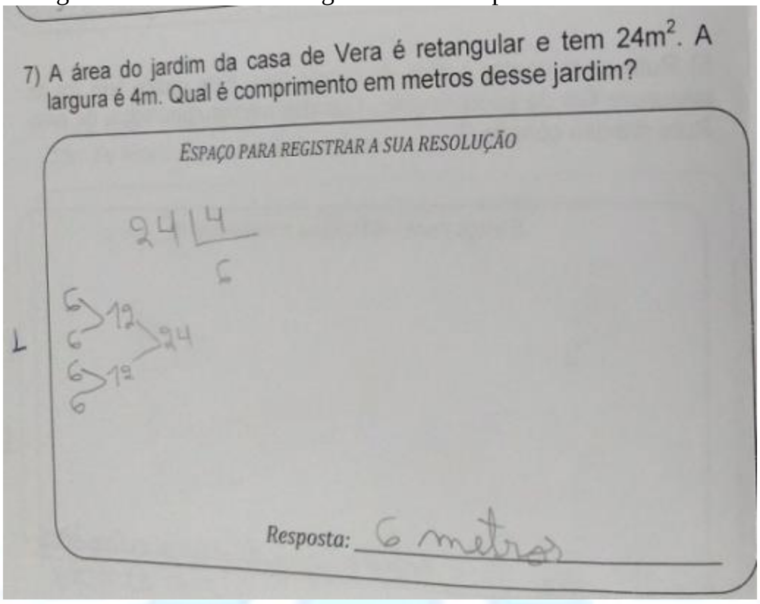
Figure 3 – Additive thinking in the Q5 / Multiplication calculation procedures



Source: Research data.

A brief observation of the protocols suggests that students, in reading the statements, opted for operations in the multiplicative field and, in doing the calculations, resorted to additive thinking. This fact can be seen in the responses of two students to Q7 and one student to Q5. In view of the knowledge present in the schemes employed by the students, we can say that the understanding that multiplication is the repeated sum of parts is a concept-in-action. In the same way, it is also a concept-in-action the recognition that division is a successive subtraction of the same number. In Figure 4, we have a student protocol that has this concept-in-action:

Figure 4 - Additive thinking in calculation procedures / Division



Source: Research data.

When interviewed, the student who produced this protocol explained: *As I wanted to divide by 4, I wondered what number I had to take four times out of 24 until I had no remainder*. *I do this and it just does not work when the numbers are too big.*

It is a concept-in-action present in a scheme, which, according to the student himself, is not very effective, because with larger numbers, which are not part of the students' numerical repertoire, it becomes impractical. Considering that our subjects have been studying the multiplicative structure for the fourth consecutive year, agreeing with Barbosa and Oliveira (2018), we consider how little such teaching has favoured the development of mental calculation procedures. The school's action of limiting the teaching of the multiplicative field to the repeated sum of parts and to the reproduction of multiplication and division algorithms prevents the construction of other concepts and properties of the multiplicative field. As Barbosa and Oliveira (2018: 14) point out, *this may also be the cause of the high number of miscalculations and the students' inability to reflect on the wrong results they have produced.*

At level 3, or level of multiplicative thinking, the strategy that the student uses necessarily passes through the multiplicative structure. Faced with problem situations, our subjects whose answers fall into this level chose multiplication or division. When they chose to multiply in Q7, for example, the students found 96 or when they got the calculations wrong, they found other numbers, but in all cases the numbers were greater than 24, the measure of the given area.

Although the choice for multiplication or division to solve the questions already suggests the existence of theorems-in-action, our attention was drawn to the fact that the students did not realise how absurd the answers were, since the known dimension of the rectangle is a number greater than 1 and, under these conditions, it is impossible for the area measurement to be a smaller number than those corresponding to dimension measurements. Even among the 8 students who opted for the division in Q7, 3 made mistakes in calculations, producing absurd results, and did not realise it. Data such as those lead us to infer that the students experience a teaching that prioritises the mechanical reproduction of algorithms and formulas. The interview with one of these students reinforces our inference:

> Researcher: What did you think when you found 96?
> Student: Nothing! I went to do question 8 at once.
> Researcher: When you find a result, don't you think if it's right or wrong?
> Student: No! I thought a lot, but a lot, but at the time I was asking the question. When I finished, I went to do the other one.

As we can see in this transcript, the student does not understand the need to think about the results he finds when he solves a problem.

In Charts 4 and 5, we present the operation chosen and the number of calculation errors per question we found:

Chart 4 - Quantitative of calculation errors per operation in Q5

Question 5		
	No calculation error	With miscalculation
Multiplication	17	0
Division	0	0

Source: Prepared by the author.

Chart 5 - Calculation errors per operation in Q7

Question 7		
	No calculation error	With miscalculation
Multiplication	7	5
Division	5	3

Source: Prepared by the author.

Still considering the calculation procedures, it is important to note that in Q5 there were no calculation errors. On this fact, Barbosa and Oliveira (2018, p.17) infer that *it is due to the numbers involved in the question. 3 and 5 are small numbers whose product appears in the tables that a good part of Brazilian students must memorise from an early age.*

Thus, it is possible that in situations where numbers are larger and are not in the usual tables, the number of errors for this type of question increases considerably.

Final considerations

The analysis of the results allows us to make two considerations: one from the quantitative point of view and the other from the qualitative point of view. Although it is not the focus of this article, regarding the quantitative point of view we highlight the high error rate in the two situations analysed, and especially in the question in which the product is given and one of the factors is requested. These data led us to emphasise how important it is for the teacher who teaches mathematics in elementary school to diversify the problem situations of the multiplicative conceptual field that propose to students not only among the classes suggested by Magina, Santos and Merlini (2014), but in the same class, to address the various possibilities of situating the unknown and the necessary data to the resolutions of the situations as suggested by Franchi (1999) and Moreira (2002).

Regarding the qualitative analysis, from the strategies employed by the students, we identified three levels of reasoning (incomprehensible, additive and multiplicative). At the level of additive thinking, we also identified two sub-levels: one in which the student randomly sums up the data present in the statement and another in which the student confuses the concept of perimeter with the concept of area. In all the questions, we observe the privilege of the numerical representations to the detriment of the pictorial representations and we infer that this fact is due to a teaching based on the reproduction of algorithms. We believe that such a phenomenon may imply, in turn, the students' reduced ability to assess the often-absurd responses they provide to problem situations.

In a superficial understanding, the results presented in this study can provoke a pessimistic view regarding the learning of the multiplicative field in the final years of elementary school. However, although students have produced many errors in the test, the analysis of their records and interviews also reveal concepts and theorems-in-action, that is, it is not correct to assume that students know nothing about multiplicative structures. Here we highlight the total and/or partial mastery of multiplication and division algorithms, the understanding of the reversibility between addition and subtraction and multiplication and division, the association of multiplication with the repeated sum of parts, and division with successive subtractions. It is evident that this knowledge is not enough to affirm that students already master the multiplicative field, however they can serve as starting point for a more conscious work from the teachers' side, to promote the construction of other concepts belonging to this conceptual field.

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