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Discovery and generalization of patterns, an exploration in the learning of Algebra

Descubrimiento y Generalización de Patrones, una exploración en el Aprendizaje del Álgebra

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ABSTRACT

The following document contains, briefly, the theoretical foundation, on which we have conceived as a basis to justify the application of the research project, and some results about the project which deals with the exploration of the potential of the 7th grade student to learn algebra through the discovery and generalization of patterns, which is the focus of the project, taking into account the historical, cognitive, pedagogical and didactic implications, and showing the most relevant results. The project is a qualitative research, using the didactics of problem solving, to try to introduce the students in the learning of the Algebra in a more productive way.
KEYWORDS: algebra, arithmetic, pattern, generalization, variable.

RESUMO

El siguiente documento contiene, de manera breve, la fundamentación teórica, sobre la que hemos concebido como base para justificar la aplicación del proyecto de investigación, y algunos resultados sobre el proyecto el cual trata sobre la exploración del potencial del alumno de 7mo grado de aprender algebra a través del descubrimiento y generalización de patrones, que es el foco del proyecto, teniendo en cuenta las implicaciones históricas, cognitivas, pedagógicas y didácticas., y mostrando los resultados más relevantes. El proyecto es una investigación cualitativa, utilizando la didáctica de la resolución de problemas, para intentar introducir a los alumnos en el aprendizaje del álgebra de una manera más productiva.
PALAVRAS-CHAVE: álgebra, aritmética, patrón, generalización, variable.

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Introduction

To begin, it is important to note that this work is an expanded version of the document presented at the International Seminar on Research in Mathematical Education (SIPEM) on November 4th, 2018, in Foz do Iguaçu-PR, Brazil, which was entitled "Exploration of the Potential to Learn Algebra through the Discovery and Generalization of Patterns ", and was published in the annals of the event, in working group 2 (GT2), and both are based on a research conducted in Honduras in 2009, as part of a research project at the Universidad Pedagógica Nacional "Francisco Morazán" (UPNFM). Once this point is clarified, we can continue to say that as students and teachers of Mathematics we have been able to observe the difficulty in the teaching-learning process of Mathematics at all levels of education, especially at the intermediate level, specifically when introducing students to the study of Algebra. Ursini (1996, p.423) points out that "Problems subsist when trying to manipulate literal symbols or use them in problems that require algebraic symbolization for their solution"; this is because students still do not understand the true meaning of variable . The problems that have to do with the teaching and learning of Algebra underscore the crisis of traditional teaching that we live in the classroom in most national public schools. It is necessary to induce them to develop strategies that allow them to understand the true meaning of mathematical concepts specifically those of Algebra. Romero and other authors (2007), in their article "Introduction to Algebra Through Functional Activities and Generalization " point out that the main cognitive obstacles of traditional teaching are linked to Arithmetic that frequently focuses on the results of the processes of calculation more than in the relational and structural aspects, and are those that oppose the development of algebraic thinking.

It is recognized that mathematical activity consists in the search of regularities and patterns in order to establish generalizations and from them to make predictions. An approach to Algebra can be achieved through generalization activities that involve the recognition and manipulation of mathematical objects. On the other hand, Ursini (1996) proposes "The need to design special environments to help students approach new mathematical notions using their previous knowledge as support". Manuel Cardona (2007) says that when analyzing the Basic National Curriculum (CNB, 2005), in the third cycle, there are Algebra units referring to equations in which the proposed methodology places special emphasis on the problem-solving strategy as a means to promote the development of mathematical thought. In addition, the CNB in the description of the Mathematics curriculum for the third cycle (Pg. 10) defines

Mathematics as a discipline that systematizes the intuitive capacity of the human being to be able to find the necessary ideas to solve problems.

The teachers of Mathematics have transmitted to the students an incomplete idea of the meaning of Algebra considering it as the manipulation of symbols with activities of simplification of Algebraic expressions or of solving equations or inequations. Of course, these activities are fundamental and constitute the historical roots of Algebra, but Algebra is more than manipulating symbols. According to the NCTM (2000) students need to understand (from Algebra) their concepts, structures and principles governing the manipulation of symbols and how they can be used to record ideas and expand their understanding of situations, also, in the later grades of the basic education should stimulate the development of Algebraic thinking of students with transitory activities between Arithmetic and Algebra. Working with symbols in the simplification of Algebraic expressions or solving equations, is just one dimension of what it means to learn Algebra, whose concept is broader; it is related to a considerable list of abilities of the mind. This ability is developed when students live meaningful learning experiences that lead to the understanding of mathematical concepts, within which is the recognition and generalization of patterns.

Identification and Generalization of Patterns

The ability to generalize refers to the possibility of finding numerical or geometric patterns; a "pattern" is a sequence of elements that has a rule or an order. They can be classified as patterns: repeated and growing. Repeated patterns are those that have a fixed sequence of repeating elements. On the other hand, increasing patterns are those that have elements that change according to a rule.

According to the Glossary of Mathematical Terms (2009), the concept of pattern is essential in mathematics. Without being aware of it, students use patterns much earlier, even before learning to add. In terms of generalizing, one can say that it is to make a judgment from the particular, that is, to take from the empirical observation of a case or particular cases and to take them to universalization. Normally when a particular case is repeated there is a tendency to generalize, the more repetition there is in the particular experience, the stronger the generalization.

Do we learn algebra by discovering patterns?

Recognizing patterns, describing them and expressing patterns in different ways is one of the keys to generalization in mathematics. They have been proposed in investigations such as John Mason (apud CASALLAS, ESTRELLA, 2001), several possible approaches that can lead students to the construction of formulas: visualization; the manipulation of the figure on which the process of generalization is based, facilitating, in relation to this, the construction of the formula; the formulation of a recursive rule that shows how to construct the following terms from the precedents; and the finding of a pattern that guides them directly to the formula.

It is important to recognize that this process of approaching from a numerical thinking to algebraic (generalization of patterns) has presented, through history, many difficulties and a break between the concrete and the abstract. Therefore, the role of the teacher consists in the design of methodological strategies that start with situations that allow problems to analyze, organize and model situations both of the student's daily life and of other scientific disciplines and that lead to processes of generalization of algebraic expressions.

The teaching of mathematics based on solving problems from the conception of "doing mathematics".

According to Polya (apud CANALES, CASTRO, 2007) on mathematics as an activity, for a mathematician, who is active in research, mathematics can sometimes appear as a game of imagination: we must imagine a mathematical theorem before testing it; you have to imagine the idea of the test before putting it into practice, and following this idea, the students should be given some opportunity to solve problems in which they first imagine and then try out some mathematical question appropriate to their level.

The use of the terms "problem" and "problem solving" has had multiple meanings over the years, a first meaning is to solve problems as part of the context, this is where the problems are used to show the value of the Mathematical in the resolution of some problems related to experiences of everyday life. That is, problem solving is seen as a facilitator of achieving other objectives. A second meaning is to solve problems as a skill, that is, that the student is able to solve non-routine problems once he has solved routine problems, this is a superior ability, which is acquired through the learning of concepts and development of math skills. And the third meaning is the one that occupies our interest, because it conceives problem solving as "Do Mathematics".

According to Vilanova (2003), this meaning is based on "believing that the work of mathematicians is to solve problems and that mathematics really consists of problems and solutions". For Rico (apud CASALLAS, ESTRELLA, 2001), from mathematical education, strategies are defined as the forms of performance or execution of mathematical tasks, which are executed on representations of concepts and relationships. The strategies operate within a conceptual structure and involve any type of procedure that can be executed, taking into account the recommendations and concepts involved. According to Cardona (2007): "With the resolution of problems it is intended that the student apply and adapt various strategies to solve a problem. The diversity of strategies that a student has will depend on their ability to connect their ideas or mathematical concepts with the situation they are solving. When students can connect their mathematical ideas, their understanding is deeper and more durable. They can see connections between mathematical topics, in contexts that relate mathematics to other disciplines and in their own interests and experiences. "

What is the emphasis that is proposed in the current conceptions of Mathematics?

The principles and standards for mathematics education (S.A.E.M., 2000), propose a comprehensive way to carry out mathematically solid curricula, train competent teachers and of course that students receive a quality mathematical training.

In these standards it is suggested that teaching programs should train students in algebra to:

- Understand patterns, relationships and functions.
- Represent and analyze situations, mathematical structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze the change in diverse contexts.

On the other hand, in Honduras the National Basic Curriculum (CNB) is the document that includes the expectations of achievement at the national level. This document points out that, in teaching, mathematics is a discipline linked to the development of logical thinking structures, the capacity for abstraction, deductive and inductive processes and the capacity for synthesis and analysis. With the appropriation of processes and methods of a quantitative, symbolic and graphic nature, there is an indispensable support instrument for the different fields of knowledge.

Methodology

This study is a qualitative research, in order to explore the potential to learn algebra that can develop in 7th grade students through the recognition and generalization of patterns. The researchers interrelate with the object of study and try to understand the actions of students when geometric and numerical patterns are presented.

The population studied were the students who attended the 7th grade "U" section of the Evangelical Institute "Desarollo Integral" of Colonia El Pedregal, in Comayagüela, M.D.C., Honduras. The sample was a group of 13 students, who were between the ages of 11 and 13 years. These students should have basic knowledge of arithmetic, that is, make proper use of basic operations (addition, subtraction, multiplication, division and calculation of powers) with natural numbers.

For the collection of information, the action research was used as a qualitative research method, a diagnostic test was applied, activities were proposed in which the students could recognize different patterns (numerical and geometric) using different strategies to generalize them. Written tests were applied, some interviews were conducted to discuss the strategies; interest ideas that arose in the interaction with the students and direct observations on their performance during the activities were recorded. In total, five meetings were held with the students, starting with a diagnostic test, continuing with three work sessions, and ending with the application of a final test.

The situation of the teaching of algebra in seventh grade was analyzed, through the inspection of some supporting texts, which are those used by teachers (some are those provided by the secretary of education, and others vary according to the teacher) , students' difficulties in manipulating variables were identified when they were introduced to the study of algebra. In addition, activities were carried out to support understanding and manipulating algebraic expressions. One of the purposes was to develop a guide of exercises with geometric and numerical patterns that can serve the teacher to awaken in students the interest, thus improving the Teaching-Learning process. With each sequence of figures or numbers, the students were allowed to work individually, then they once discussed in teams, and always discussed with the group in general. Finally, a test was applied at the end of the whole process.

Description of the Information Collection Instruments

Diagnostic Test: This was done in order to know if the students had the basic knowledge of arithmetic, such as addition, subtraction, multiplication and division with natural numbers, as well as the hierarchy of these combined operations. The test included two pattern recognition exercises (“Easy"), two exercises with combined operations, a division of a three-digit number by a two-digit number and four enhancement exercises.

Exercise guide: included seven pattern identification exercises that were developed in the diagnostic test, during the visits and in the final test.

With direct observations: each event of interest was recorded in the performance of the students throughout the process and the important ideas discussed. The activities that were carried out included the problem statement on the board and the students worked individually on a sheet of paper that they delivered at the end of each visit. At the beginning of each visit a group discussion was held, where all the strategies used in the solution of the exercises raised in the previous visit were exposed.

Final Test: consisted of two problems about the recognition and generalization of patterns, one of medium level and the other of greater difficulty.

Analysis of data

A qualitative analysis of the diagnostic test, the final test and of the activities carried out in each visit, as well as direct observations were made looking for evidences of how students can develop a potential for the learning of algebra.

Such analysis consisted on:

Diagnostic test: each test was carefully reviewed, observing each of the strategies used by students in solving the problems presented. The data are presented in a double entry table and in their respective analysis the strategies used and the difficulties in each problem are described.

Learning activities: a comparison was made between the different data recorded through direct observations and worksheets; to observe the changes in the strategies used in the resolution of the problems during the whole process of the visits.

Final test: it was analyzed according to the strategies used by the students to know if in fact they show certain ability to work in the recognition of patterns and the generalization of these.

Diagnostic test

The interest was to observe what kind of strategies the students used to arrive at the solution, it was not so important to obtain a correct answer but what process they followed. In the case of those exercises in which it was intended to explore their previous knowledge, it was only considered important to show notions about:

- Use of the hierarchy of arithmetic operations
- Correct operations with natural numbers
- Potentiation Properties
- Division of whole numbers, particularly by a two-digit number.

The student's performance was also analyzed when confronted for the first time with problems of pattern discoveries, in order to induce them to the concept of pattern and prepare them for the next session.

Below is a table summarizing the results obtained by the students in the exploration exercises of their previous knowledge and problems with patterns. All the data shown in the tables and figures were taken directly from the data collection instruments worked with the students in the sessions.

Table No. 1: Number of students who show or do not have some prior knowledge to develop algebraic thinking.

Know-ledge / Evidence what:	Use the Operations Hierarchy	They operate with natural numbers	Express a number in the form of a power	They find the power of a number	Divide by a two digit number
Not Dominate	11	1	4	2	8

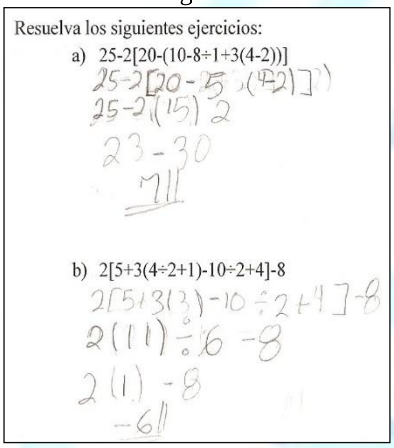
Dominate	2	12	9	11	5

Source: Research Data

At first, it was noticed that the students showed some rejection towards the test, situation that was dissipating as time went by. Possibly because several factors have intervened such as our presence (unknown) for the first time and the psychological impact of an unexpected test. Some evidences about the difficulties encountered are presented below:

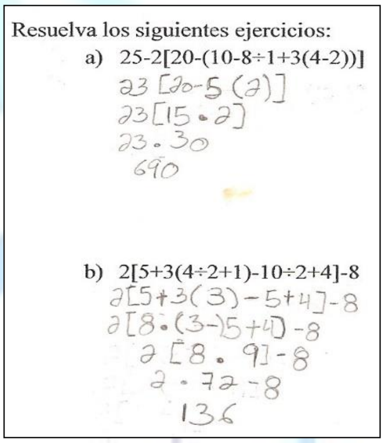
According to the results presented in the table above, it can be said that students have difficulties with the hierarchy of mathematical operations. Many students performed the addition and subtraction before the products and divisions, as can be seen in the following images:

Figure 1



Source: Research Data

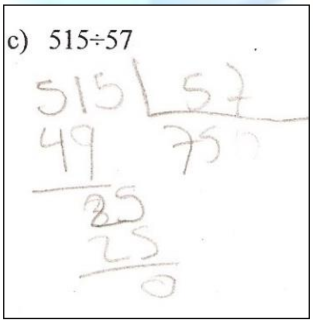
Figure 2



Source: Research Data

Some students still had difficulties in the division by a two-figure number, making mistakes such as not taking into account if the first two figures of the dividend are smaller or not than the divisor, in addition, they only took into account one of the divisor figures to make the product with the quotient, as shown in the figure:

Figure 3



Source: Research Data

The most encouraging result is the fact that all but one handle operations with natural numbers and the vast majority the properties of empowerment.

Below are the results that were obtained when students solve two pattern identification exercises.

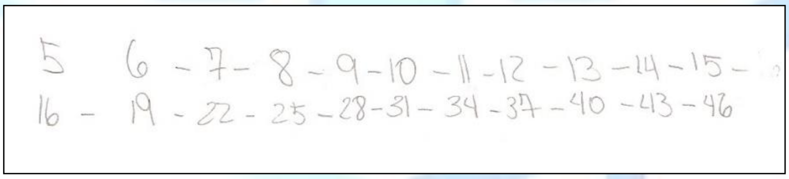
The first proposed problem asked to find the number of calls made on a given day, the students were given the following table:

Day	1	2	3	4	…
Number of calls	4	7	10	13	…

They were asked: How many calls will be made on day 5? How many on day 6? And finally, how many on day 15?

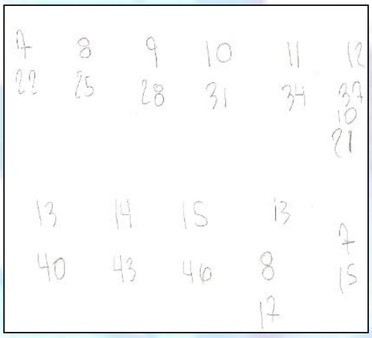
Two students solved it in the following way:

Figure 4



Source: Research Data

Figure 5



Source: Research Data

As noted, the students discovered that the number of calls increased by three, so they decided to add no matter how many times they had to do it.

By making a thorough analysis of the actions of each student, the general results can be seen in Table No. 2.

Table No. 2: Description of the results on the exercise: number of calls.

Number of students	Types of answers and actions of students to solve the problem
8	They noted that the number of calls increased by three units.
6	They found the answer to the first two questions by adding three.
1	The student did the sum wrong to find the number of calls on day five (the student add two units to the number of calls on day four).
1	The student wrote a wrong answer to the third question for the strategy he used, we observed that he counted on his fingers, and left no written evidence.
2	They thought that as the number of days increased, so did the number of calls, giving the wrong answer to the third question (multiplying by three).
3	They found the answer to each question by adding three to find the next one.
5	They did not solve the problem.

Source: Research Data

From the previous table, we can see that most of the students had a good performance in solving the first problem. The difficulty lies basically in error when counting on the fingers. In general, the students understood the problem and began to become familiar with this type of exercise. The second problem was to present the student with the following sequence of figures:

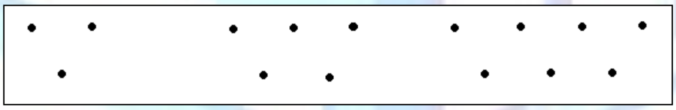
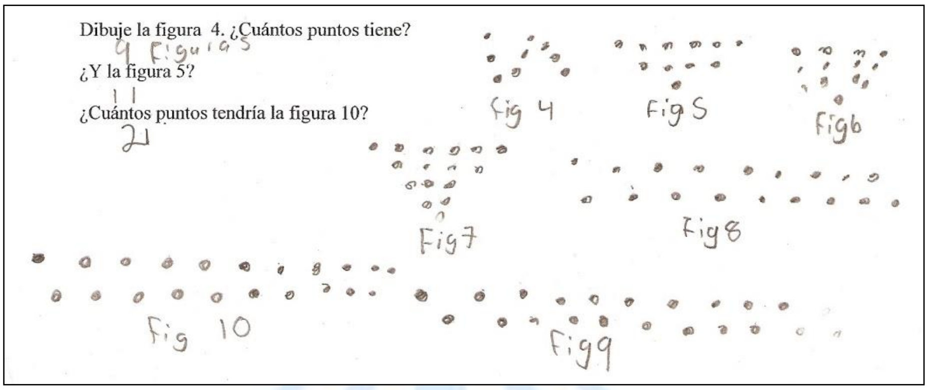


Fig. 1 *Fig. 2* *Fig. 3*

And he was asked to draw figure number 4 and say how many points he would have; likewise number 5, finally they was asked, how many points would fig. 10 have?

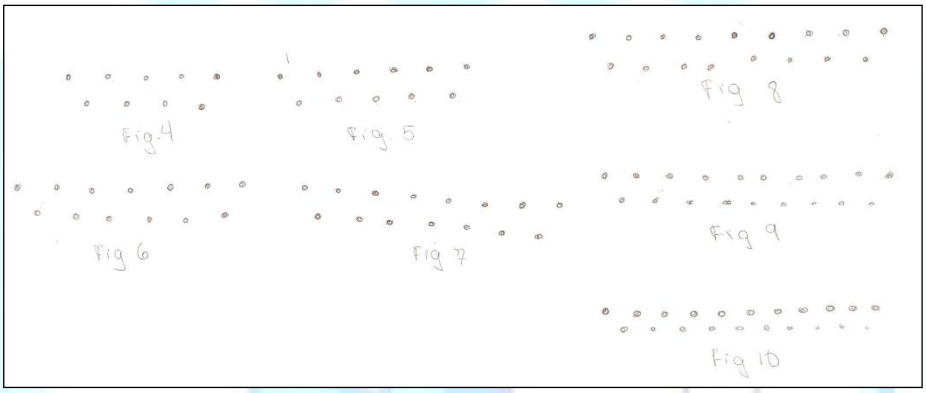
Some students worked in the following way:

Figure 6



Source: Research Data

Figure 7



Source: Research Data

If we observe figures 6 and 7 carefully, we note that these two students have the same number of points in the corresponding figure of the given sequence. This makes us think that one of them found the pattern in the increase of the points but not in its graphic representation. In contrast, the other student noticed the geometric pattern and relied on it to find the number of points.

Below is a table in which we can see the general results obtained by the students in the second problem with patterns.

Table No. 3: Description of the results on the exercise: number of points in the figures

Number of students	Types of answers and actions of students to solve the problem
9	They recognized that the number of points each figure had increased by two.
9	They found the answer to the first two questions by adding two.
8	They found the answer to the third question by adding two units to find the next one.
1	The student found the correct answer to each question, but does not leave evidence of the work done.
1	The answer of the third question is incorrect, the student does not leave evidence of the work to solve it.
6	They arrived at the answers making the drawing of each figure.
1	He found the answer to each question although he drew each figure in a unique way, because he did not recognize the geometric pattern.
4	They did not solve the problem.

Source: Research Data

Table No 3, shows that among the actions that students took to solve the problem was to use as a strategy the graphic representation of each figure and then count their points. Others noticed that the number of points increased by two and it was enough for them to add two to the number of points in the previous figure. The difficulties encountered were errors in making the calculations.

Due to what has been mentioned in previous paragraphs and the information contained in the tables, it is noted that there was greater acceptance and comfort of the students when solving the pattern exercises, than those who tried to verify the previous knowledge, this could be because the exercises of patterns do not require a mechanical procedure or follow any specific rules or steps.

The majority of the students recognized the pattern and found the right answers to the questions. It is possible that this interest shown by the students, to try to find the correct result to these problems, either because it is the first time they were faced with this type of problems or maybe they were motivated by not having a certain method to perform them. In addition, no problem had specific questions regarding the topics that are being addressed in class.

The previous results indicate that the students are interested and possess the knowledge necessary to develop in them the adequate skills for the learning of algebra through the discovery of patterns. Next, we will see relevant results obtained in some of the sessions:

First session:

In the first session with the students, there was a discussion (15 min) of the two problems about the pattern exercises proposed in the diagnostic test in order to explore the considerations they made to solve them.

Analysis of the first proposed problem

The problem consisted of the sequence of following figures:

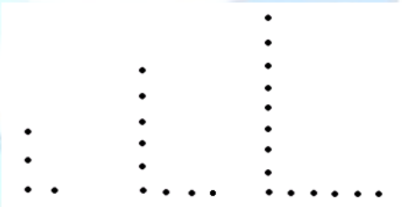


Fig 1 Fig 2 Fig 3

The aim was that the students could discover the pattern to find the amount of points in each figure and be able to predict others, finding first the number of points in fig. 4, after fig. 5 and fig. 11, finally, to observe if they were able to elaborate an expression to know how many points had a figure that belongs to that sequence.

The first question: How many points does fig. 4 have?, it was answered correctly by all students so it can be said that they did not have any difficulty to find how many points had that figure, the strategy they used was to add 5 to the number of points in the previous figure (having previously discovered that the number of points increased by 5).

Results to the second question: How many points does fig. 5 have?

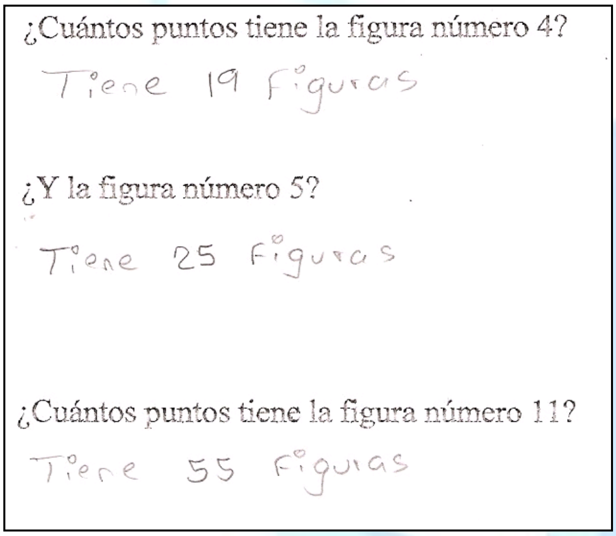
The majority of the students (10) correctly answered the second question. Of the three students who did not answer correctly, two of them commented having had the error when making the sum. And the other student said he had not found the pattern and decided to place any number.

Results to the third question: How many points does fig. 11 have ?

In this most students (8), found the correct answer to this question, of the five students who did not answer correctly two made the mistake by adding five to the wrong result they got in the second question; two students made the mistake when adding and the other said they had given a random answer.

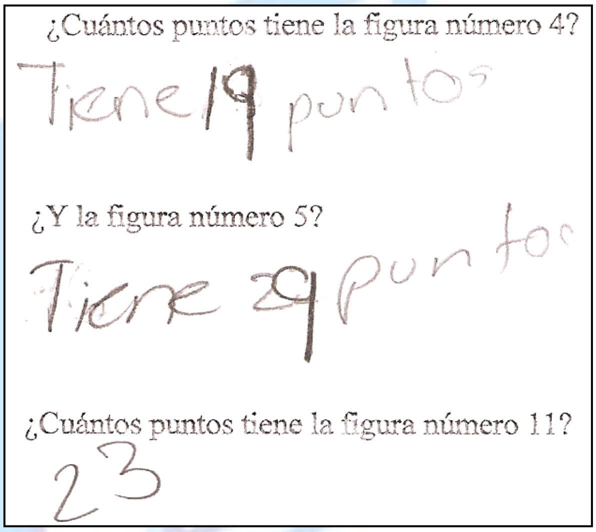
As we see in the following images:

Figure 8: Error adding



Source: Research Data

Figure 9: Random response



Source: Research Data

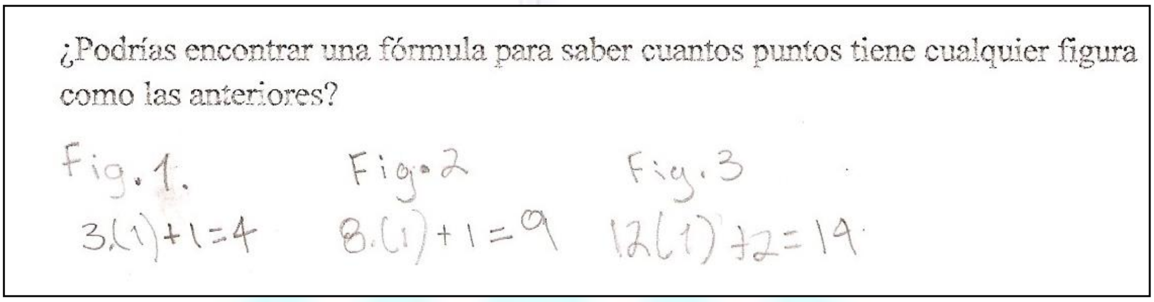
In general, the students did not present difficulties to arrive at the correct answer, the majority identified the pattern and managed to find the adequate result. Those who failed in the attempt made the mistake in calculating the sum.

Results to the last question: Find a formula to know how many points any figure in the sequence has.

In this only two students managed to find the relationship between figure number and number of points, another made an attempt to get to it, the others left no sign of having even tried.

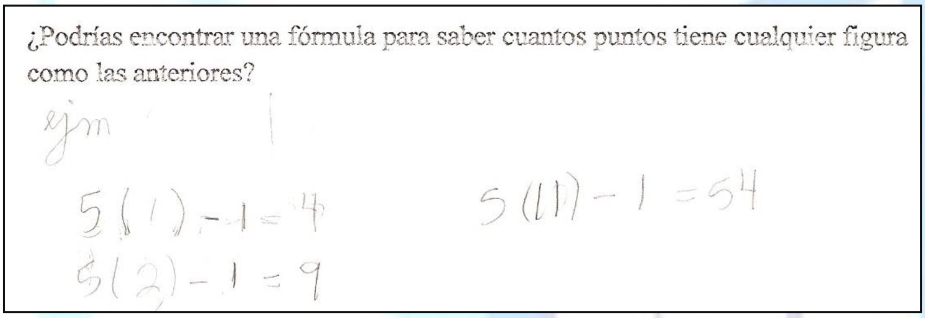
We can observe the work that the students did in the following images:

Figure 10: Tried to generalize



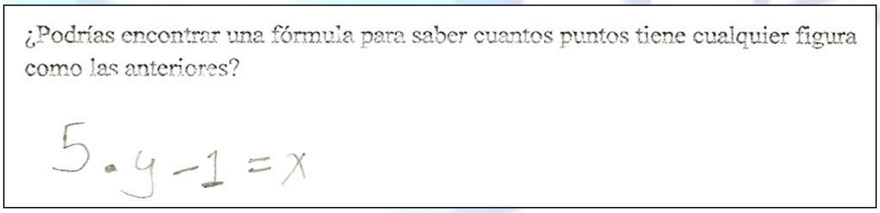
Source: Research Data

Figure 11: Found generalization, but not its algebraic expression



Source: Research Data

Figure 12: Managed to generalize



Source: Research Data

From this first session we can conclude that students actually use strategies they already know, by identifying the pattern, most chose to add five to the number of points in the previous figure and some drew to confirm their conjecture. It can be said that the mistakes made were not necessarily difficulties in understanding the problem but in the calculations that were made.

Then, two more sessions were held, with the main objective of having the students achieve generalization, in which the following results were obtained:

In the second session, the students worked in four teams, of which three managed to reach generalization. Therefore, we noticed that the results improved when the students worked in teams.

In the third session, the students worked individually only five students came to the generalization successfully. Those that did not, presented difficulties to raise it, that is, they were not able to carry out registry changes, in other words, they remained with the purely arithmetical approach and failed to identify a more complex one as an algebraic expression that met the requirements that would solve the problem. Furthermore, this difficulty is not exactly an absence of knowledge, but rather a prior knowledge that does not admit generalization to a new field and therefore leads to error.

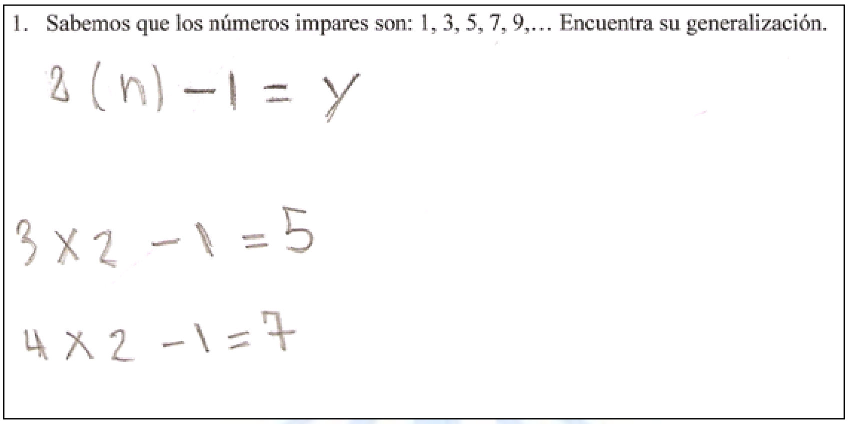
After a process of familiarization of the students with the successions of numbers and figures, and remembering that at the end of each session a discussion of the proposed problems was made, it was decided to evaluate the progress of these, by means of a test. In this test two problems were posed, in which the students should use the skills acquired in solving the problems proposed in each of the sessions. The generalization of the first was similar to those of the first and second sessions, and the second was related to the problem of the third session, which induced the students to make use of the strategies used throughout the process.

Final test:

Analysis of the first problem proposed in the final test

The first problem raised was the following. We know that the odd numbers are: 1, 3, 5, 7, 9, ... and the students were asked to generalize the odd numbers and find the next two numbers using that generalization. There were five students who came to the generalization without any difficulty. Like shown in the next figure:

Figure 13: Achieves generalization


1. Sabemos que los números impares son: 1, 3, 5, 7, 9,… Encuentra su generalización.

2(n) − 1 = y

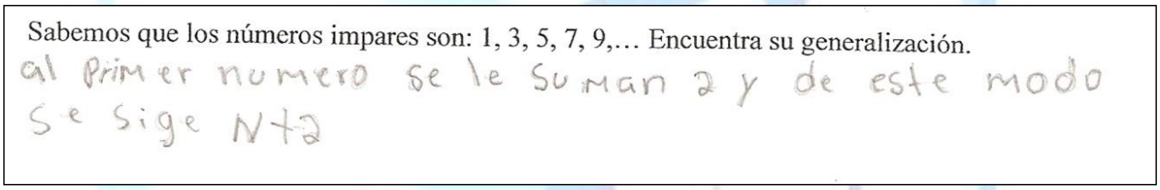
3 x 2 − 1 = 5

4 x 2 − 1 = 7

Source: Research Data

The remaining eight failed to state the generalization, having difficulty in discovering the correct algebraic expression, they discovered a certain pattern but recursively, as shown in the following figures.

Figure 14: Discover the pattern but fail to generalize


Sabemos que los números impares son: 1, 3, 5, 7, 9,… Encuentra su generalización.

al primer numero se le suman 2 y de este modo se sige N+2

Source: Research Data

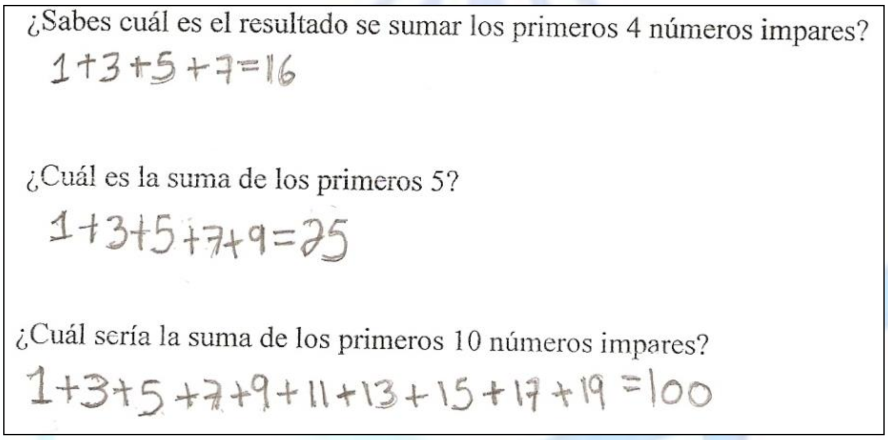
In the second part, the difficulty was related to the fact of not having found the generalization in the first paragraph, and therefore it was not possible to find the following two odd numbers. Among the strategies used we find, once again, trial-error. As well as the particularization and generalization. Of the strategies and difficulties observed, it will be discussed in detail in later paragraphs, because there is a similarity with those of the second problem.

Analysis of the second problem proposed in the final test

The second problem was the following: We need to know the sum of the odd numbers. If we add the first 2 odd numbers we would get: 1 + 3 = 4. Now let's see what happens if we add the first 3 odd numbers, the result is: 1 + 3 + 5 = 9. The problem contained four questions whose results are presented below.

All the students found the correct answer to the first, second and third questions. The question were: Do you know what is the result of adding the first 4 odd numbers? What is the sum of the first 5? And what would be the sum of the first 10 odd numbers? Everyone found the answers by adding each of the odd numbers, as shown in the following figures.

Figure 15


¿Sabes cuál es el resultado se sumar los primeros 4 números impares?
1+3+5+7=16

¿Cuál es la suma de los primeros 5?
1+3+5+7+9=25

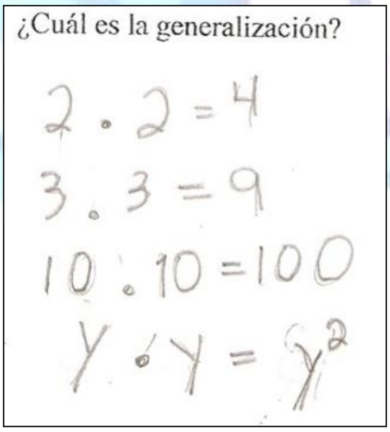
¿Cuál sería la suma de los primeros 10 números impares?
1+3+5+7+9+11+13+15+17+19=100

Source: Research Data

Results to the last question: What is the generalization?

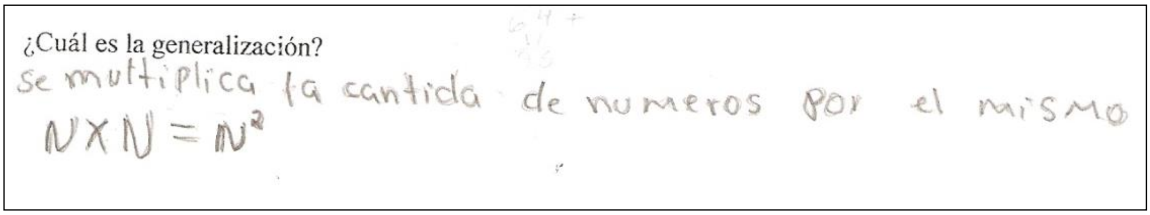
In this case the majority could reach generalization, and some were able to discover the pattern but did not write the expression that produces the answer. This is manifested in the following figures.

Figure 16


¿Cuál es la generalización?
2 . 2 = 4
3 . 3 = 9
10 . 10 = 100
Y . Y = Y²

Source: Research Data

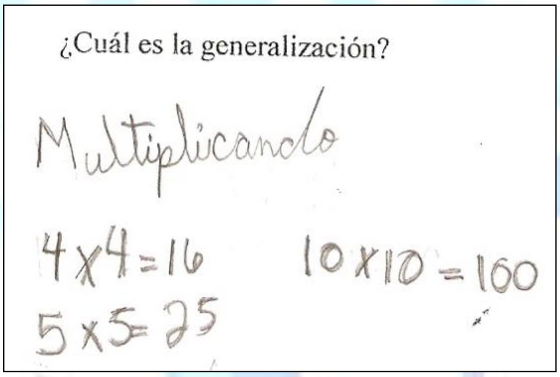
Figure 17



Source: Research Data

Those who did not reach the generalization was because they did not make record changes, that is, they could not go from the arithmetic to the algebraic record. Like shown in the next figure.

Figure 18



Source: Research Data

In general, the students used several strategies that were discovered throughout the process in each of the sessions, one of the most outstanding was trial and error, the students tested with an algebraic expression that fulfilled the necessary conditions to achieve generalization, when it was not the correct value they proved otherwise by modifying the operations with the variables. Another prominent strategy was particularization and generalization, sometimes the problem seen as a whole is unapproachable, so to start with, you can tackle a part of it that seems simpler, the problems raised allowed the students to face them in this way and thus many were successful. In addition, it is necessary to mention that within the strategies used were the search for problems similar to those worked on in the sessions. As for the difficulties, the one that stands out is that related to the passage from arithmetic to algebra; This is because it was not only necessary to discover the pattern and know how to do the arithmetic calculations, but also to make a change of representation in which variables are used, that is, to use the algebraic language.

Final considerations

In summary, it can be said that the learning of algebra is more effective when adequate activities are carried out so that students assimilate better the algebraic language. It is worth mentioning that the problems raised in the worksheets and in the final test, as well as the discussions held with the students, have served as a guide in the development of their math skills. It is important to point out that the teacher plays a fundamental role in creating opportunities in which students are confronted with situations in which they must conjecture, compare and make use of different representations, that is, they must be able to use both the arithmetic language as the algebraic.

Students use different strategies to generalize a pattern, using, first of all, their previous knowledge, after obtaining some tools such as understanding what a letter represents in different contexts in mathematics, most were able to manipulate certain operations with numbers and letters to get to generalize a pattern. The most commonly used strategies were trial and error and tackling the problem starting with the easiest. Those students who failed to state the generalization had difficulty carrying out record changes, were able to see the pattern, know what their behavior was and even predict successive terms, but were unable to tell how they behaved in a general way using, no longer a purely arithmetic language, but the algebraic language.

The connections made by these students to generalize a pattern lie in the associations they make between problems that they had already solved in the first sessions and those that they had to solve later without leaving aside their previous knowledge of arithmetic. They look for similarities between the behavior of one pattern and another, as well as the manner in which each element of the same sequence, either numbers or figures, differs in the proposed problems.

The activities carried out supported the potential to find relationships in the majority of students to understand the use of letters in representation of general numbers in mathematics. Each activity motivated the students to create for themselves an expression that involved literals and operations with them; The proposed problems and discussions confronted the students with the need to find a pattern and be able to "describe" the behavior of the student in a general way, which makes it easier to learn the algebra. Those who have not been able to do so can do so whenever a little more time is devoted to the stages of observing and writing an expression that represents a pattern.

Obviously, there was progress in students both in their motivation to solve problems and to make use of a new language in which not only numbers play a fundamental role but also the use of variables. To confront the students to new situations in which they are the creators of the algebraic expressions and, in addition, to verify if in fact said expressions fulfill the requirements raised in some problem develops in them autonomy and stimulates the development of logical-mathematical skills that enhance your learning of algebra.

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