PERSPECTIVAS DA EDUCAÇÃO MATEMÁTICA

REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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Teacher, is the Class Music or Math?

Professora, a Aula é de Música ou Matemática?

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ABSTRACT

This work presents and discusses, in the light of the Theory of Records of Semiotic Representation, the results of a research whose objective was to analyze the strategies used by a group of students when working with fractions in the study of music theory. The qualitative research included the participation of a group of students from the music theory discipline of a music school located in Santa Vitória - Triângulo Mineiro. The results show that, although the students presented difficulties in keeping the times and, consequently, in identifying the Music described in the music score, the articulation of the different representations and the transformation of representation records made them understand the musical and mathematical concepts involved, which allowed them to perceive the close relationship between Music and Mathematics and not to distinguish what was Music and what was Mathematics at the time of the activities.
KEYWORDS: Music. Mathematics. Representation Records. Teaching Music and Mathematics.

RESUMO

Este trabalho apresenta e discute, à luz da Teoria dos Registros de Representação Semiótica, os resultados de uma pesquisa cujo objetivo foi analisar as estratégias utilizadas por um grupo de estudantes ao trabalhar com frações no estudo de teoria musical. A pesquisa de cunho qualitativo contou com a participação de um grupo de estudantes da disciplina de teoria musical de uma escola de Música localizada em Santa Vitória – Triângulo Mineiro. Os resultados mostram que, apesar de os estudantes apresentarem dificuldade na marcação dos tempos e, consequentemente, na identificação da Música descrita na partitura, a articulação das diferentes representações e as transformações de registros de representação fizeram com que eles compreendessem os conceitos musicais e matemáticos envolvidos, o que permitiu que eles percebessem a estreita relação entre Música e

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Matemática e não fizessem distinção do que era Música e do que era Matemática no momento da realização das atividades.

PALAVRAS-CHAVE: Música. Matemática. Registros de Representação. Ensino de Música e Matemática.

Introduction

The relationship between music and mathematics cannot be denied. However, when we listen to a song, we come across sounds that produce melodies, but we can hardly analyze how much Mathematics exists in the sounds and melodies that please our ears.

The value of Mathematics in Music manifests itself in an essential conception of what is musical sound and what is rhythm. Through very precise mathematical relationships, it is possible to produce sounds, creating Music and, when associated in certain ways, these sounds can produce very pleasant results.

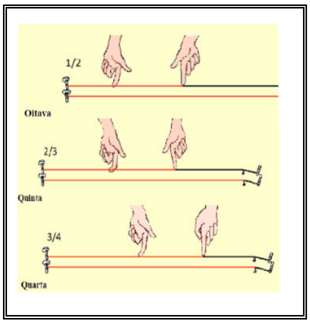
Pythagoras is attributed the advent of the heptatonic or diatonic scale. He conducted studies through the experiment with the monochord, an instrument of a single string, noting that a string stretched and put into vibration produces a sound. If the length of this string is reduced by half, a louder sound is produced, which has a very interesting relation with the first one (one octave above), and, reducing the initial length of the string to 1/3, the vibration of the remaining 2/3 (one fifth) is related to the vibration of the first and the second, and, reducing again the length now to 1/4, the vibration of the remaining 3/4 (one fourth) is in a harmonic relation with the others. The sounds produced by the 1/2.2/3 and 3/4 vibrations of the string correspond to the C, G and F notes, respectively.

According to Menezes (2013), from the combinations achieved at the time as consonants, now known as octave, fifth, fourth and unison, are respectively the proportions: 2:1; 3:2; 4:3; 1:1.

The Pythagorean tuning was based on the octave and the fifth, the first two intervals of the harmonic series. The construction of the whole Pythagorean scale originated from the induction of the proportions from the octaves and the fifths relative to the consonant intervals.

The image below illustrates how such scale is characterized by making use of the proportions obtained from a stretched string.

Figure 1: Subdivisions of the string to obtain the heptatonic scale



Source: Authors' archive.

The calculations were done for whole number reasons. In this way, the preparation of the "sound" was historically initiated, that is to say, to elevate and tune the condition of a musical scale. The tuning of the sound is defined as the height that will determine the intensity and the musical note played.

Thus, it should be noted that the use of the concepts adopted by Pythagoras to perform such an experiment can cooperate with the learning of mathematical concepts such as fractions and proportions.

As far as learning is concerned, it cannot be denied that cognitive aspects are involved in the process that directly interfere with the subject's relationship with the object, whether mathematical or not. In this sense, considering the existing relationship between Music and Mathematics, it is possible to notice that, as in Mathematics, Music is permeated with specific registers that represent it and, when codified, can be transformed into sounds or other registers inherent to the musical area.

As in Mathematics, Music also allows productions to be analyzed from the records used when making or studying music theory, that is, if we want to realize how much the individual understands about music theory, the records of representation used by him need to be examined.

From this perspective, using the lens of the Theory of Records of Semiotic Representation, this work presents and analyzes the strategies adopted by a group of students when working with fractions in the study of music theory, seeking to highlight the close relationship between Music and Mathematics.

Music and Mathematics: a harmonic relationship

Music is the art of combining sounds and silences. Rhythm is the combination of sounds over time. Harmony is the combination of simultaneous sounds. We could say that rhythm is "horizontal" and harmony is "vertical" - exactly as it is represented in a musical score.

To appreciate the function of rhythms in music it is necessary to have some mathematical knowledge that allows us to perform some operations with numbers and understand the meaning of numerical writing in the musical context. Some interpretations require sensitivity to proportions, which in some cases can be quite complex (GARDNER, 1994).

Music can motivate and cause students to create links with what is being studied, which facilitates the construction of knowledge. In more advanced levels, for example, in High School, Music can help in understanding content with functions, logarithms, progressions, among others.

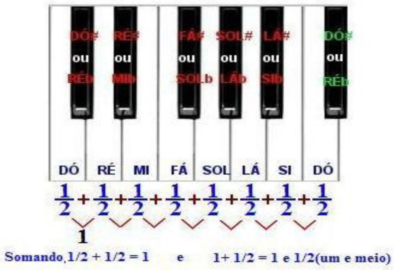
If we resort to history to look for some of these contents, Pythagoreans observed that grades differentiated by octave intervals (from eight to eight grades) presented a certain similarity, and could be defined as an equivalence class, in which two grades become equivalent, if the existing interval between them is an integer number of octaves, being able to reduce different octaves in only one, having equivalent grades in all other octaves (ABDOUNUR, 1999 p. 9).

The next step was to divide this octave into sounds that would determine the sound alphabet we know until today. This was possible due to the simplicity in the reasons of fifth and octaves, making possible the construction of a scale with seven notes, by means of divisions by fifth. Thus, the sequence F, C, G, D, A, E, Si was formed, consisting only of fifth. Consequently, the sound alphabet (cipher) was created, which continued to be the object of study by mathematicians who later organized the seven notes as follows: C, D, E, F, G, A, B.

Scales are a sequence of notes that obey certain patterns, ranging from one note of a certain frequency to another with double. On the musical scale, there are seven different notes, repeating the first with the last, although this one is twice as frequent as the first (or half the string length), as shown in Figure 1, and is one octave above, i.e., to the right. The octave means, therefore, that a note becomes the octave from the first, that is, after the seven notes, the octave is the repetition of the first, but with a sharper tone.

A musical interval between two notes is determined by ordinal numbers that relate the position between the note and the first of the scale to which it belongs. Therefore, the C-E interval is a third greater, because E is the third note on the C major scale.

Figure 2: Representation of a musical break on the piano keys



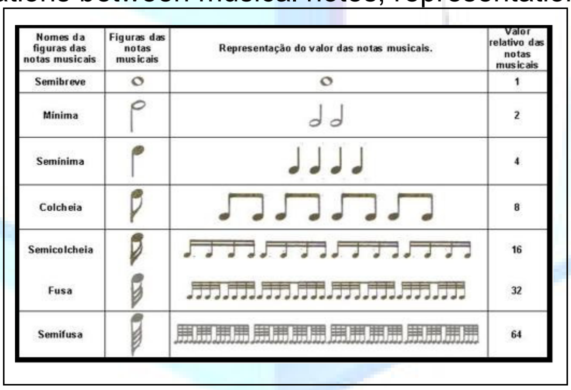
Source: https://www.google.com/search?hl=pt-BR&tbm=isch&source=hp&biw=1517&bih=730&ei=ja2UXcXnH-WP0AbI96mYCA&q with adaptations.

The C-F range is a fourth as much higher as it is lower, because F is the fourth grade on both the C and Cm scale, and the F-C' range is a fifth (C' refers to the first C after F).

Mathematics is also present in Music in the scores which are formed by a pentagram (set of five horizontal lines) and the figures of musical notes, and each figure has a value referring to the notes they represent.

The symbols used in the scores represent the musical notes and their respective pauses, each with its corresponding value. This value can be a natural or rational number, which form proportions. The Semibreve is associated with the value 4, the Minima has half of that value, the Quarter note half of the Minima value and so on. This relation can be better understood in the following image:

Figure 3: Relations between musical notes, representations and values



Nomes da figuras das notas musicais	Figuras das notas musicais	Representação do valor das notas musicais.	Valor relativo das notas musicais
Semibreve			1
Minima			2
Seminima			4
Colcheia			8
Semicolcheia			16
Fusa			32
Semifusa			64

Source: Authors' class notes.

Music can be a tool for teaching Mathematics, and an example would be to start studying fractions with a simple song like Happy Birthday to you. When singing the song, a group beats each syllable in the quarter note. Another group claps in the first half of each bar, another group in the first and third half, a fourth group claps eight times in each bar. In this way, students can compare the whole, half, fourth and eighth and establish a relationship. In this case, the mathematical relationship in a song is more easily understood and becomes less abstract.

The Theory of Semiotic Representation Records and Learning

The issues related to human knowledge are directly linked to the object of knowledge and its representations. Often regarded as "imitations" of the object, the representations are confused with the object itself.

In Mathematics, this duality also happens and can interfere negatively with the learning process. If we ask a primary or high school student if he knows what 4/5 is, he will probably answer that it is a fraction, rather than saying that it is the representation of the numerical register of a fraction.

Unlike other areas of knowledge in Mathematics, representations assume a fundamental role in the constitution of this science, whose all objects of knowledge are abstract and that we can only manipulate their representations, being therefore essential to know several representations of the same object.

Regarding the duality between object and representation, Duval (2011, p. 16-17) emphasizes:

> The first scheme of knowledge analysis was developed based on the epistemological opposition between the representation of an object and the object represented. Knowledge begins when we no longer adopt the representation of the object in place of the object itself.

In this sense, knowledge is constructed from the moment the individual does not confuse the representation of an object with the object itself and understands that representations are means of access to knowledge related to the object in question. In other words, knowledge is only constructed when the individual encounters the representation of the object and knows that he is in contact with a representation, and not with the object itself.

When we think about teaching, we usually establish immediate correspondence with the learning process, and from this point of view questions like: How to understand the students' difficulties often come to mind. What is the origin of these difficulties? In order to obtain coherent answers to these questions and also to interpret these answers in an appropriate way, it is necessary to look at these questions from the point of view of cognition.

The mathematical routes present a fundamental characteristic that is centered on the transformations of semiotic representations obtained within the context of a proposed problem. According to Duval (2011), this is where mathematical referrals are distinguished from referrals in other sciences, such as Physics, Biology, Geology, etc. In Mathematics, we work only with semiotic representations and, whenever necessary, transforming them into others. Therefore, in Mathematics, a semiotic representation becomes interesting as it can become another representation.

According to Duval (2003), the originality of a cognitive approach is not in observing the mistakes made by students and from them determining their "conceptions" and the origin of their difficulties in a certain mathematical subject. The originality of this approach lies in trying to describe the cognitive functioning that enables the student to understand, carry out and control the diversity of the mathematical processes that are proposed to him in the various teaching situations.

For a look from the cognition point of view to mathematical activity, the semiotic representations deserve a prominent place in this discussion because, according to Duval (2003), it is enough to observe the history of the development of mathematics to realize that the evolution of semiotic representations was a primordial condition for the evolution of mathematical thought.

In agreement with Duval (2003), we will call the wide variety of semiotic representations used in Mathematics "registers" of representation. These registers allow mathematical objects, which are abstract, to be represented to help their understanding.

As for mathematical activity, Duval (2003, p. 14) points out that

> The originality of mathematical activity lies in the simultaneous mobilization of at least two registers of representation at the same time, or in the possibility of exchanging representation registers at any time.

The mathematical richness of semiotic representations lies in the transformations that can be made with them, and not in the representation itself. The diversity of transformations that can be accomplished with the representations of a mathematical object constitutes a very powerful arsenal for mathematical activity.

The possibility of transforming the semiotic representations into mathematical activity determines the path to be followed by the individual in solving problems, which may be easier or more difficult, provide other triggers or discoveries, everything will depend on the choice made.

In this direction, Duval (2011) states that the procedures to perform a task, the ways to solve a problem, change according to the type of representation in which they are included. Therefore, it is plausible to say that the paths to solving a problem depend on the type of representation with which one works, and the developments are consequences of the transformations that occur during the resolution process.

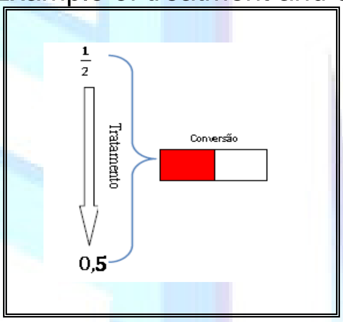
As for the types of transformations in semiotic representation, they are of two radically different types: treatments and conversions. The treatments consist in the transformation of a semiotic representation into another, remaining in the same system, that is, a transformation of internal representation to a representation register or system.

Within the register of natural language, a treatment occurs when a paraphrase is performed, since it has the function of reformulating what has been stated with the intention of substitution or explanation, that is, saying in other words what has already been said. In Mathematics, we can exemplify the treatment when, from the algebraic representation of a function, we find the equation of the line tangent to the curve that represents the given function at a given point of tangency. In the example quoted, all treatment is done within the same representation register, in this case, the algebraic register.

Conversions, in turn, consist of transforming a semiotic representation into another by changing the system, but keeping the reference to the same object. The conversion, therefore, is an external transformation in relation to the starting record.

The following figure presents an example of treatment next to a conversion, in the attempt to highlight the difference between these two transformations.

Figure 4: Example of treatment and conversion



Source: Authors' archive.

In Figure 4, the fractional representation of a number has undergone a treatment-type transformation and the decimal representation of the same number has been transformed. Although the representation underwent a transformation, the register remained in the same semiotic system, that of the numerical representation.

As any of the numerical representations of the rational number is transformed from the figurative representation, we can say that the representation of the object underwent a transformation of the conversion type, altering the system of representation while preserving the characteristics of the object.

Considering the conversion in a broader sense, not limited to the numerical and figurative representations of a rational number, the translation of the data of the enunciation from a natural language problem into symbolic, numerical or algebraic writing is a conversion of the different linguistic expressions into other symbolic expressions. However, conversion requires us to understand the difference between the content of a representation and what is being represented. In other words, in a conversion it is fundamental to know how to distinguish the object and its representation.

It is also worth noting that the rules of conversion are not the same in the sense in which the change of register is being made. A conversion of the numerical register to the figurative register requires the individual to mobilize strategies completely different from those required in the conversion in the opposite direction.

Still in this direction, Duval (2009, p. 63) points out that

> the lack of coordination between different registers very often creates a deficiency for conceptual learning. Conversely, learning specifically focused on changing and coordinating different registers of representation produces spectacular effects on macro tasks of production and understanding.

From a mathematical point of view, conversions are necessary in order to choose the register with which treatments will be carried out more easily, that is, to obtain a register that will serve as a support for the treatments. This idea is also valid in the teaching process, which often pays more attention to treatments than to conversions, because the former determines a right or wrong response.

The change of representation record is something indispensable in the resolution of mathematical problems, usually the first action performed by the individual when faced with a problem.

The conversion between the representation registers presents two distinct phenomena, congruence and non-congruence. When the conversion is almost immediate, we have the phenomenon of congruence, being possible to observe in both directions of the conversion a thermo-forward correspondence between the significant units of departure and arrival. However, when conversion does not take place in an almost immediate manner, and the significant units made available are not sufficient to carry out the conversion in an immediate manner, we have the phenomenon of non-congruence.

Most of the difficulties presented by students in mathematical learning are in the conversions that present the phenomenon of non-congruence, because the semantic equivalence of the units of meaning is not explicit in the records of departure and arrival.

Methodology and description of the study

When conducting a research, the researcher confronts data produced and collected with the purpose of making discoveries or validating conjectures in order to achieve its objectives and perform a theoretical review on the subject related to the research theme (LÜDKE; ANDRÉ, 1986). In general, this occurs from the study of a problem that arose from the curiosity of the researcher in search of answers. In the specific case of this work, the curiosity lies in the relationship between Music and Mathematics.

The approaches of analysis of a research work are useful for understanding and relating the objects being studied. This research is characterized as a qualitative study.

Bogdan and Biklen (1994) argue that this type of research has five basic characteristics:

1) In qualitative research, the researcher is the main instrument and spends much of the time in the research site, as in schools, seeking to understand educational issues.

2) The data collected are descriptive, the researcher analyzes them observing the details and respecting to the maximum the way they were registered.

3) The main interest of the research is not the result, but the process.

4) The forms of analysis are constructed from the data collected and grouped.

5) The reality and the context in which the subjects are inserted are taken into consideration in this type of approach.

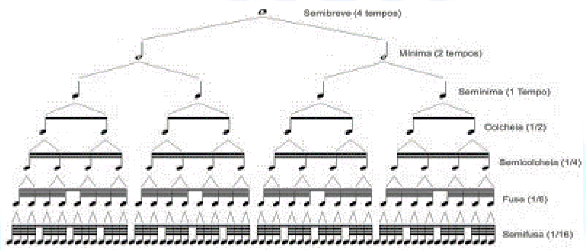
This research contemplates the five characteristics described by Bogdan and Biklen (1994) in the sense that we spent a good part of the time conducting the study inserted in the music school; the analysis was carried out in a descriptive manner; the greater interest was in the learning process of the students, and not in the final result of the application of the activities; the analyses were undertaken from the data produced and collected with the researched students; finally, the reality of the students was taken into account both in the elaboration of the tasks and in the process of executing the sequence.

The study was developed in a music school in the city of Santa Vitória - MG, with a group of students between 8 and 16 years old. To ensure the anonymity of the participants, they were assigned fictitious names.

For the theoretical teaching of Music, the students need some skills with arithmetic calculations, necessary for the formation of grades. As it was found that some students had difficulty, the sequence of activities was proposed so that they could better understand the relationship between one grade and another through fractions.

The activities took place in four classes of 50 minutes each, by means of cards, whose activities and objectives involved will be presented below.

Activity 1**:** First steps in music theory



I – - ANALYZE THE TREE OF MUSICAL NOTES AND COMPLETE THE FOLLOWING SENTENCES:

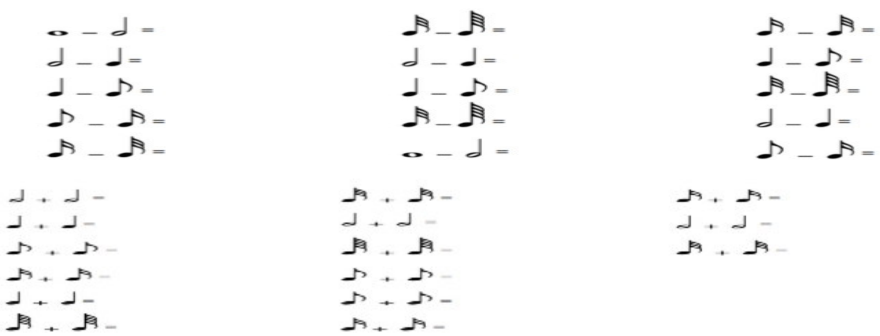
A) THE MINIMUM IS HALF OF ____________________.
B) THE MINIMUM IS HALF OF___________________.
C) THE EIGHTH NOTE IS THE HALF OF THE_______________________.
D) THE SEMICOLCHEA IS THE HALF OF ________________
E) THE FUSA IS THE HALF OF THE_______________________________________.
F) THE SEMI-FUSE IS THE HALF OF____________________
G) I NEED ______ MINIMUM TO GIVE A SEMIBREVE
H) I NEED ______ SEMINARIANS TO GIVE A MINIMUM
I) I NEED ______ EIGHTH NOTES TO GIVE A MINIMUM
J) I NEED ______ SEMICOLLEYS TO GIVE AN EIGHTH NOTE
K) I NEED ______ FUSAS TO GIVE A SEMICOLCHEIA
L) I NEED ______SEMIFUSAS TO GIVE A FUSA
M) THE SUM OF THE TWO MINIMUMS IS A______________
N) THE SUM OF TWO SEMINIMINUTES IS ONE______________
O) THE SUM OF TWO EIGHTH NOTES IS ONE_______________
P) THE SUM OF TWO SEMICOLLEYS IS A ___________
Q) THE SUM OF TWO MERGERS IS A___________________ THE SUM OF TWO SEMIFUSES IS ONE _______________

II – ANSWER:

A) HOW MANY MINIMUMS ARE NEEDED FOR A SEMIBREVE?
B) HOW MANY SEMICOLLEYS ARE NEEDED FOR A MINIMUM?
C) HOW MANY SEMI-FLIES ARE NEEDED FOR A WHOLE NOTE?
D) HOW MANY EIGHTH NOTES ARE NEEDED FOR A MINIMUM?
E) HOW MANY SEMI-FUSIONS ARE NECESSARY FOR AN EIGHTH NOTE?
F) HOW MANY SEMI-FUSIONS ARE NECESSARY FOR A MINIMUM?
G) HOW MANY SEMI-FLIES ARE NECESSARY FOR A SEMICOLVE?
H) HOW MANY FUSAS ARE NECESSARY FOR A SEMI-FULL?
I) HOW MANY SEMI-MINUTES ARE NECESSARY FOR A MINIMUM?
J) HOW MANY SEMICOLLEYS ARE NECESSARY FOR A SEMICOLLEYS?

K) HOW MANY SEMICOLLEYS ARE NECESSARY FOR A MINIMUM? III - SEPARATE WITH BARS

IV – OBSERVE THE MUSICAL NOTE TREE AND PERFORM THE INDICATED OPERATIONS



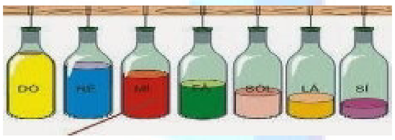
Activity 1 is composed of four items that sought to work on the initial notions of music theory, exploring the mathematics involved. The first and second items are performed with a simple treatment, as they consist of an interpretation of the note tree, and both the enunciation (source record) and the answer (target record) are given in the mother tongue. With these items it was expected that students would relate the note figures and observe the equivalence between them.

In item three, the intention was to explore the relationship between the time signature given at the beginning of each pentagram just after the treble clef and the time of each bar. To perform the proposed task the student would need to perform a congruent conversion, since the time signature is given in the numerical symbolic register and the student, through the equivalence of note figures, should separate the bar intervals by means of a vertical bar.

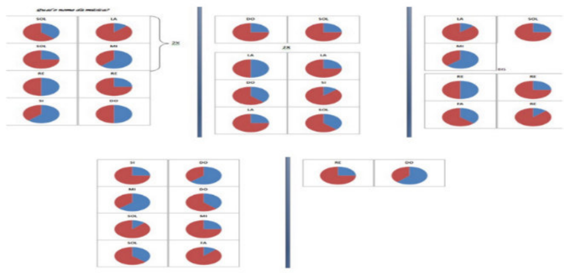
The last item of this activity could be performed in two ways, using the figure record or the numerical record. If the student makes use of the figure record, a treatment will be performed and, if a numerical record is used, a conversion will be performed.

Activity 2: Applying theory in practice

I - OBSERVE THE BOTTLES, THE MUSICAL NOTE THAT EACH ONE REPRESENTS AND RELATE THE NOTE TO THE AMOUNT OF LIQUID IN THE BOTTLE.



II – RELATE THE NOTES REPRESENTED IN THE CIRCLE SCORE TO THOSE PRESENT IN THE BOTTLES.

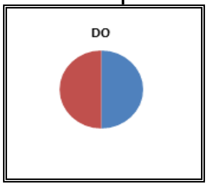


III - PRODUCE THE SEQUENCE OF SOUNDS DESCRIBED IN THE CIRCLE SCORE AND FIND OUT WHAT THE MUSIC IS.

The purpose of this activity was to have the students associate the fractions represented by means of circles with the musical notes and produce a sequence of sounds, whose melody is from a famous Christmas music.

In order to carry out the first item of the activity, the student needs to understand that the circle in Figure 5 represents two tenses (the complete integer represents four tenses, so half of the integer represents two tenses), and that two tenses represent the value of the minimum.

Figure 5: Circles representing notes



Source: Authors' files.

In the sequence, students need to relate the note represented in each circle on the score to those arranged in the bottles by playing the note holding the value of their time, for example, in Figure 5 two tenses are represented. And finally, in the third item relating the notes and obeying the tenses, students should form a sequence of sounds, following the beat to identify the Music.

Analysis and discussion of results

For the analysis of the data, the answers given in the activities by the participants were verified under the point of view of the Theory of Records of Semiotic Representation, in an attempt to understand how the cognitive process of these students took place when carrying out the activities.

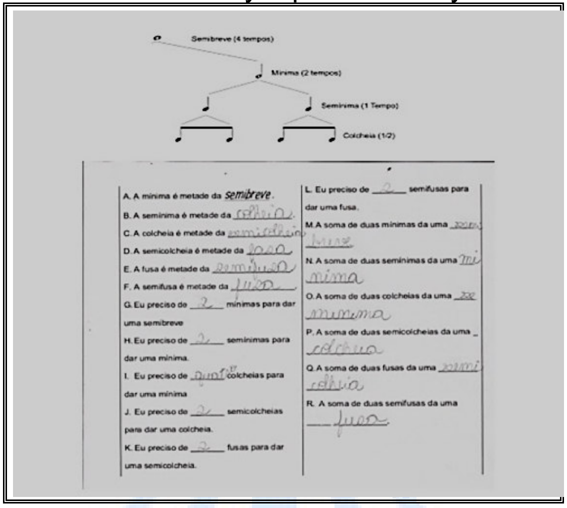
In an attempt not to lengthen the text of the analysis too much, the most interesting answers were selected, either the one that represents the majority of the students, or the most curious, in which the strategy used brings interesting elements that deserve to be discussed.

In the first item of Activity 1, the students had to observe the note tree that related the value figures. It was possible to notice that most of the records presented were correct, which demonstrates that they understood the idea of how these figures related to each other through counting.

This item was performed very calmly, almost immediately, since it was enough for them to observe the number of notes necessary for the formation of another one. The type of transformation performed by the students is a congruent conversion, since, according to Duval (2003), in this type of transformation it is possible to establish a thermo-forward relationship between the significant units of the register of departure and those of the register of arrival.

However, in some registers, the students presented conversion errors, in the sense that they did not correctly associate the figure with its value, as shown in the image below.

Figure 6: Item I of Activity 1 performed by student Jose



Source: Authors' files.

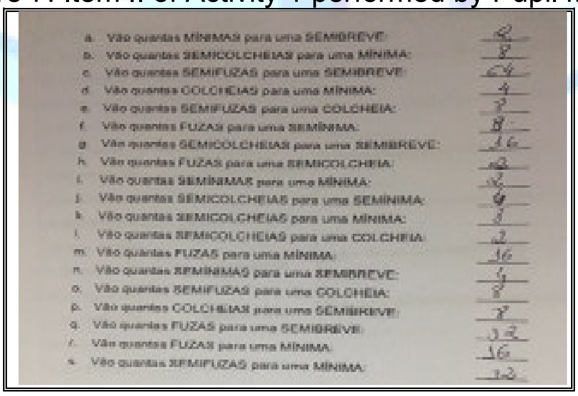
In this case, the student inverted the figure tree by reading from the bottom up. The minimum is half the minimum, not the eighth.

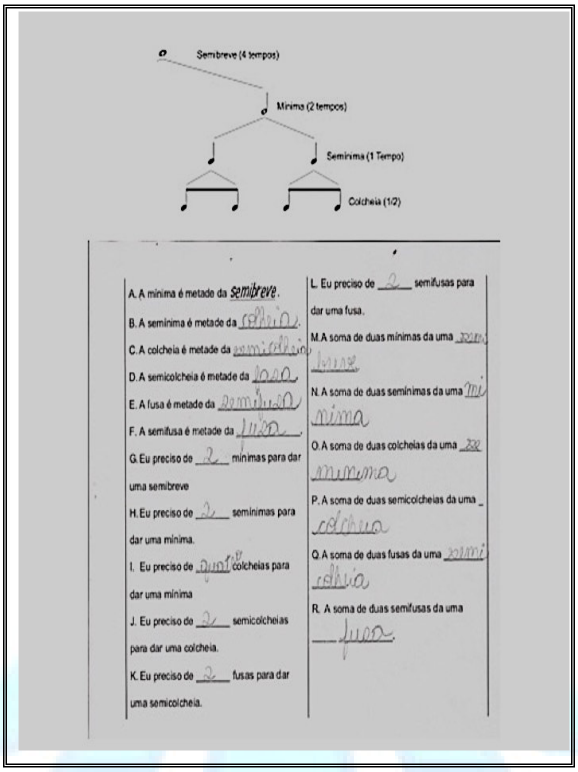
For this student, the fact that two eighth notes appeared related to one eighth note indicated that the eighth note would be the half of the eighth note.

The second item of this activity is very similar to the previous one, however, while the first deals with the equivalence of the note figures, the second explores the numerical equivalence between them, also requiring a simple count that makes it possible to perform a congruent conversion between the representations: start record (figure) and finish record (numerical), respectively.

The extract that represents the accomplishment of this item is present in Figure 7.

Figure 7: Item II of Activity 1 performed by Pupil Maria





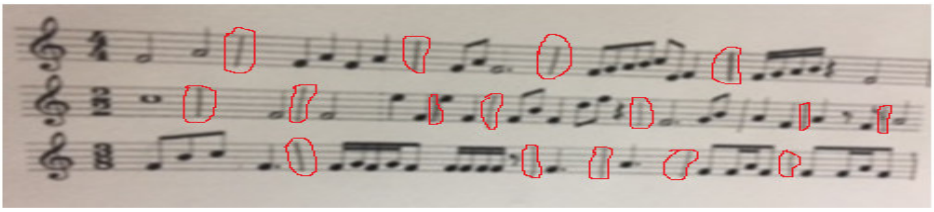
Source: Authors' files.

The transformation carried out in this type of activity is the conversion from the mother tongue to the numerical symbolic record, because the activity requires the student to establish an equivalence relationship between the figures of grades, which in the activity are presented in the mother tongue. For Duval (2011), the activities that require a congruent conversion demand from the student lower cognitive cost.

Unlike the previous items, the third required treatment in the figure record. In order to carry out this transformation, the student had to sum the numerical values of the note figures (note figure tree), thus marking each bar with a bar. It is worth mentioning that to mark bars it is necessary to observe the time signature formula at the beginning of the score, for example, the first score of item III starts with two four numbers superimposed, and the top number indicates the time signature, which in this case is quaternary, and the bottom one is the time signature that fills the time, which in this case is semibreve.

The realization of what was requested in this item required a conversion, because by placing the time bars on the score from the formula presented at the beginning, the student should transform the numerically represented time bar into bars to separate sets of note figures. The protocol that represents the realization of this item is present in Figure 8.

Figure 8: Item III of Activity 1 performed by student Anna



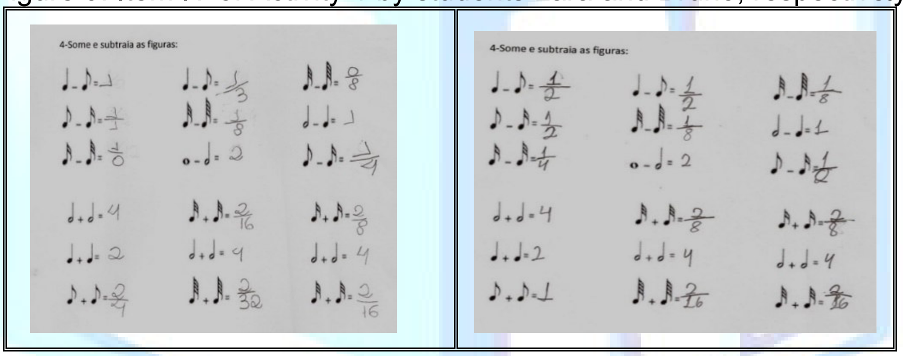
Source: Authors' archive.

In Figure 8, the bars of bar inserted by Anna were highlighted, and it is possible to notice that she includes the bars correctly, with the exception of the second pentagram, where, after the second note figure, she would have to put a bar. She even did it initially, but, perhaps out of insecurity, she ended up erasing it.

In item IV of Activity 1, it was necessary to perform a conversion that could be done both in the figurative sense to numeric representation and to relate the figures, keeping the figurative register and only the final answer be transformed into the numeric symbolic representation.

The example of how the students performed this item is in Figure 10.

Figure 9: Item IV of Activity 1 by students Lara and Bruno, respectively



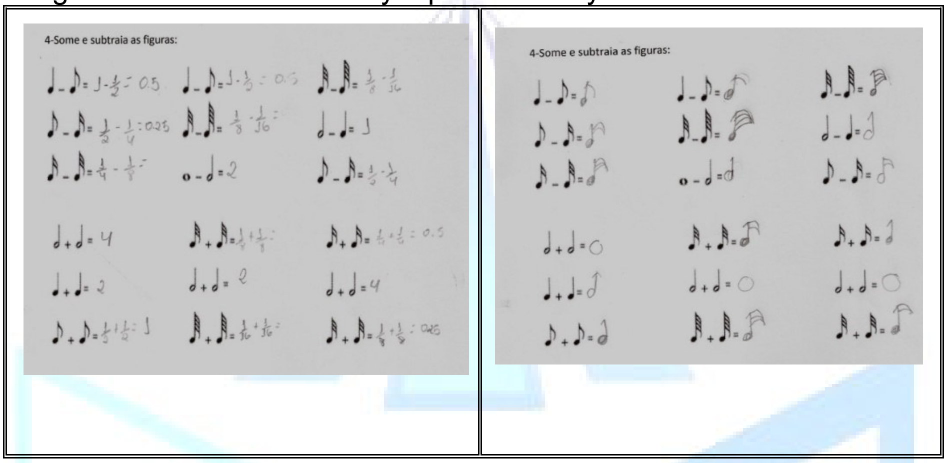
Source: Authors' archive.

It is possible to notice the presence of the zero denominator in the records of student Lara, which shows that the concept of fraction was not part of the previous knowledge of some students, because the division by zero is an indetermination and is not defined.

The accomplishment of the task required in item IV of Activity 1 required the conversion of the figurative representation to the numerical one and, during the execution of the transformation, it was possible to notice that the students understood that each musical figure corresponded to a fraction of another figure, which made it possible for those who mastered the concept of fraction and the operations to make the calculations without greater difficulties.

However, the realization of item IV of Activity 1 also involved other very interesting strategies that helped the students in the conversion of the figurative representation to the symbolic numeric one. These strategies can be seen in Figure 10.

Figure 10: Item IV of Activity 1 performed by students Luan and Julia



Source: Authors' files.

Figure 10 shows the answers given by students Luan and Julia, respectively. By analyzing the protocols, it is possible to see that the students used different strategies to accomplish what was requested. While Luan, in most situations, converted each figure to a numerical representation in order to perform the arithmetic calculation, Julia made a treatment giving the answers in the figure representation. The two strategies highlight the importance of the student recognizing the object in its different representations and corroborate Duval's (2011) words, when he emphasizes that it is of utmost importance that the learner recognizes the object of study in its different representations.

In Activity 2, the proposal was to bring the students some bottles of water so that they could understand the sound represented by each note and find out which song is described in the score. The goal was not only to play the instrument, but also to understand the notes, the times they have to be held and the distribution of the notes in the score (bar).

For that purpose, the score was represented by graphs, in which each graph represented the time unit value of each note, as illustrated in the figure from Activity 2 presented above.

The conversions performed by students in Activity 2 are more complex than in the previous activities, because it was necessary to perform three conversions (1st Figural transformation for the graph; 2nd Transformation from fraction to proportion in relation to note time; 3rd Relate each graph to the notes on the bottle).

Some students understood the objective but could not make the immediate conversion. They presented difficulties in the rhythmic part, so it was necessary for the teacher to do the pulsation with them counting the times in order to identify the Music. It was possible to notice the interest of the students in discovering which Music was represented in the score.

In this task, the students needed to mobilize all the knowledge built in the previous activities. They had to go through four conversions, presenting more difficulty in the last one (the moment of execution of the Music), because the student started from the figurative representation in which the score was presented and reached what we call "sound representation", articulating the different sound representations, producing the melody.

When the conversion does not happen in an almost immediate way, we have the phenomenon of non-congruence. According to Duval (2011), in this phenomenon it is not possible to make a term to term association of the elements of the register of departure with those of the register of arrival, and also, for this same author, it is in the non-congruence that resides most of the difficulties of the students in learning mathematics.

In general, it was possible to notice that the students understood the present relationship between Music and Mathematics, despite the difficulties presented, as could be seen in the comments made by them after classes, in which many said that without mathematical knowledge it would not be possible to carry out the activities and others emphasized that it was a time to learn Mathematics.

Another way of working fractionally in music is to use recyclable materials to build a percussion instrument, thus working with various rhythms represented by note figures and pauses, each with its own value. This could perhaps ease the difficulties presented by the students in Activity 2 and help identify the Dingo Bells Music described in the score.

If we understand that Music motivates and causes students to create links with what is being studied, it seems to us that this activity has helped to understand the notions of fractions, for example, equivalent fractions and operations with fractions, and to understand the relationship of Mathematics with other areas of knowledge, such as music theory.

Final considerations

This work is the fruit of discussions and experiences among the authors inside the University during study meetings about mathematical learning. Music has always permeated these meetings, since the second author is a teacher of music theory and the first author is a student of music at a conservatory in the Triângulo Mineiro. This fact has motivated the authors to research and build a teaching proposal, which would involve these two themes. The fractions content was chosen because the students presented difficulties related to the identification of times in musical notes, since the time of one note is formed from the times of other notes.

It can be said that the objective of proposing and analyzing this material was achieved. From the students' records, we identify their greatest difficulties and the relationships built between different types of conversions.

The students who participated in this research showed, during the classes, a greater interest in Mathematics, which was provoked by the teaching of Music. During the process, those who still had no knowledge about fractions were able to develop an intuitive idea of the concept, making mental calculations. In turn, those who had knowledge about fractions were able to further improve the formalized concept, expanding the known relationships.

In these activities, the Theory of Records of Semiotic Representation can be an excellent instrument of analysis on the part of the teacher because, besides allowing him to reflect on how the cognitive process of learning the concept of fraction occurs, it contributes to the understanding of their languages and the properties of mathematical objects and Music, when the notions of fraction are approached.

The difficulties presented by the students showed how much we still have to explore the concept of fraction and that Music can become a useful tool for teaching and learning Mathematics.

Some items of the activities could have been better explored, for example, the formation of the fraction concept. One proposal, before starting the intervention for a better understanding, would be to take up some notions about the concept of fraction and then propose new activities involving this concept. The idea suggested in this work was not to explore specifically the formation of the concept, but rather the operations involving fractions, that is, some notions of fraction.

Also, it was possible to notice the total involvement of the students, mainly in the activity "what is the Music?", in which they had to put into practice all the knowledge built in the previous classes. In this sense, it became evident how rewarding it is to awaken the taste, motivation and interest for Mathematics in a playful way, approaching other areas of knowledge, such as Music.

Finally, it became evident the possibility that the work developed with Music and Mathematics in this research will become an important tool to be used by the Basic Education teacher, since the students start to give a different meaning not only to the subject of Mathematics, but also to the relationships between concepts, procedures and forms of reasoning, as it happened with Pythagoras in Antiquity.

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